CPSC 531: System Modeling and Simulation

Carey Williamson
Department of Computer Science
University of Calgary
Fall 2017

Quote of the Day

- "The generation of random numbers is too important to be left to chance" - Steve Park (R. Coveyou)

- Main messages:
  - Need great rigour in design and use of (P)RNG
  - Need great care in RVG process as well (avoid GIGO!)
  - Verification and validation apply here as well!

Outline

- Common Discrete Distributions
- Common Continuous Distributions
- RVG Testing
  - Uniformity
  - Independence
  - Mean and variance
  - Central tendency: mean, median, mode
  - Extreme values: min and max
  - Visual appearance: pdf and CDF
  - Autocorrelation properties

Common Discrete Random Variables

- Discrete Uniform(a,b)   (also called EquiLikely(a,b))
  - Choosing at random from a finite set of discrete items
  - Examples: dice, cards, balls in urn, socks in drawer
- Bernouilli(p)
  - Binary outcome from an experiment: success (p) or failure (1-p)
  - Examples: coin toss, defective component, packet error
- Geometric(p)
  - Often arises from <u>counting process</u> for a Bernouilli RV
  - Example: how many tosses before the first 'Tail' occurs
- Binomial(n,p)
  - Another type of counting process applied to Bernouilli RV
  - Example: how many 'Heads' in n tosses of a coin
- Poisson(λ)
  - Often arises from counting process for an Exponential RV
  - Limiting case of Binomial RV when n approaches infinity
  - Example: how many traffic accidents in Calgary yesterday

Summary: Common Discrete Random Variables

Type	pdf	CDF	Mean	Variance
EquiLikely(a,b)	$1/(b-a+1)$	$(x-a+1)/(b-a+1)$	$(a+b)/2$	$((b-a+1)^2-1)/12$
Bernoulli(p)	$p^x(1-p)^{1-x}$	$(1-p)^{1-x}$	p	$p(1-p)$
Geometric(p)	$p^x(1-p)$	$1-p^{x+1}$	$p/(1-p)$	$p/(1-p)^2$
Binomial(n,p)	$\binom{n}{x} p^x(1-p)^{n-x}$	See textbook	np	$np(1-p)$
Poisson(λ)	$\lambda^x e^{-\lambda}/x!$	See textbook	λ	λ

Common Continuous Random Variables

- Continuous Uniform(a,b) (note that U(0,1) is a special case!)
  - Choosing at random from a specified range of (continuous) values
  - Examples: temperature, rainfall, message size, weight of a package
- Exponential(λ)
  - Often a good model for "random" events (arrivals, duration)
  - Single parameter λ represents "rate", while mean $\mu = 1/\lambda$
  - Examples: accidents, earthquakes, lightning, hole-in-one, phone calls
- Standard Normal(0,1)
  - The classic "Bell Curve" with zero mean and unit variance
  - Examples: statistical noise, normalized residual errors
- Normal(μ,σ)
  - A generalized Gaussian with mean μ and standard deviation σ
  - Often arises when summing other RVs (via central limit theorem)
  - Examples: height, weight, IQ, test scores of a (human) population

Summary: Common Continuous Random Variables

Type	pdf	CDF	Mean	Variance
Uniform(a,b)	1/(b-a)	(x-a)/(b-a)	(a+b)/2	$(b-a)^2/12$
Exponential(λ)	$\lambda e^{-\lambda x}$	$1 - e^{-\lambda x}$	1/λ	$1/\lambda^2$
Normal(0,1)	See textbook	Φ(z)	0	1
Normal(μ,σ)	See textbook	Φ((x-μ)/σ)	μ	σ^2

RNG and RVG Testing

- Uniformity: Chi-square test (discussed last week)
- Independence: KS-test (discussed last week)
- Other tests and utilities:
  - avg.c: sample mean, sample variance, sample std deviation
  - buckets.c: compute histogram (pmf or pdf) of data
  - Check the central tendencies: mean, median, and mode
  - Check the extreme values: minimum and maximum
  - Plot the pdf and look at it visually: does it look right?
  - Plot the CDF and look at it viually: does it look right?
  - autocorr.c: compute autocorrelation coefficients to see if RV is correlated with itself at different time lags