Rewriting Aspects of Differential Linear Logic

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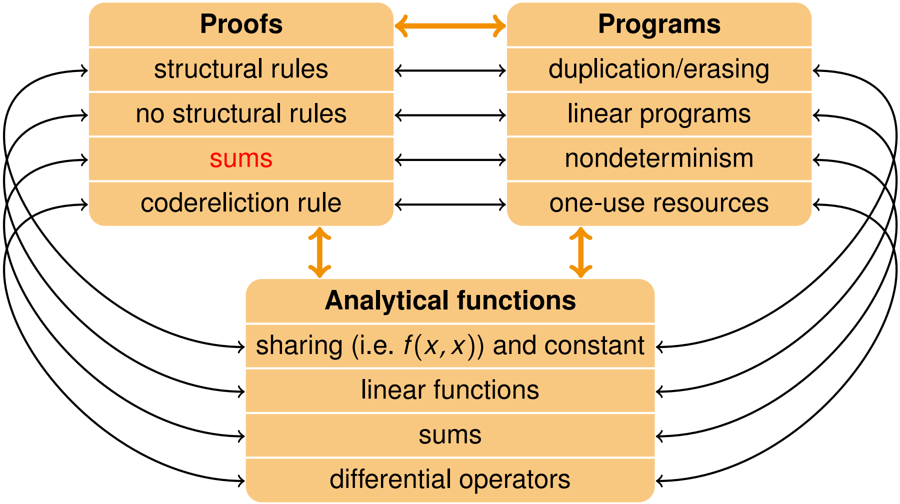
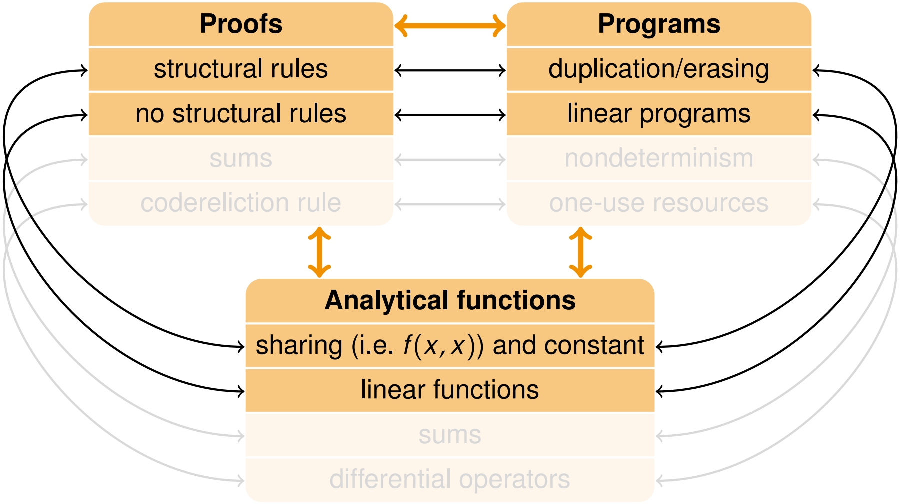
Proofs/programs as analytical functions



Ordinary proof nets



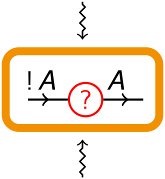
Proof nets as string diagrams

- Proof nets are a parallel syntax that abstracts away topologically commutation of rules, giving a neat account of cut-elimination.
- We can see them as string diagrams for monoidal categories with equality of morphisms $\leftrightsquigarrow$ topological equivalence.
- Linear category $\rightsquigarrow$
  monoidal comonad $! +$ isomorphism $!(A \mathbin{\&} B) \cong {!A} \otimes {!B} + \dots$
  $\rightsquigarrow$ further equalities of morphisms.
- Following will be a brief presentation of MELL proof nets with categorical and analysis intuitions.
- $A \to B \rightsquigarrow$ linear functions, $!A \to B \rightsquigarrow$ analytical functions.

Dereliction

!'s counit:

$$\epsilon : !A \to A$$



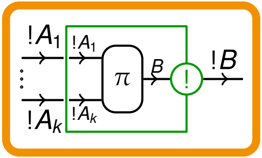
the identity x seen as analytical

Exponential box

!'s functoriality, multiplication (digging) and monoidalness:

$$\delta : !A \to !!A \qquad m : !A \otimes !B \to !(A \otimes B) \qquad \frac{\sigma : A \to B}{!\sigma : !A \to !B}$$

$$\pi^! : \bigotimes_i !A_i \xrightarrow{\otimes\delta} \bigotimes_i !!A_i \xrightarrow{m} !\left(\bigotimes_i !A_i\right) \xrightarrow{!\pi} !B$$



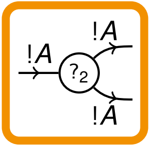
composition of analytical f's is analytical
box packages function to be ready for plugging

Structural rules

!'s functoriality, diagonal, terminal object, exponential isomorphism:

$\Delta : A \to A \,\&\, A$

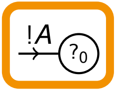
$d : {!A} \xrightarrow{!\Delta} {!(A \,\&\, A)} \cong {!A} \otimes {!A}$



$f(x, y)$ analyt. $\Longrightarrow$ $f(x, x)$ analyt.

$T : A \to \top$

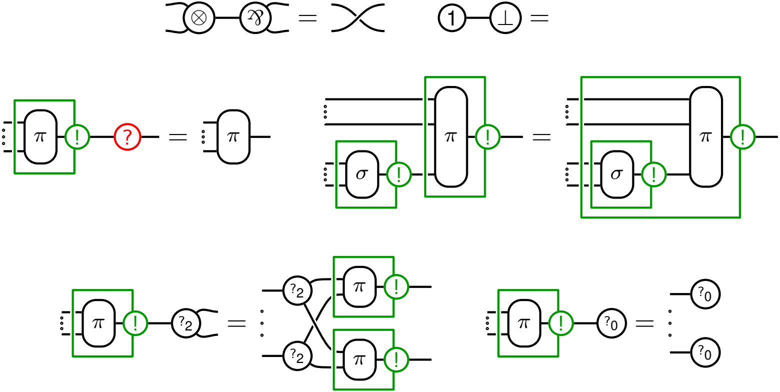
$e : {!A} \xrightarrow{!T} {!\top} \cong 1$



constant is analyt.

Contraction and weakening make $!A$ a commutative comonoid.

From equalities. . . to rewriting

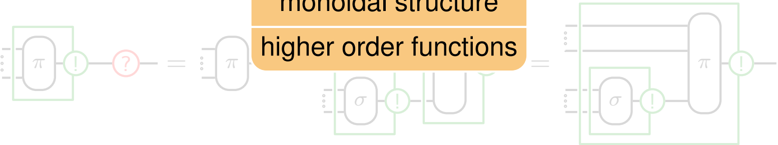


from string diagrams. . . to interaction nets!

From equalities. . . to rewriting

monoidal structure

higher order functions



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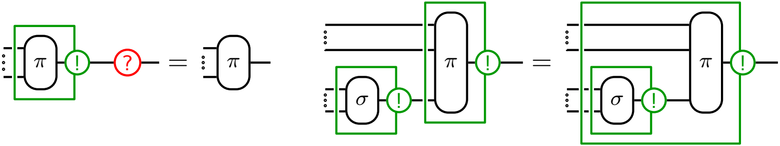
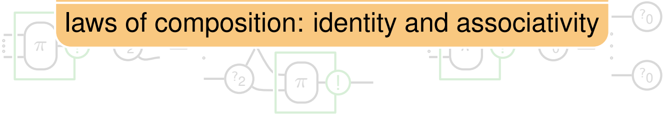


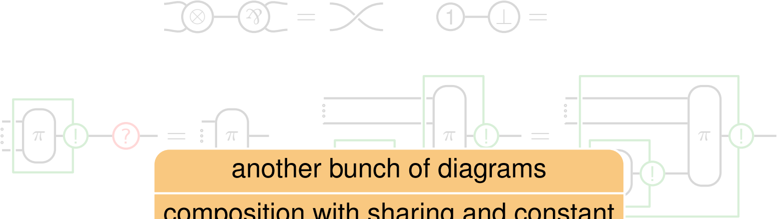
lots of diagrams

laws of composition: identity and associativity



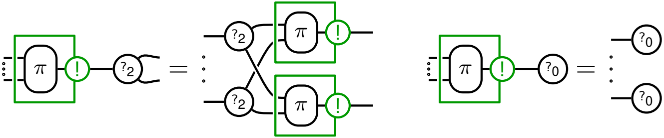
from string diagrams. . . to interaction nets!

From equalities. . . to rewriting



another bunch of diagrams

composition with sharing and constant



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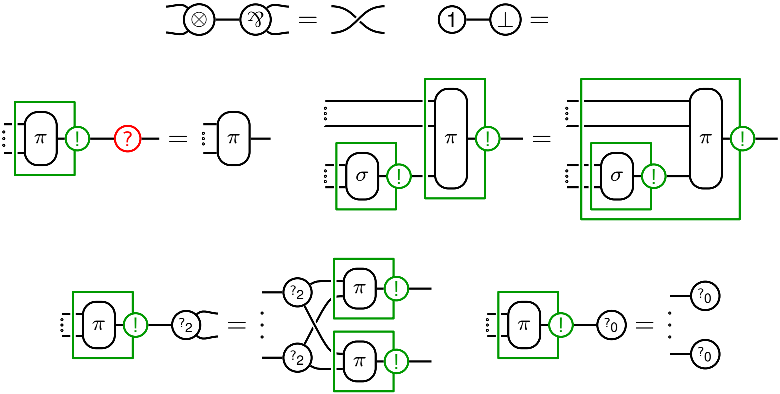

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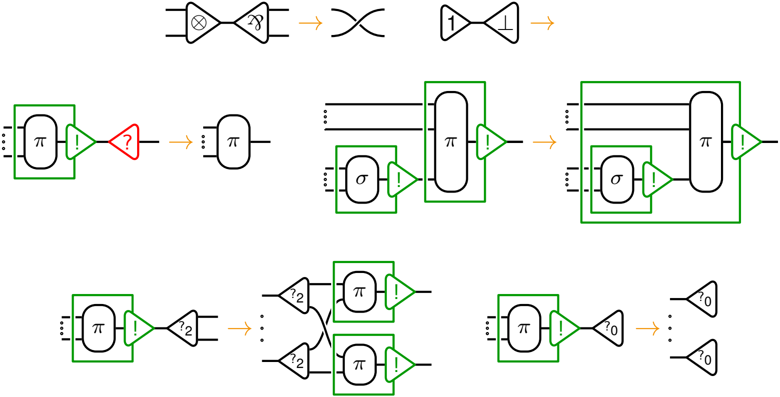
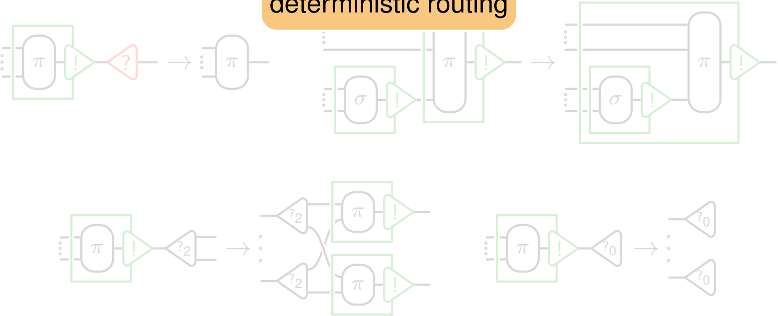


from string diagrams. . . to interaction nets!

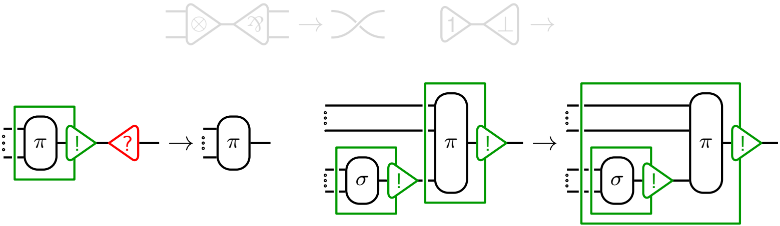
From equalities. . . to rewriting

deterministic routing



from string diagrams. . . to interaction nets!

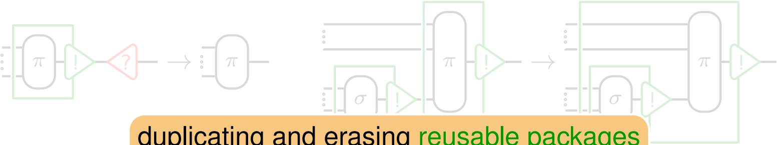
From equalities. . . to rewriting



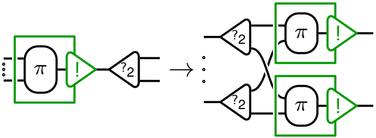
querying and fusing reusable packages

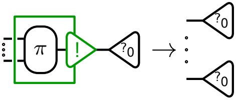
from string diagrams. . . to interaction nets!

From equalities. . . to rewriting



duplicating and erasing reusable packages





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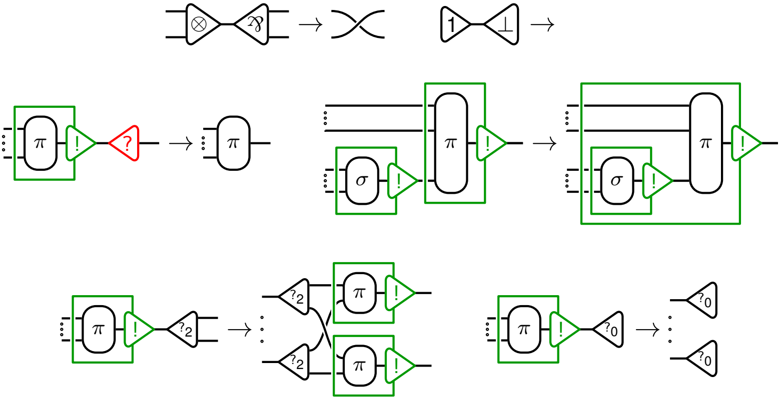
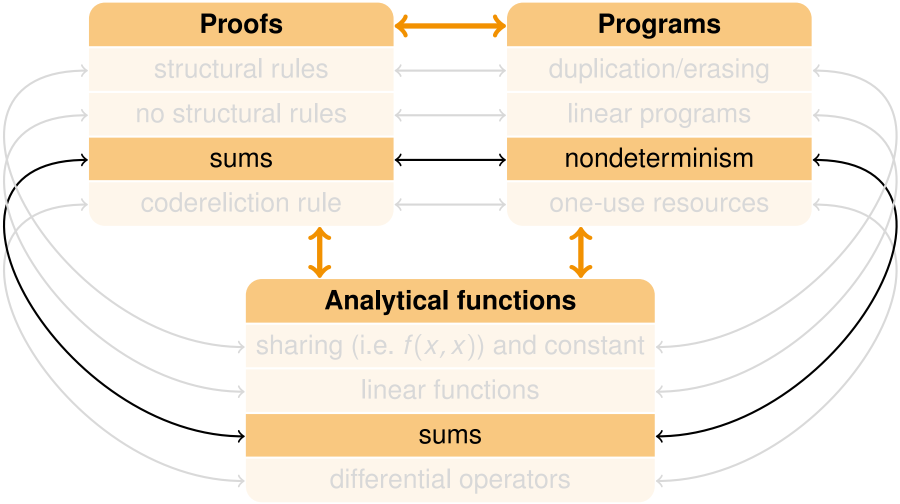


from string diagrams. . . to interaction nets!

Adding nondeterminism



Sums from biproducts

- What happens if product & and coproduct $\oplus$ are equal?
- We have the biproduct $\divideontimes$ (and $0 = \top$).
- Then morphisms $A \to B$ get a commutative monoid structure: in short, can be summed.

$$\pi + \sigma : A \xrightarrow{\Delta} A \divideontimes A \xrightarrow{\pi \divideontimes \sigma} B \divideontimes B \xrightarrow{\nabla} B, \qquad 0 : A \to \top \to B.$$

- Sum distributes on composition:

$$\left(\sum_i \pi_i\right); \left(\sum_j \sigma_j\right) = \sum_{ij}(\pi_i; \sigma_j),$$

as morphisms are linear!

Sums from biproducts

- What happens if product & and coproduct $\oplus$ are equal?
- We have the biproduct $\ast$ (and $0 = \top$).
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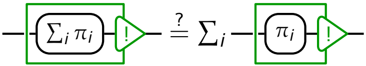
as morphisms are linear!

Sums, nondeterminism, boxes

- Computationally, $\pi + \sigma$ can be viewed as internal choice between the two, or as independent parallel computations.
- What about boxes?



- Application of analytical functions/ordinary programs is linear in the function, but not in the argument:

$$(\lambda x.M)(N_1 + N_2) \neq (\lambda x.M)N_1 + (\lambda x.M)N_2.$$

Sums, nondeterminism, boxes

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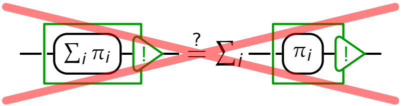

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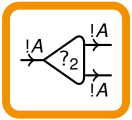
Costructural rules

!'s functoriality, codiagonal, coterminal object, exponential isomorphism:

$$\Delta : A \to A \ast A$$

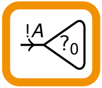
$$d : !A \xrightarrow{!\Delta} !(A \ast A) \cong !A \otimes !A$$



$f(x,y)$ analyt. $\Longrightarrow$ $f(x,x)$ analyt.

$$A \to \top$$

$$e : !A \to !\top \cong 1$$



constant is analyt.

Cocontraction and coweakening make $!A$ a commutative comonoid.
Together they make $!A$ a commutative bialgebra.

Costructural rules

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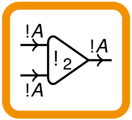
$$\nabla : A \ast A \to A$$

$$\rightsquigarrow$$

$$m : {!A} \otimes {!A} \cong {!(A \ast A)} \xrightarrow{!\nabla} {!A}$$

$$\rightsquigarrow$$



$$\leftarrow\!\!\rightsquigarrow$$

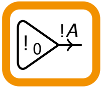
$f(x)$ analyt. $\Longrightarrow$ $f(x+y)$ analyt.

$$\top \to A$$

$$\rightsquigarrow$$

$$u : 1 \cong {!\top} \to {!A}$$

$$\rightsquigarrow$$



$$\leftarrow\!\!\rightsquigarrow$$

$f(x)$ analyt. $\Longrightarrow$ $f(0)$ analyt.

Cocontraction and coweakening make $!A$ a commutative monoid.

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Costructural rules

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$$\Downarrow$$

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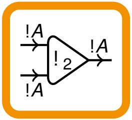
$$\Downarrow$$



$$\Uparrow$$

$f(x)$ analyt. $\implies$ $f(x+y)$ analyt.

$$\top \to A$$

$$\Downarrow$$

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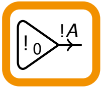
$$\Downarrow$$



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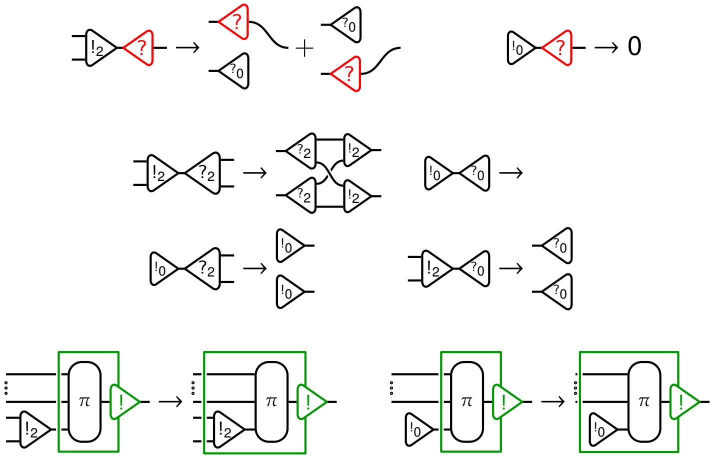
New equalities/reductions



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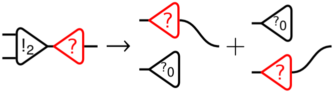


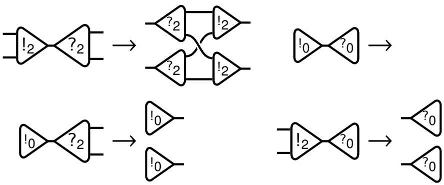
by definition (+ diagrams. . .)

$x + y$ and 0 as analytic functions

a query meets choice (or no choice)

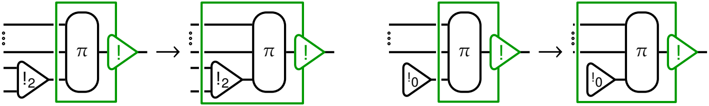
New equalities/reductions

- bialgebraic structure of $!A$
- commutation of sharing and sum
- nondeterministic routing

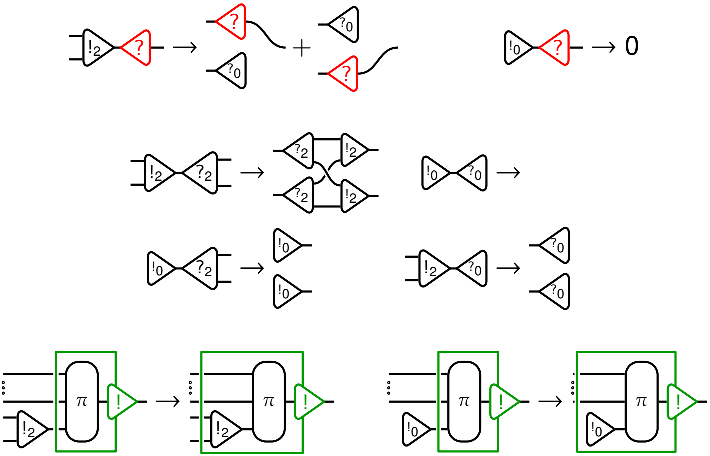


New equalities/reductions

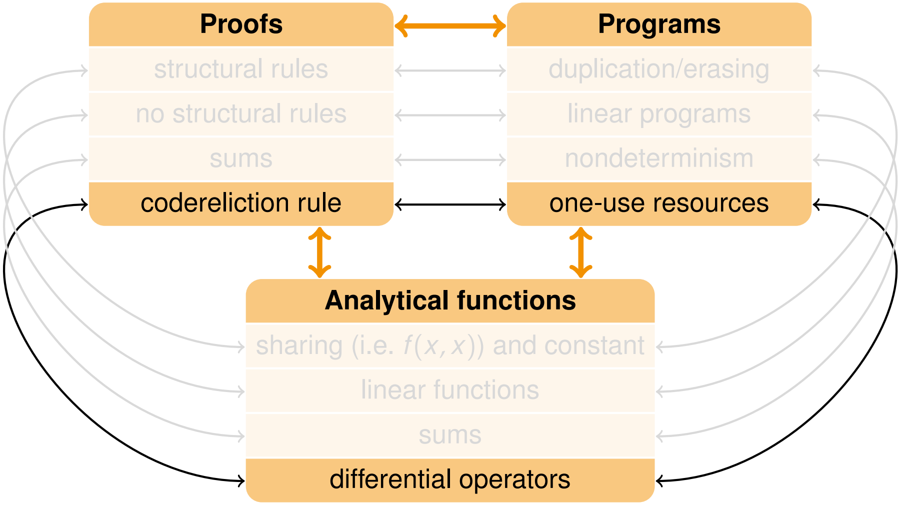
cocontraction and coweakening are as boxes



New equalities/reductions



Adding derivation

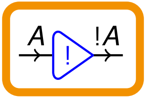


Codereliction

Symmetric to dereliction:

$$\eta : A \to\, !A$$

$\rightsquigarrow$

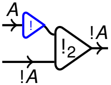


$\leftsquigarrow$

turning $f(x)$ into the linear map $\left.\dfrac{\partial f}{\partial x}\right|_{x=0}$.

Derivation in 0 is all that's needed:

$\frac{\partial f}{\partial x} = \left.\frac{\partial f(y+x)}{\partial y}\right|_{y=0}$, i.e.



.

Derivation in computation

Repetita iuvant…

Q: What is the derivative of a function?

A: Its best linear aproximation.

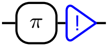
Q: What is the derivative of a program?

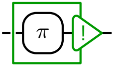
A: Its best linear aproximation!

.i.e. the (nondeterministic) aproximation using its input exactly once

⇝

One-use resources!

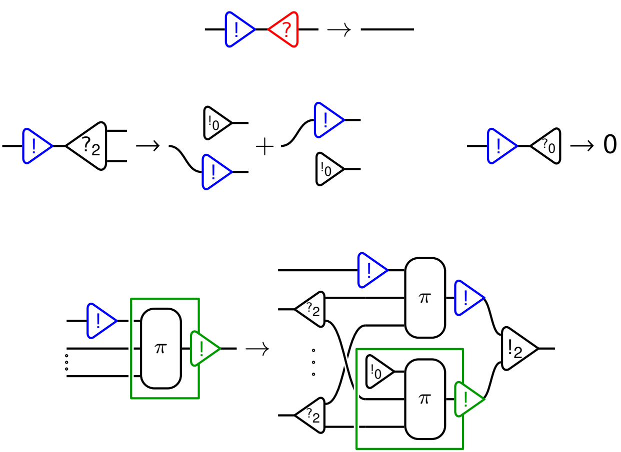


can go where


can, but won't be duplicated.

(or erased)

The new reductions



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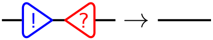
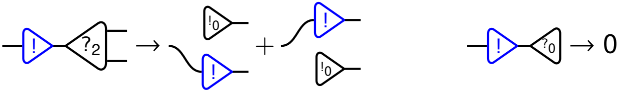


codereliction is right inverse of dereliction

$$\frac{\partial x}{\partial x}\Big|_{x=0} = \mathrm{id}$$

a query meets a one-use resource and is answered



...

$$\frac{\partial f(x,x)}{\partial x}\Big|_{x=0} = \frac{\partial f(y,z)}{\partial(y,z)}\Big|_{(y,z)=(0,0)} \cdot \frac{\partial(x,x)}{\partial x}\Big|_{x=0} = \frac{\partial f(x,0)}{\partial x}\Big|_{x=0} + \frac{\partial f(0,x)}{\partial x}\Big|_{x=0}, \qquad \frac{\partial C}{\partial x} = 0$$

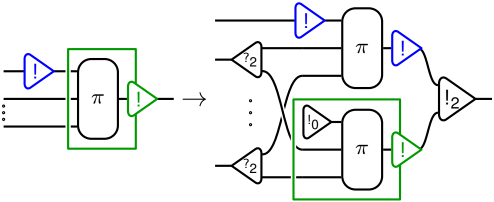
a one-use resource is contended by multiple (or no) queries

The new reductions

. . .

$$\left.\frac{\partial f(g(x))}{\partial x}\right|_{x=0} = \left.\frac{\partial f(y)}{\partial y}\right|_{y=g(0)} \cdot \left.\frac{\partial g(x)}{\partial x}\right|_{x=0},$$

a one-use resource is asked by a reusable one,
of which exactly one copy gets it



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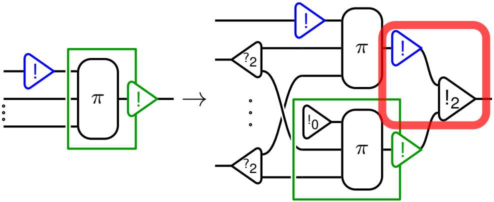

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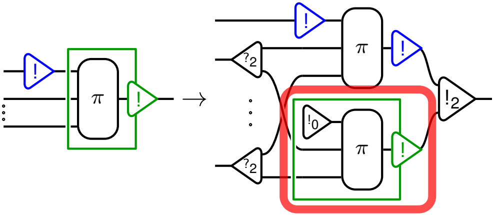

The new reductions

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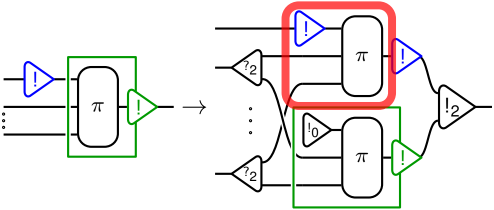
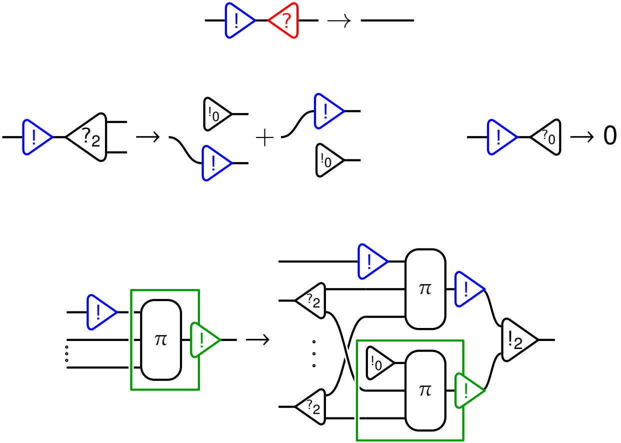


The new reductions



Are differential nets a good rewriting system?

Confluence? (aka Church-Rosser)

Is nondeterminism truly internal?
Do the possible outcomes depend on the reduction strategy?

Finite developments?

If taking a snapshot of possible reductions of a net and we reduce those ones only (and their copies thereof), does this terminate?

Termination?

Do all nondeterministic branches terminate?

Conservation?

Do non-erasing reductions preserve potential infinite reductions?

Standardization?

Can all reduction chains be rearranged in increasing depth?

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If taking a snapshot of possible reductions of a net and we reduce those ones only (and their copies thereof), does this terminate?

Termination?

Do all nondeterministic branches terminate?

Conservation?

Do non-erasing reductions preserve potential infinite reductions?

Standardization?

Can all reduction chains be rearranged in increasing depth?

Summary of results

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Untyped DiLL						
Second order DiLL						
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Stating the finite developments theorem

DiLL°: the system blocking "new" redexes

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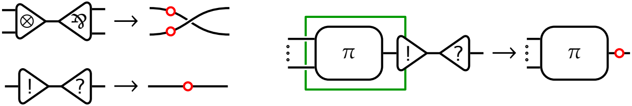

Theorem (Finite developments)

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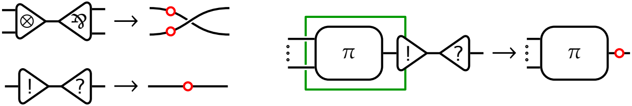

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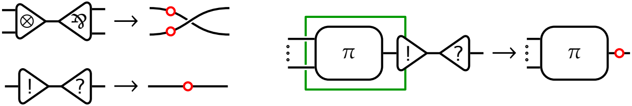

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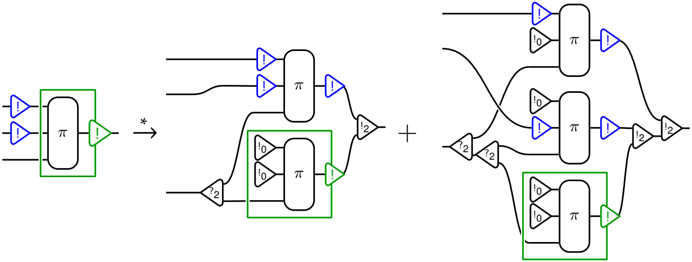

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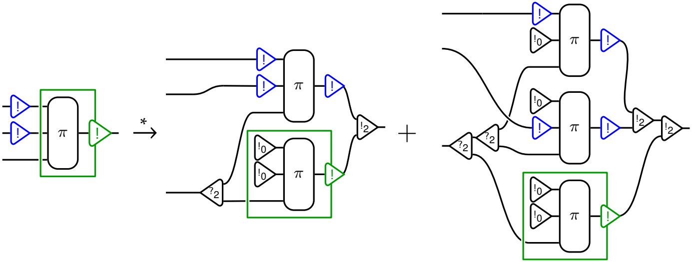

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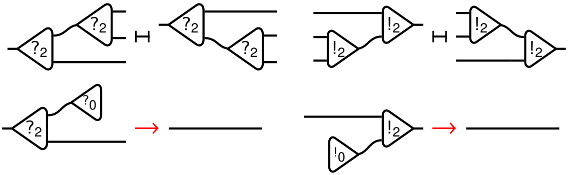
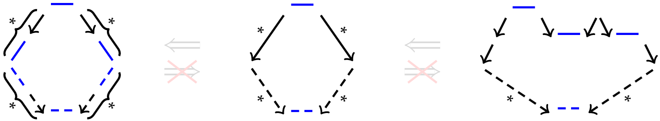
Associative equivalence and neutral reduction



- associative equivalence $\sim = \mapsto^*$
- neutral reduction (if reversed arbitrary (co)contraction trees can be generated)
- other (optional) equivalences can be added...

Reduction modulo equivalence

- Confluence properties in presence of an equivalence relation $\sim$:



confluence of $\to / \sim \; = \; \sim\to\sim$

confluence modulo

Church-Rosser modulo (CR$\sim$)
$(\to \cup \leftarrow \cup \sim)^* \subseteq \xrightarrow{*}\sim\xleftarrow{*}$

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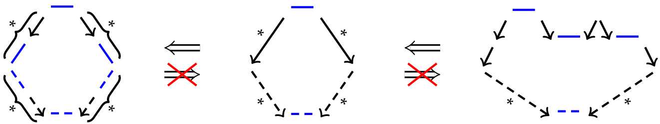

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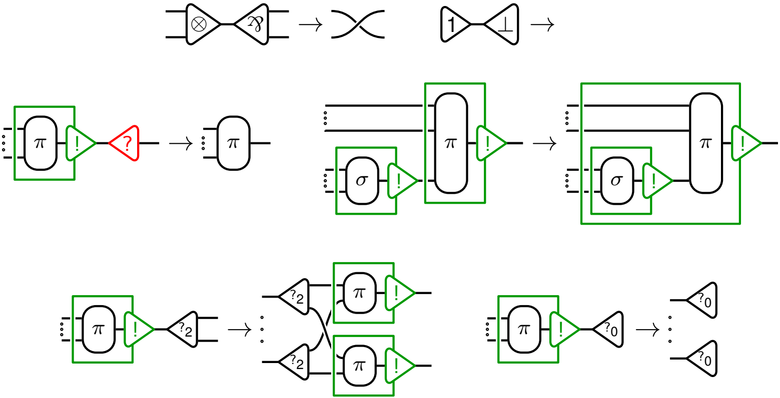

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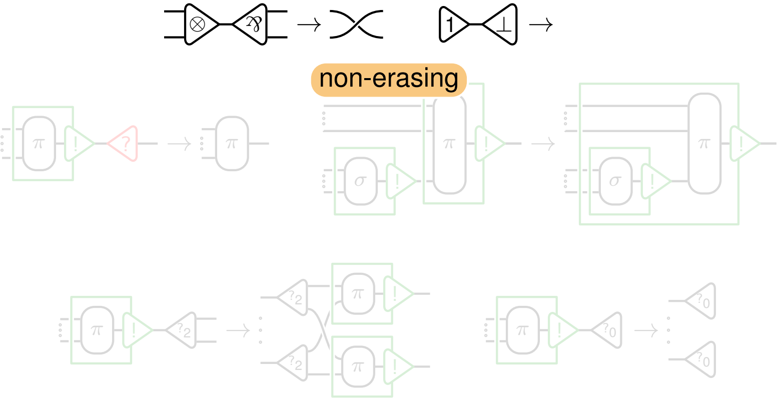

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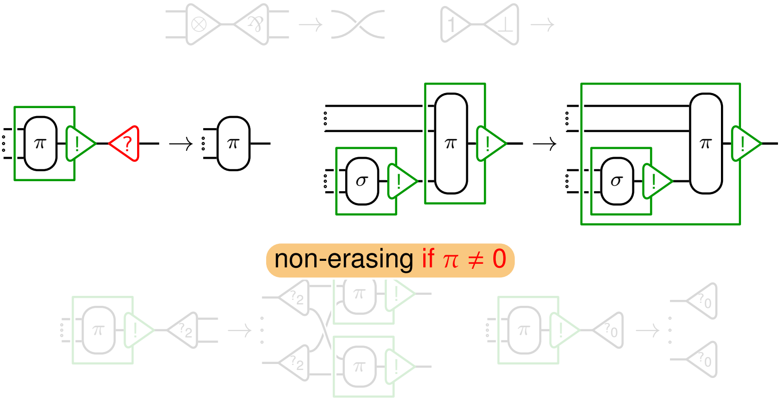

non-erasing if $\pi \neq 0$

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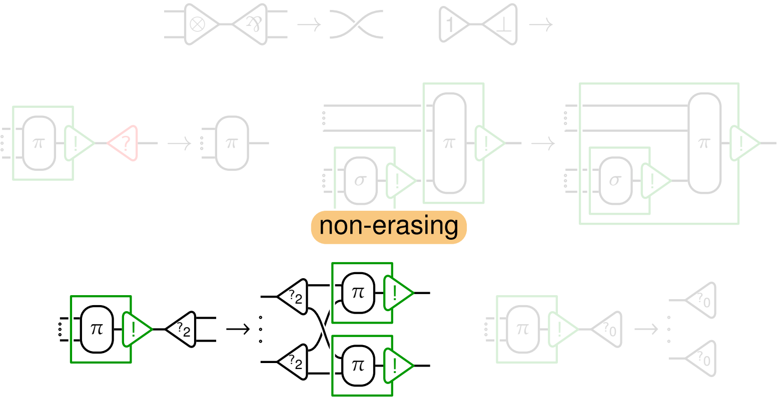

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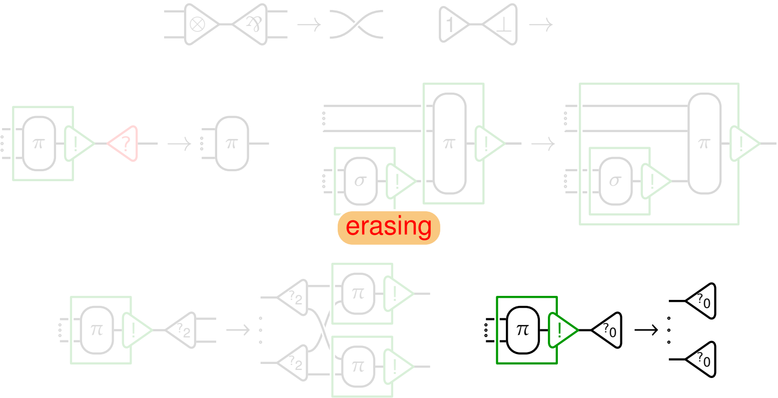

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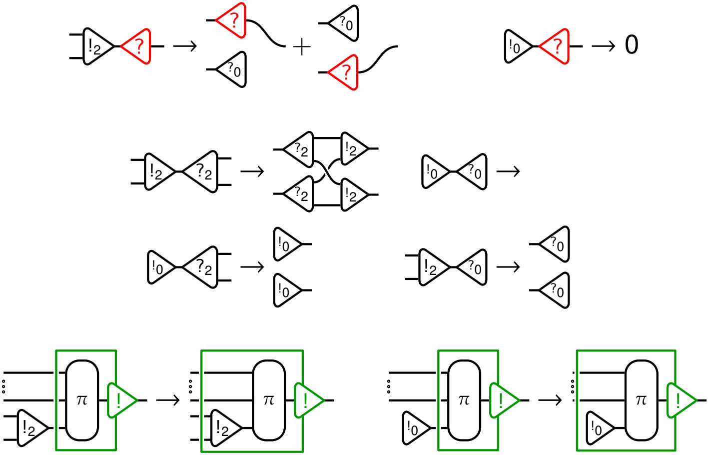

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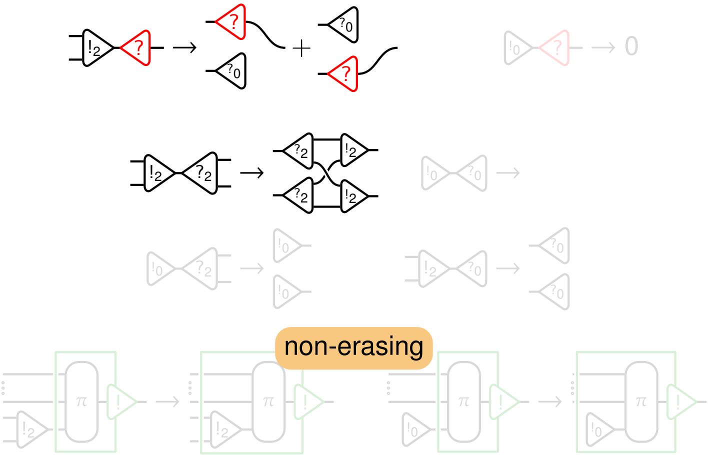

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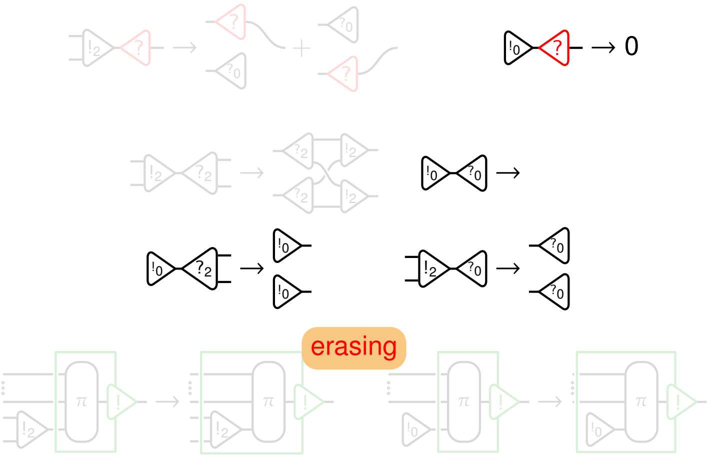

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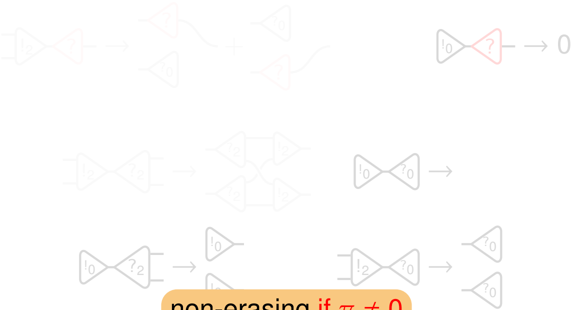

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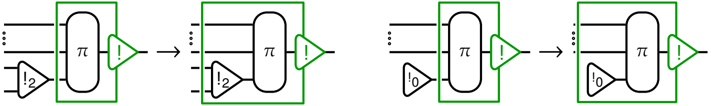

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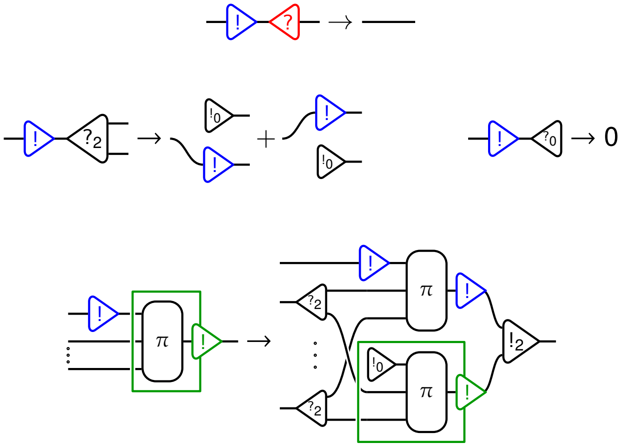

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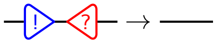

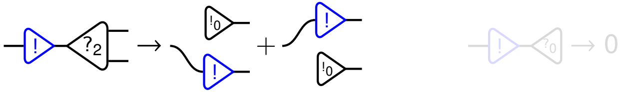

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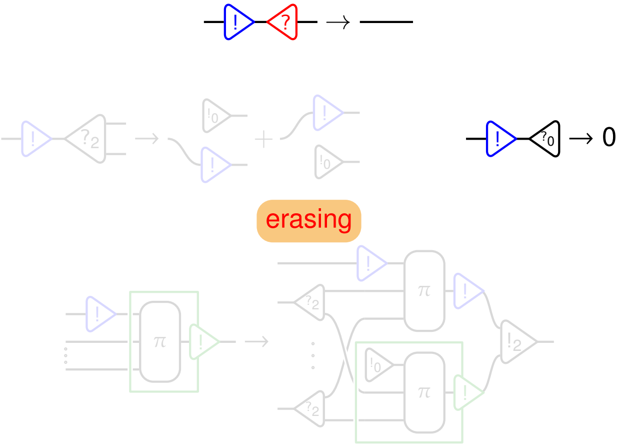

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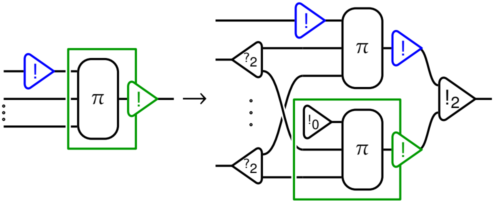

Two problems

We want:

Theorem

If $\pi \xrightarrow{\neg er} \sigma$ *and* $\pi \notin \mathrm{SN}\sim$ *then* $\sigma \notin \mathrm{SN}\sim$.

1. In the purely untyped case, this is false (contrary to LL)
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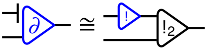
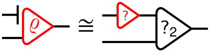
Enter $\mathrm{DiLL}_{\partial\varrho}$

- Asymmetry between dereliction and codereliction. . .
- . . . what if we switch to partial derivation as primitive, with a symmetric counterpart for dereliction?

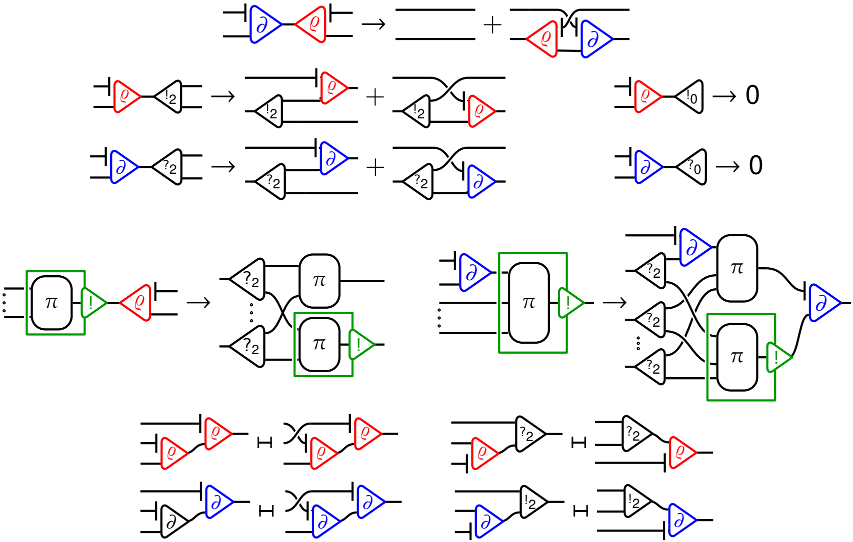


linear substitution



linear query

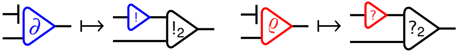
DiLL$_{\partial\varrho}$



Properties of $\mathrm{DiLL}_{\partial\varrho}$

- It enjoys confluence (both for regular and non-erasing)
- It enjoys conservation
- As already seen, there is the translation

$\mathrm{DiLL} \xrightarrow{(-)^{\vartriangle}} \mathrm{DiLL}_{\partial\varrho}$:



- $\pi^{\vartriangle}$ simulates π (if $\pi \leadsto \sigma$ then $\pi^{\vartriangle} \leadsto \sigma^{\vartriangle}$)
- Moreover, if π is $\mathrm{WN}_{\neg\mathrm{er}}$ then so is $\pi^{\vartriangle}$ (not trivial!)

$$\pi \in \mathrm{WN}_{\neg\mathrm{er}} \implies \pi^{\vartriangle} \in \mathrm{WN}_{\neg\mathrm{er}} \implies \pi^{\vartriangle} \in \mathrm{SN}_{\sim} \implies \pi \in \mathrm{SN}_{\sim}$$

So conservation holds for both DiLL and $\mathrm{DiLL}_{\partial\varrho}$

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