1. Consider the *Roberts basis vectors* below:

$$\mathbf{w}_1 = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \qquad \mathbf{w}_2 = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \qquad \mathbf{w}_3 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\mathbf{w}_4 = \frac{1}{2}\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$$

It is possible to represent any 2-by-2 image region as a linearly combination of $\mathbf{w}_1$ through $\mathbf{w}_4$.

(a) Show that the image region

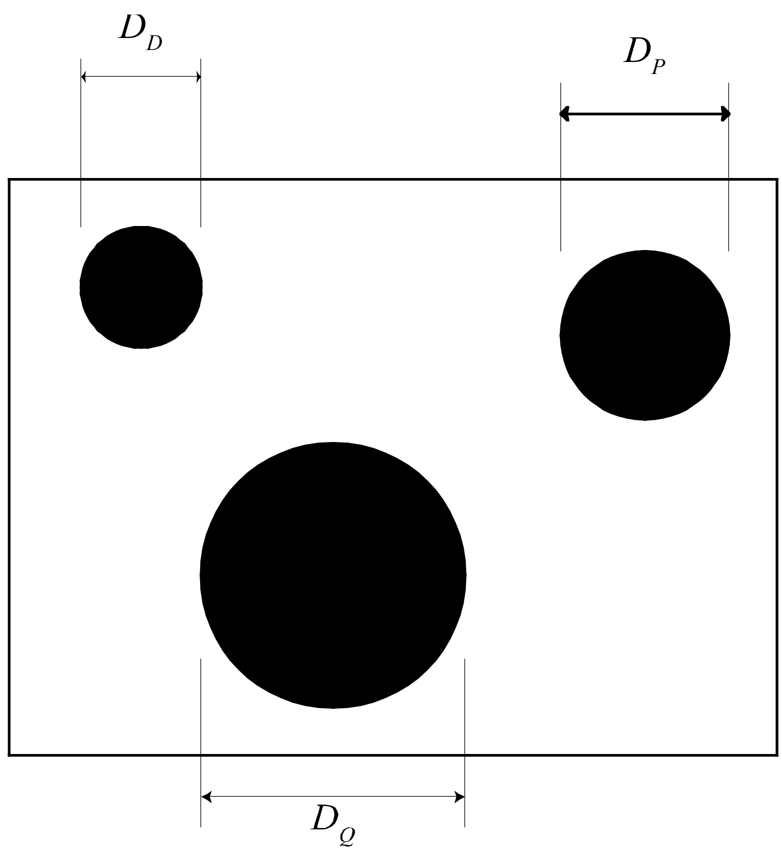
$$\begin{bmatrix} 10 & 5 \\ 5 & 0 \end{bmatrix}$$

is a linear combination of $\mathbf{w}_1$ through $\mathbf{w}_4$. Hint: the *amount* of each basis vector can be determined by matching pixels.

(b) Show that the above statement is true, i.e., that it is possible to represent any 2-by-2 image region as a linearly combination of $\mathbf{w}_1$ through $\mathbf{w}_4$.

(c) Describe, in words, what kind of image features correspond to strong matches with each of the four bases. Use this to describe the 2-by-2 region in part (a).

2. A camera takes a image, *I*, of a penny, a dime, and a quarter lying on a white background, not touching one another, their bounding boxes do not overlap. Thresholding is used to successfully create a binary image, *B*, with **1** bits for the coins and **0** bits for the background. You know the diameters of the coins D_P, D_D, and D_Q for the penny, dime and quarter respectively. Using the morphological operators *dilation* and *erosion*, and pixel-wise logical operators *AND*, *OR*, *NOT*, and *XOR*, show how to produce three binary output images: *P*, *D*, *Q*. *P* should contain just the penny (as **1**s), *D* should contain just the dime, and *Q* should contain just the quarter.



3. The Prewitt *masks* (convolution kernels) for measuring image gradient in edge detection are:

$$M_x = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix} \qquad \text{and} \qquad M_y = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}.$$

In the following steps, show that the Prewitt masks gives the weights for a best-fitting plane (in the least-squares sense) approximating the intensity surface in a 3-by-3 image neighbourhood.

Let $I_{r,c}$ be the image intensity at position r,c in the neighbourhood, so you have following 9 image intensities .

$$\begin{matrix} I_{-1,-1} & I_{0,-1} & I_{1,-1} \\ I_{-1,0} & I_{0,0} & I_{1,0} \\ I_{-1,1} & I_{0,1} & I_{1,1} \end{matrix}$$

(a) Given the equation of a plane

$$z = pr + qc + z_0 ,$$

each point in the neibourhood defines an equation of the form

$$I_{r,c} = pr + qc + z_0 .$$

Write a linear system of 9 equations for the entire neighborhood of the form $A\mathbf{x} = \mathbf{b}$, where $\mathbf{x}$ are the unknown parameters of the plane p, q, z_0, and $\mathbf{b}$ are image intensities.

(b) The least-squares solution to $A\mathbf{x} = \mathbf{b}$ is given by

$$\mathbf{x} = (A^T A)^{-1} A^T \mathbf{b}$$

Solve for $\mathbf{x}$. Show your work.

(c) From the result in (b), show that convolution with the Prewitt masks give p, and q.

(d) What is the significance of z_0?