CPCS449 Tutorial

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Principle of structural induction for lists

To prove a property P(xs) holds for all finite lists, we must prove two things:

1. Base case: Prove P([]).
2. Induction step: Prove P(x:xs) on the assumption that P(xs) holds.

Proof format

Proof.

We want to prove two goals of the induction proof:

1. For base case, we have to prove: (base)
2. For induction step, we have to prove: (ind)
   on the assumption that: (hyp)

Base:

Left-hand side:

Right-hand side:

Induction:

Left-hand side:

Right-hand side:

□

Structural induction examples P208

```
sum []     = 0                          -- (s1)
sum (x:xs) = x + sum xs                 -- (s2)

[] ++ zs = zs                           -- (++1)
(w:ws) ++ zs = w:(ws ++ zs)             -- (++2)
```

Prove:

- sum (xs ++ ys) = sum xs + sum ys
- xs ++ [] = xs
- xs ++ (ys ++ zs) = (xs ++ ys) ++ zs

Participation question 04

Write a structural proof to show: sum(reverse xs) == sum xs.