CPCS449 Tutorial

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Principle of structural induction for binary trees

To prove a property P(x) holds for all finite nodes, we must prove two things:

1. Base case: Prove P(Empty).
2. Induction step: Prove P(Node x left right) on the assumption that P(left) and P(right) holds.

Principle of structural induction for algebraic types

To prove a property P(e) holds for all finite elements of an algebraic type T, we must prove two things:

1. Base case: Prove P(C) for all non-recursive constructors C of T.
2. Induction step: Prove P(CR $e_1 \ldots e_n$) for all recursive constructors CR of T on the assumption that P(e_1) and ... and P(e_n) holds.

Textbook exercises P354 14.44

```
data NTree = Nil | Node Int NTree NTree

depth Nil = 0
depth (Node x l r) = 1 + max (depth l) (depth r)

size Nil = 0
size (Node x l r) = 1 + size l + size r
```

Prove :

- size tr $< 2^{(\text{depth tr})}$