

An Analytical Model for Wireless Networks with Stochastic Capacity

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Abstract

In wireless networks, the system capacity can vary unpredictably with time, due to mobility of users and dynamic channel assignment protocols. This variation in capacity with time, known as ‘stochastic capacity’, can have a major impact on the performance measures such as call blocking probability and queueing delay, of the wireless networks. The dependence of the stochastic capacity on the interaction between the input traffic and the wireless networks makes performance analysis for wireless networks a challenging task. In this paper, we present an analytical framework for the performance analysis of wireless networks with stochastic capacity variation. A capacity-traffic composite model is presented to reflect the interactions between the traffic process and the capacity variation process. With the assumption of general distributions for capacity variation process and exponential distributions for traffic process, the expressions for the performance measures are derived by using semi-Markov process and Markov regenerative theory. It is observed that high variance in the effective capacity of the underlying system degrades its performance. The analytical results are validated by extensive simulations.

Keywords: Stochastic capacity, Cellular networks, Call blocking probabilities, Markov regenerative process, semi-Markov process.

1. INTRODUCTION

In many systems, the available system capacity can vary unpredictably with time. Two simple examples are Web server farms and high-performance computing centers, where the failure of a computing node can result in the loss of jobs from the system. In addition, the removal of computing nodes from the system, even if scheduled in advance to avoid job losses (i.e., planned maintenance), has an impact on the blocking rate or queueing delay seen by other jobs.

Many other examples of stochastic capacity systems appear in the context of networks [5, 19, 24]. For example, in a reservation-based network with multiple priority levels, high priority calls such as emergency services may take precedence over ordinary traffic. The network capacity available for low priority traffic thus varies with time based on high priority traffic demands. In wireless ad-hoc networks, system capacity is strongly dependent on the number of hops

in the routing path [15]. As routes change in response to node mobility, the effective system capacity also varies. In cellular networks, capacity variation arises from the mobility of users, dynamic channel assignment protocols, and the time-varying characteristics of the wireless propagation environment [23]. This phenomenon also applies to wireless LANs and CDMA systems.

Investigating performance in such systems requires a new approach that considers not only the input traffic demands but also the stochastic characteristics of the capacity variation process. The stochastic capacity variation may even depend on the interaction between the traffic and the system itself. For example, in a CDMA system, there is a “soft capacity” limit determined from intra-cell and inter-cell interference from the active calls [3]. It is this “soft limit”, rather than the “hard limit” (i.e., number of channel elements) that determines the effective capacity of the system.

Evaluating the effective capacity measures (e.g., blocking and outage performance) in stochastic capacity wireless networks requires the combination of telecommunications models for channel variability and user-level models for subscriber traffic and mobility. This approach integrates the traffic and queueing models to capture both capacity and traffic characteristics. In such situation, the capacity of the system is a random variable rather than a fixed value. The performance analysis of such networks is of interest since it may yield deeper insights into traffic control and network design issues.

In this paper, we develop a capacity-traffic composite model for the wireless networks to study the impacts of the capacity variation on network performance. We factor the stochastic parameters into the performance analysis to show how they interact with stochastic characteristics of traffic. We study the impacts using analytical modeling and simulation modeling, allowing (where possible) generally distributed traffic and capacity models. This model provides greater flexibility to investigate impacts from stochastic system characteristics than is possible with a continuous time Markov chain model for the pure capacity model. Furthermore, the proposed model is carefully constructed for utilizing tractable solution techniques.

The rest of the paper is organized as follows. In Section 2, we present the analytical model for wireless networks

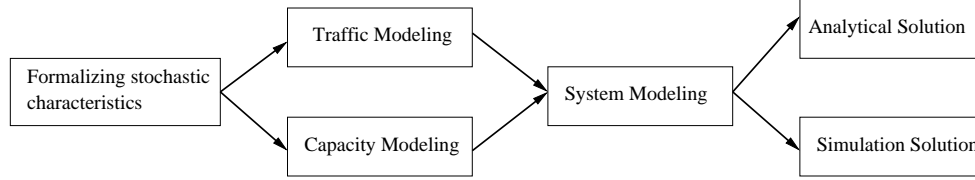


Figure 1. Overview of modeling methodology

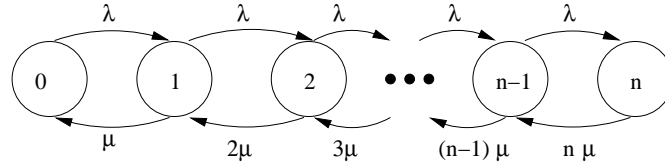


Figure 2. State transition diagram for the traffic model

with stochastic capacity, wherein we first give the system overview, then the capacity and the traffic models are presented and finally the capacity-traffic composite model is constructed. Analytical solution of the proposed model is provided in Section 3. In Section 4, numerical and simulation results are presented. Finally, the conclusions and pointers to future research are given in Section 5.

2. ANALYTICAL MODEL

2.1. System Overview

The overview of methodology for evaluating the performance of wireless networks with stochastic capacity is shown in Figure 1. First, we formalize the underlying stochastic characteristics of system capacity. Based on these characteristics, we model the variation in the traffic and the capacity through stochastic processes such as continuous time Markov chains (CTMC), semi Markov process (SMP), and Markov regenerative processes (MRGP). Finally, we combine the processes for the traffic variation and the capacity variation and obtain the composite model. Under the suitable assumptions, this composite model can be viewed as a two-dimensional stochastic process $\{(C(t), N(t)), t \geq 0\}$, where the first dimension $C(t)$ corresponds to the stochastic capacity and the second dimension $N(t)$ represents the traffic occupancy of the network at time t . The stochastic process $\{(C(t), N(t)), t \geq 0\}$ can be analyzed by applying analytical as well as simulation modeling techniques. In this paper, we first present the analytical solutions for the stochastic process of systems with stochastic capacity and then validate the analytical model through the simulations.

It is possible to adapt our analytical model for any wireless protocol. As stated in the introduction, effective capacity of a network may vary randomly with time because of stochastic traffic effects, the channel status, and the dynamics of protocols used for channel assignment, bandwidth allocation, rate control, and mobility management. The capacity

variation may happen in any network with different protocols. In this section, we abstract our methodology by formalizing the traffic and capacity variations without indicating any characteristics from specific networks.

2.2. Traffic Model

In this section, we develop the traffic model corresponding to the stochastic process $\{N(t), t \geq 0\}$ in the two-dimensional stochastic process $\{(C(t), N(t)), t \geq 0\}$. The network occupancy by the traffic at any time t , denoted by $N(t)$, depends on the traffic arrivals and service time of the ongoing calls. In wireless networks, the traffic arriving at the network is expected to exhibit different stochastic properties. These are well-studied in the literature [21]. The service time of each ongoing call is independent and can follow exponential or general (non-exponential) distributions. Based on the characteristics of the traffic arriving into the network, the traffic arrivals can be modeled as Poisson process, Markov modulated Poisson process, and Markov arrival process.

The assumption of Poisson arrivals and exponential service times reduce the complexity involved in solving the traffic model. Given that the network capacity is n , $(0 \leq n \leq M)$, we present the traffic model with arrivals according to a Poisson process with rate λ and exponential service times with rate μ . Then, the stochastic process $\{N(t), t \geq 0\}$ is a homogeneous continuous time Markov chain (CTMC) and can be analyzed as an $M/M/n/n$ queueing system. Figure 2 shows the state transition diagram for the traffic model. Each state in Figure 2 represents the traffic occupancy at any time t . By solving the system of homogeneous equations corresponding to this Markov chain, the steady state probability of the system oc-

cupancy is given as [4]:

$$\lim_{t \rightarrow \infty} P\{N(t) = k\} = \frac{\left(\frac{\lambda}{\mu}\right)^k / k!}{\sum_{i=0}^n \left(\frac{\lambda}{\mu}\right)^i / i!}, \quad k = 0, 1, \dots, n$$

For mathematical tractability, we assume that the interarrival time is follow an exponential distribution. However, our methodology can be extended to non-exponential interarrival times. On relaxing the assumptions of Poisson arrivals and exponential service times in the traffic model, the complexity in solving the underlying stochastic process increases. It is a cumbersome task to obtain the steady state probabilities.

2.3. Capacity Model

After describing the traffic model $\{N(t), t \geq 0\}$ in the previous section, we next proceed to present the top level capacity variation model represented by stochastic process $\{C(t), t \geq 0\}$. The random variable $C(t)$ represents the capacity of the system at time t . The capacity of the network may vary randomly with time because of the channel status, the dynamics of protocols used for channel assignment, bandwidth allocation, rate control, and mobility management. Therefore, the time between capacity changes is generally (non-exponential) distributed with cumulative distribution function $G(\cdot)$ and density function $g(\cdot)$. Note that the change in the network capacity is independent of the traffic variation. Hence, the stochastic process $\{C(t), t \geq 0\}$ is modeled as a semi-Markov process (SMP) with the state space $E = \{C_0, C_1, \dots, C_M\}$. Figure 3 shows the state transition diagram for the capacity model.

Let the transitions of the process $\{C(t), t \geq 0\}$ occur at epochs (or instants of times) τ_0, τ_1, \dots with $\tau_0 < \tau_1 < \dots < \tau_n < \tau_{n+1}$. Let X_n denote the state of the system at epoch τ_n such that the Markov property is satisfied at each time epoch τ_n . Then the pair $\{X_n, \tau_n\}$ constitutes a Markov renewal sequence on state space E . Note that $\{X_n, n = 0, 1, \dots\}$ constitutes a discrete time Markov chain (DTMC) with state space E . Thus, the stochastic process $\{C(t), t \geq 0\}$ is a SMP where,

$$C(t) = X_n \text{ on } \tau_n \leq t < \tau_{n+1}, \quad n = 0, 1, \dots$$

Here, the Markov chain $\{X_n, n = 0, 1, \dots\}$ is said to be an embedded discrete time Markov chain (DTMC) of the SMP $\{C(t), t \geq 0\}$. To analyze a SMP, we need to deal with two sets of parameters:

- (i) mean sojourn time $M(i)$ in state $i \in E$ and
- (ii) the transition probabilities $H(i, j)$ between different states $i, j \in E$.

The mean sojourn times are expressed as follows:

$$\begin{aligned} M(i) &= E[\tau_n | X_n = i], \\ M(i, j) &= E[\tau_n | X_n = i, X_{n+1} = j]. \end{aligned}$$

Let

$$H(i, j) = \lim_{t \rightarrow \infty} H(i, j, t) = P\{X_{n+1} = j | X_n = i\}, \quad (1)$$

where

$$H(i, j, t) = P\{X_{n+1} = j, \tau_{n+1} - \tau_n \leq t | X_n = i\}.$$

Since the process is time homogeneous, the transition probabilities $H(i, j, t)$ are independent of n .

The steady state probability vector $\mathbf{H} = [H(C_0), H(C_1), \dots, H(C_M)]$ is obtained by solving the matrix form of Kolmogorov forward equations for the embedded DTMC:

$$\mathbf{H} = \mathbf{H}\mathbf{Q}_c,$$

where $\mathbf{Q}_c = [Q_c(i, j)]$ is the transition probability matrix of this embedded DTMC such that $Q_c(i, j) = \lambda(i) [H(i, j) - I(i, j)]$ with $\lambda(i) = \frac{1}{M(i)}$, and $I(i, j) = 1$ if $i = j$, and $I(i, j) = 0$, otherwise.

The steady state probabilities $\{\pi_i\}, i \in E$ of the process $\{C(t), t \geq 0\}$ are expressed in terms of steady state probabilities $H(i), i \in E$ and the mean sojourn times $M(i), i \in E$ as follows:

$$\pi_i = \frac{H(i)M(i)}{\sum_j H(j)M(j)}, \quad i \in E. \quad (2)$$

The above SMP model for the capacity variation system can be extended to a Markov renewal modulated Poisson process. In this case, the stochastic process $\{C(t), t \geq 0\}$ will be defined on a Markov chain, say, $\{N(t)\}$ where transitions in $C(t)$ occur according to a Poisson process with intensity $\alpha_j^{(r)}$ whenever the Markov chain is in state j . Thus, every Markov renewal interval includes a Poisson process. With this model, the impacts from large-scale capacity variation (modeled as a renewal process) and small-scale capacity variation (modeled as a Poisson process) can be analyzed. This extended model will be presented in Section 2.4. The distribution function of the sojourn time of the renewal process given that it is in state r is represented by $M^{(r)}$. This sojourn time does not represent the time period that the capacity stays at a state, but it represents a renewal interval that may include many capacity transitions. To distinguish the difference from the Poisson process, we use $\alpha_j^{(r)}$ to represent the arrival rate of the system at state j given that the renewal process is in state r . It can be expressed as:

$$\alpha_j^{(r)} = \frac{1}{M^{(r)}(j-1)} \left(1 - \frac{(\rho^{(r)})^j}{j!} e^{-\rho^{(r)}} \right), \quad j \in E. \quad (3)$$

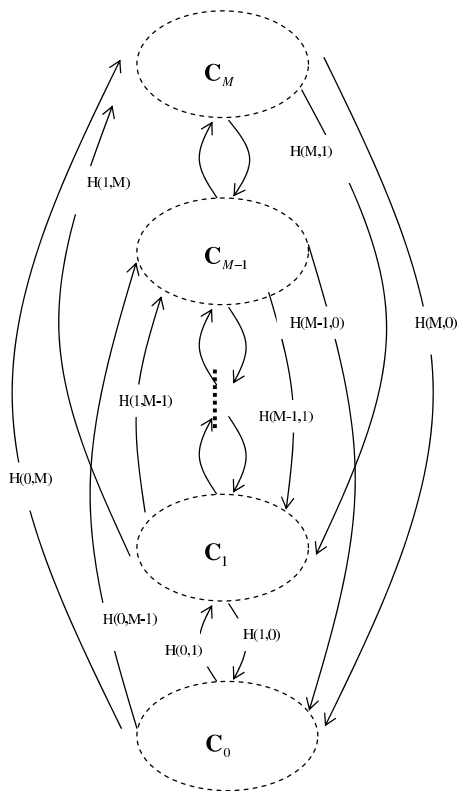


Figure 3. State transition diagram for the capacity model

where $\rho^{(r)}$ is the mean value of the subordinate capacity process given that the renewal process is in state r . $M^{(r)}(j-1)$ is the mean sojourn time of the subordinate capacity process for the state $j-1$. Each current state of the renewal process governs the current arrival rate of the Poisson process. Therefore, the correlated behavior is constructed by letting the states of the modulator correspond to different levels of activity represented by the conditional arrival rate.

2.4. Capacity-Traffic Composite Model

In this section, we present the analytical model for capacity and traffic variation as a Markov regenerative process (MRGP), where the renewal instants of the capacity process are related to the regeneration points in the MRGP. Between the regeneration points, the traffic occupancy in the system changes. The steady-state solution of the system model can be determined by calculating global and local kernel matrices. The global kernel matrix denotes the conditional probability described by the Markov renewal sequence. The local kernel matrix describes the behavior of the MRGP between two transition epochs of the subordinate process.

We are analyzing a stochastic capacity network with developing traffic and capacity model without specifying network protocols. We are also assuming that a general system can be mapped to such a model. An example is a wire-

less network. The large-scale capacity variation arises from large-scale channel fading, which could be factored into the semi-Markov capacity process in our analysis with its fading following a lognormal distribution. The small-scale capacity variation is affected by small-scale channel fading, which could be a Markov process with its fading following a Rayleigh distribution.

2.4.1. MRGP Model

Consider the wireless network system with underlying stochastic process denoted by $\{(C(t), N(t)), t \geq 0\}$, where $C(t)$ denotes the capacity of the system and $N(t)$ corresponds to the traffic occupancy at any time t . $\{(C(t), N(t)), t \geq 0\}$ is a continuous time discrete state two-dimensional stochastic process with state space $\Omega = \{(i, j); 0 \leq i \leq M, 0 \leq j \leq i\}$ and the state transition diagram shown in Figure 4. This stochastic process is not a CTMC since the sojourn time in each state is not exponentially distributed. Furthermore, $\{(C(t), N(t)), t \geq 0\}$ is not a SMP since between any two capacity changes, the state of the system can change due to call arrivals or departures. We observe that the underlying process satisfies the Markov property at time instants $\{\tau_n\}$ of capacity changes only. Define τ_{n+1} as time epoch of $(n+1)st$, $(n > 0)$ capacity variation and let $\{(C(\tau_n), N(\tau_n)) = (i, k)\}$, $i \in \Omega' = \{C_1, C_2, \dots, C_M\}$ and $0 \leq k \leq i$. Here, the el-

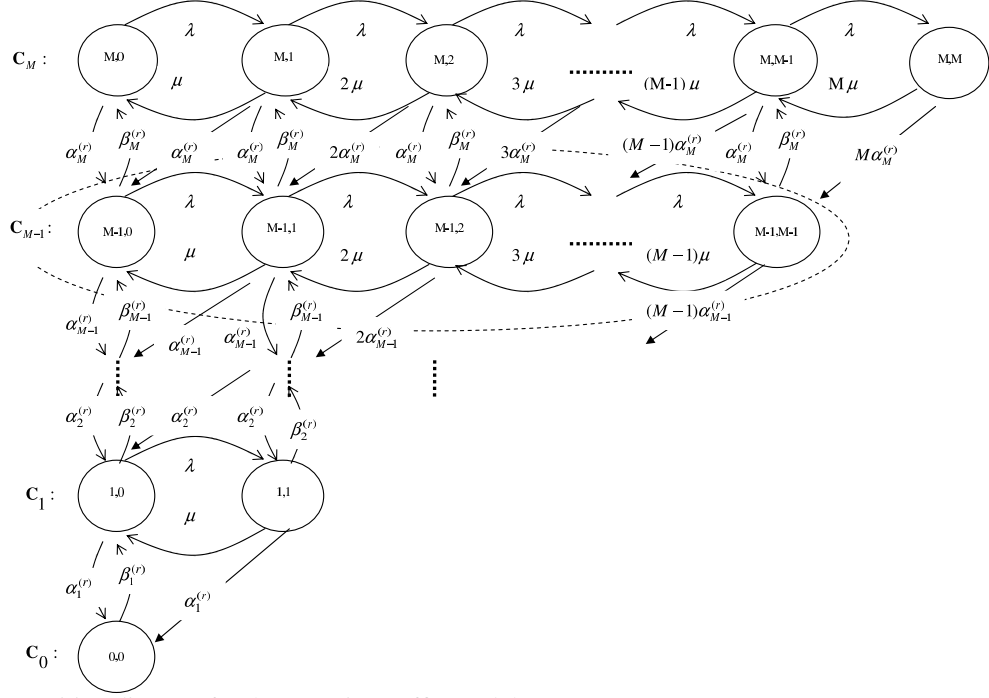


Figure 4. State transition diagram for the capacity-traffic model

elements of Ω' are called regeneration points. Consider the set of epochs $\{\tau_n, n = 0, 1, \dots\}$ at which the system is observed. Note that the state of the system can change between τ_n and τ_{n+1} because of call arrival or departures. We observe that the sequence $\{(C(\tau_n), N(\tau_n)), \tau_n\}$ is a Markov renewal sequence and $\{(C(t), N(t)), t \geq 0\}$ is a MRGP.

3. ANALYTICAL SOLUTION

Following the solution approach for MRGP [1], [2], [13], the expressions for global and local kernel matrices are obtained. We first obtain the expressions for global kernel matrix $K(t) = [K((i,k), (j,l), t)]$. The interpretation for the elements of matrix $K(t)$ is as follows: $K((i,k), (j,l), t)$ denotes the probability that the system will be in state (j,l) , $0 \leq l \leq j$, at the time of the next capacity change (i.e., regeneration instant) which occurs on or before time t , given that the system was in state (i,k) , $0 \leq k \leq i$ and $(i,k), (j,l) \in \Omega'$ just after the previous capacity change instant. Also, if we let $\{(X_n, Y_n), n = 0, 1, 2, \dots\}$, where X_n is the capacity of the system and Y_n is the traffic occupancy at the time instant τ_n , then $\{(X_n, Y_n), n = 0, 1, 2, \dots\}$ will be an embedded DTMC with transition probability matrix $K(\infty)$.

The matrix $E(t)$ describes the dynamics of the process during the time between two consecutive regeneration points starting from a state of regeneration points. The explanation for the elements of matrix $E(t)$ is as follows: $E((i,k), (j,l), t)$ denotes the probability that the system will be in state (j,l) , $j \in \{0, 1, \dots, M\}$, $0 \leq l \leq j$, at time t and the next

capacity variation occurs after t given that the system was in state (i,k) , $i \in \{1, 2, \dots, M\}$, $0 \leq k \leq i$.

To obtain the steady state probabilities for MRGP, we need to know the expressions for $K(t)$ and $E(t)$.

Closed form expressions for $K(t)$

Let Ω_i ($i = 1, 2, \dots, M$) be the set of states reachable from state i in which the subordinate CTMC can spend a non-zero time before the next embedded DTMC transition, where $\Omega_i = \{(i,k), 0 \leq k \leq i\}$. The evolution of the MRGP between two Markov regeneration epochs can be described by a CTMC infinitesimal generator matrix Q , as the only state transitions that take place during this time are due to exponentially distributed events. The matrix Q of the corresponding subordinate CTMC with initial state (i,k) is given as

$$Q = \begin{pmatrix} -\lambda & \lambda & & & & \\ \mu & -(\lambda + \mu) & \lambda & & & \\ & 2\mu & \dots & \lambda & & \\ & & & -(\lambda + (i-1)\mu) & \lambda & \\ & & & i\mu & -i\mu & \end{pmatrix} \quad (4)$$

where λ is the arrival rate and μ is the departure rate in the traffic process. Following the solution approach for MRGP [1, 2, 13], the kernel functions of the MRGP are obtained first. Let $p_{(i,k),(j,l)}(t)$ be the probability that the subordinate CTMC will be in state (j,l) at time t given that it was in state (i,k) initially. Let $\mathbf{p}(t)$ be the corresponding probability row

vector. Then

$$\frac{d}{dt}\mathbf{p}(t) = \mathbf{p}(t) Q(t), \quad p_{(i,k),(i,k)}(0) = 1, \quad i \in E \quad (5)$$

From the definition of the kernel $K(t)$ of an MRGP, we write the set of elements of the global kernel $K(t)$ as follows:

$$K_{(i,k),(j,l)}(t) = \int_0^t p_{(i,k),(j,l)}(t) dG(x), \quad 1 \leq l \leq M; \quad 0 \leq j \leq M, \quad (6)$$

where $p_{(i,k),(j,l)}(t)$ are obtained by solving equation (5).

The local kernel matrix $E(t)$ describes the behavior of the process during the time between two consecutive Markov regeneration epochs. Therefore, for $i = 1, 2, \dots, M$, $0 \leq k \leq i$; $j = 0, 2, \dots, M$, $0 \leq l \leq j$, the elements of the local kernel are given as:

$$E_{(i,k),(j,l)}(t) = p_{(i,k),(j,l)}(t) [1 - G(t)]. \quad (7)$$

To obtain the steady state solution, we define two new variables: (i) $\alpha_{(i,k),(j,l)}$, the mean time the MRGP spends in state (j, l) between two successive regeneration instants, given that it started in state (i, k) after the last regeneration:

$$\alpha_{(i,k),(j,l)} = \int_0^\infty E_{(i,k),(j,l)}(t) dt, \quad (8)$$

and (ii) the steady state probability vector $\mathbf{v} = (v_k)$ of the embedded DTMC:

$$\mathbf{v} = \mathbf{v}P, \quad \sum_{k \in \Omega'} v_k = 1, \quad (9)$$

where $P = K(\infty)$ is the one-step transition probability matrix of the embedded DTMC. Following the methodology in [10], the steady state solution for this MRGP is given as:

$$\begin{aligned} \pi_{(i,k)} &= \lim_{t \rightarrow \infty} P\{C(t) = i, N(t) = j\} \\ &= \frac{\sum_{(i,k) \in \Omega'} v_k \alpha_{(i,k),(j,l)}}{\sum_{(i,k) \in \Omega'} v_k \beta_{(i,k)}} \end{aligned}$$

$$\text{where } \beta_{(i,k)} = \sum_{(m,r) \in \Omega} \alpha_{(i,k),(m,r)}.$$

4. NUMERICAL AND SIMULATION RESULTS

In this section, we give the analytical and simulation results. We illustrate the performance changes of the systems with the different stochastic characteristics of capacity and traffic.

Table 1. Factors in Simulations

Factor		Distributions
Traffic Model	Arrival process	Poisson
	Holding time	Exponential
Capacity Model	Time epochs $\tau(n's)$	Deterministic, Exponential
	values of X_n	Normal

Table 2. Workload Parameters in Simulations

Parameters	Values	
Traffic arrival rate	1.0	
Mean holding time (sec)	30	
Period of capacity changes (sec)	10, 15, 30, 60, 120	
Capacity value (Erlang)	Mean	30, 40, 50
	Variance	2, 5, 10

4.1. Simulation Model

Our simulation work is carried out using call-level simulation. The two inputs provided to the simulation are a call workload file and a network capacity file. These correspond to the traffic and capacity stochastic processes described in Section 2.2. and Section 2.3. respectively.

The call workload file contains a time-ordered sequence of call arrival events. Each call specifies its source node, destination node, arrival time, and duration. Each call requires one unit of network capacity. We use workload files with 100,000 calls. We consider this trace length adequate to highlight differences in performance for various stochastic characteristics of the traffic model. The network capacity file contains a time-ordered sequence of capacity change events. A capacity file with 10,000 capacity change events is used. Both the call workload file and the capacity file are generated according to the corresponding distributions listed in Table 1. The system parameters used for simulation are summarized in Table 2. In our study, we consider the call blocking probability as the primary performance metric for analyzing the stochastic capacity wireless networks.

4.2. Results

The simulation results are shown for a single representative simulation run with 100,000 calls. To obtain the results, we set the mean call holding time to be 30 seconds. Further, the network capacity varies stochastically, with a capacity change every 120 seconds. The capacity (in calls) is drawn from a normal distribution with a mean of 40, and a specified standard deviation of either 2 or 10. We use the notation $DN(X, Y)$ to denote that capacity changes on a deterministic (D) schedule, with the capacity value drawn from a normal $N(X, Y)$ distribution with mean X and standard deviation Y .

Figure 5 shows the call blocking probability versus the offered traffic load in Erlangs. It is observed that high load leads to an increase in call blocking probability. Figure 6 shows the

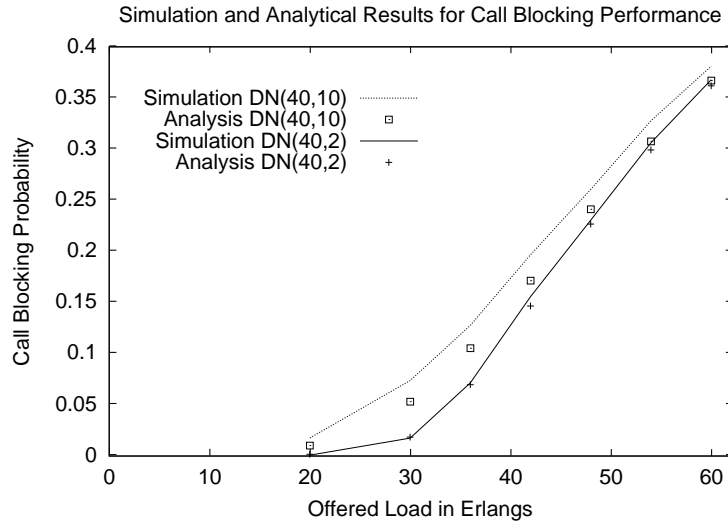


Figure 5. Blocking Probability versus Offered Load in Erlangs

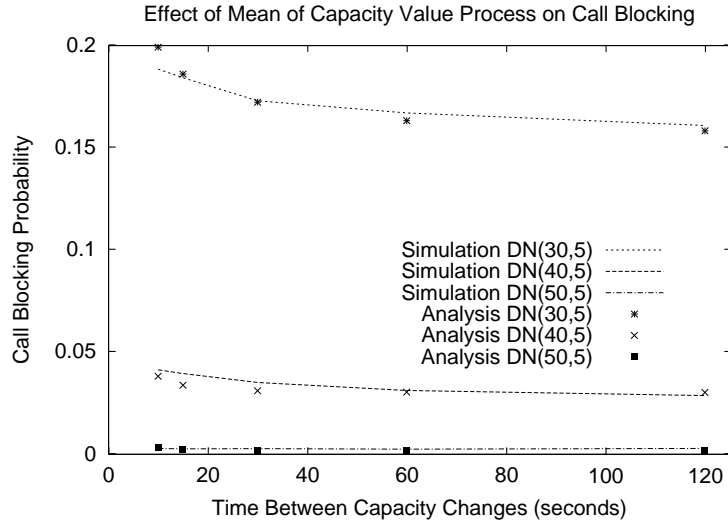


Figure 6. Effect of Mean of Capacity Value Process

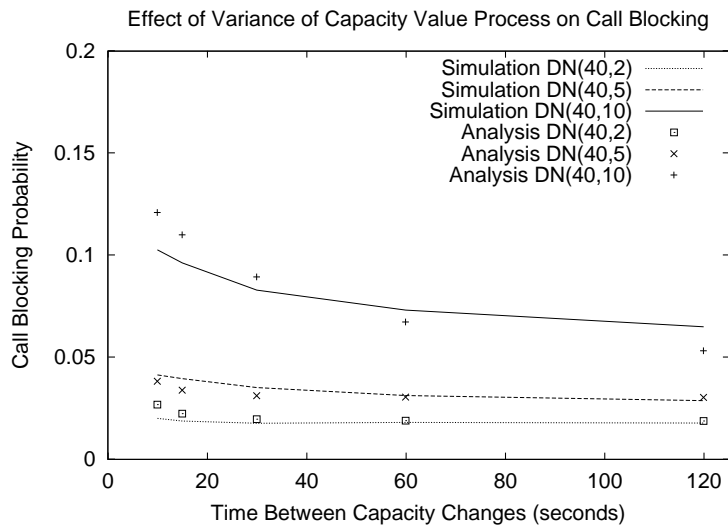


Figure 7. Effect of Variance of Capacity Value Process

results for three different values of mean capacity (30, 40, and 50 calls), while Figure 7 shows the results for low, medium, and high variance in the capacity value process. For these results, the network capacity values are drawn from a normal distribution with the indicated mean and standard deviation. It is inferred from the results of Figures 6 and 7 that while call blocking probability is inversely related to the capacity mean, it is directly related to the capacity variation. Further, it is noticed that the frequency of capacity changes has a noticeable impact on call blocking when the load is high (the DN(30,5) case in Figure 6) or when there is high variance in the capacity process (the DN(40,10) case in Figure 7).

To validate our analytical model, we carry out extensive simulation. The results of Figure 5-7 show that the analytical results match with simulation results very well. These results show that the effective capacity of a stochastic capacity system is lower than that in a fixed capacity system. The reduction in effective capacity is more acute when the capacity is highly variable. Higher variability could arise from higher variance in the capacity value process, higher frequency capacity changes, or both.

4.3. Remarks and Observations

Constructing the generator matrix Q of the subordinate process is a critical part for solving the Markov regenerative stochastic process. For Poisson traffic, the elements of this matrix are constants. For a generally distributed traffic process, however, this matrix is time-dependent, which complicates the solution process. Therefore, the tractability of the analytical solution depends on the complexity of the subordinate processes in the MRGP. In the proposed capacity-traffic composite model, the subordinate process is the traffic process, where the inter-arrival times and call holding times are exponentially distributed. However, the assumption of non-exponential distributions on transitions in other than the traffic process restricts its applicability. Therefore, it is worthwhile to study non-exponential cases [22].

In this section, based on our current knowledge of stochastic analysis, we categorize the problems in analysis of stochastic capacity wireless networks. Table 3 lists the solvable cases for various traffic and capacity models. The relative complexity of the solution process is indicated in parenthesis.

Various models for capacity processes are listed in column 1. Regeneration points in these capacity processes can be mapped into the renewal process in MRGP. Note that the Markov process case is a particular case of Markov renewal process. We first consider traffic processes with exponential inter-arrival times, generally distributed call holding times and infinite servers. Further, for the case of generally distributed inter-arrival times and call holding times with infinite servers, we investigate the solvability of the

capacity-traffic composite model for continuous phase-type arrival and Bernoulli-Poisson-Pascal (BPP) process. For the capacity model, we consider the Markov process, Markov renewal process, and Markov modulated Poisson process. The tractability of the composite capacity-traffic model depends on the traffic processes listed in Table 3.

A few of the cases, such as $M/M/n/n$ traffic process and Markov process, Markov renewal process or Markov modulated Poisson process for capacity, are solvable. Also, $M/G/n/n$ traffic process and Markov process for capacity variation forms another MRGP and hence is solvable. While these few cases can be solved through MRGP, the remaining cases mentioned in the table can only be solved when traffic stochastic characteristics (e.g., transient transition probability, sojourn time distribution etc.) are available. Namely, they can only be seen as equivalent MRGP when the generator matrix of the subordinate process is available. The processing complexity is induced by the general stochastic characteristics of traffic. In general, if the stochastic traffic characteristics are available, then the regenerative process is solvable. The transient stochastic characteristics of most tractable traffic types can be found in the literature [20].

For $G/GI/n/n$ traffic processes, the capacity-traffic model is solvable for some particular analytically tractable distributions. One example is the Markovian Arrival Process (MAP) which is a broad and versatile subclass of Markov renewal processes. In a MAP, the inter-arrival times of calls are phase type distributed, which is a convolution of many exponential distributions and hence is analytically tractable [18]. Phase type distributions reflect more detailed information about the traffic stochastic parameters. Therefore information like the transient transition probability and sojourn time distribution can be available. With this information, the regenerative process can be solved. Column 4 of Table 3 lists the complexity in solving the capacity-traffic model for the capacity processes given in column 1.

Another example is the Bernoulli-Poisson-Pascal (BPP) process [9, 11], (column 5 in Table 3). The BPP approximates $G/GI/n/n$ traffic using a linear state-dependent arrival process. The BPP is characterized by two parameters α and β . When k channels are occupied, Poisson arrival intensity is given as $\alpha + k\beta$. For $\beta = 0$, it reduces to a Poisson process. For $\beta < 0$, it represents a process with lower variability than Poisson. For $\beta > 0$, it represents a process with higher variability than Poisson process.

Given the equivalent mean L and variance V of a general traffic process [12, 21], the occupancy distribution can be approximated by a BPP with the same mean and variance. The parameters α and β are chosen as:

$$\beta = (1 - \frac{1}{Z})\mu \quad \alpha = (\mu - \beta)L \quad (10)$$

Table 3. Examples of Solvable Stochastic Capacity Systems

Capacity \ Traffic	M/M/n/n (complexity)	M/G/n/n (complexity)	G/GI/n/n (complexity)	
			Continuous phase type arrivals	BPP Approx.
Markov process	Solvable (low, precise)	Solvable (low, precise)	Solvable (medium, precise)	Solvable (medium, limitation)
Markov renewal process	Solvable (low, precise)	Solvable (medium, precise)	Solvable (high, precise)	Solvable (high, limitation)
Markov modulated Poisson process	Solvable (medium, precise)	Solvable (high, precise)	Solvable (high, precise)	Solvable (high, limitation)

with arrival rate λ and mean holding time $1/\mu$, where:

$$Z = 1 - \frac{\lambda}{\mu} + 2\mu \int_0^{\infty} W_2^c(x) dU(x) \quad (11)$$

represents peakedness and $U(x)$ denotes the renewal function of the arrival process, which is defined as the expected number of arrivals in a time interval starting just after an arrival with size x . $W_2^c(x)$ denotes the autocorrelation function of the complementary holding time distribution. The study of these traffic models jointly with the capacity model can lead to more general observations for the non-Markovian system.

5. CONCLUSION AND FUTURE WORK

In this work, we develop an analytical model for evaluating the performance of wireless networks by considering stochastic capacity characteristics. Appropriate capacity models are constructed to represent stochastic characteristics in the wireless networks. By joining it with the traffic model, an integrated system is formed and is solved using Markov regenerative process. Stochastic parameters that influence performance in such networks are investigated in the study.

Future research will consist of developing stochastic capacity models for future generation wireless networks based on CDMA technologies. Performance of these systems will be investigated in terms of call blocking probabilities and delay by considering factors such as power control and dynamic channel assignment protocols. In our paper, no mobility parameters were considered. Mobility can be factored into the consideration about the capacity variation. We have not specified the impact from mobility.

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