Poster: $\triangle$-Graphs: Flexible Topology Control in Wireless Ad Hoc Networks

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1. INTRODUCTION

This paper introduces a notion of Δ-graphs as a solution to the topology control problem in wireless ad hoc networks. Most algorithms strive to reduce energy consumption by creating a sparse topology with few long-distance links. However, in a sparse topology, the average path length is relatively large (increasing end-to-end delay), and the number of vertex-disjoint paths between source-destination pairs is relatively small (reducing fault-tolerance). Unlike traditional algorithms that generate a single topology with a certain property, we propose a distributed algorithm that generates a family of topologies (called Δ-graphs) with a range of characteristics. The network designer can choose a suitable topology by simply tuning a single parameter Δ (power savings threshold), trading off energy savings for other features such as low latency and fault-tolerance.

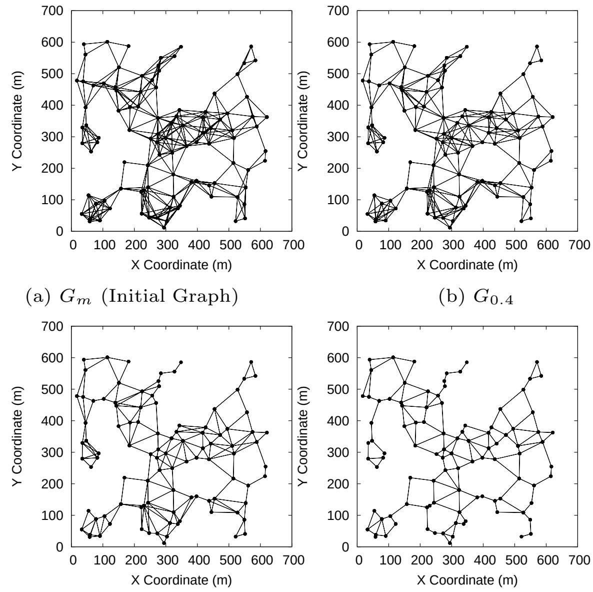
2. $\triangle$-GRAPH CONSTRUCTION

To send a message from node s to node t at distance d, the minimum transmission power is approximated by $P_{s\to t} = Kd^{\alpha}$ where $\alpha \in [2,4]$ is the path loss factor, and K is a global constant. The minimum power required using a node r as relay is: $P_{s\to r\to t} = P_{s\to r} + P_{r\to t}$. The benefit of using a relay r can be quantified by measuring the *fraction of power saved* (which is denoted by Δ):

$$\Delta = \frac{P_{s\to t} - P_{s\to r\to t}}{P_{s\to t}} = 1 - \frac{P_{s\to r\to t}}{P_{s\to t}}$$

Any n-node, ad hoc network can be modelled as a graph $G_m = (V, E_m)$ with the vertex set V representing the nodes, and the edge set, $E_m = \{\langle s,t\rangle \mid \langle s,t\rangle \in V \times V \wedge s \neq t \wedge d(s,t) \leq R_m\}$, where R_m is the distance covered by the maximum power. Given a topology G_m, the distributed algorithm constructs (based on one hop neighbor information) a *connected* subgraph $G_\Delta = (V, E_\Delta)$ where an undirected edge $\langle s,t\rangle \in E_\Delta$ iff there does not exist any relay node r between (s,t) such that relaying through r saves at least Δ fraction of the power required to send directly.

Figure 1 illustrates the central idea in our work. Figure 1(a) shows the initial topology of 100 nodes, with the link connectivity induced by setting $R_m = 100m$. Figure 1(d) shows the well-known *Gabriel graph*, which prunes all network links for which the use of an intermediate relay node provides a lower-energy path for communication. In between these two extremes, our Δ-graph approach defines a family of network topologies, with the level of link pruning controlled by Δ, which provides tradeoffs between path length, delay, fault-tolerance, and energy consumption.



(a) G_m (Initial Graph)
(b) $G_{0.4}$
(c) $G_{0.2}$
(d) G_0 (Gabriel Graph)

Figure 1: Δ-graphs for a network of 100 nodes

3. ANALYTICAL MODEL

We develop a mathematical expression for the sparsity of Δ-graphs for $\alpha = 2$. Consider a multi-hop network with n nodes uniformly distributed over a rectangular region with area A. The maximum transmission radius of each node is R. Then the expected fraction $\mathcal{F}$ of neighbors that are eliminated from a node's neighbor set in Δ-graphs can be approximated as:

$$\mathcal{F} = f(\Delta, n, A, R) = \frac{\pi n\,(1-2\Delta)\,R^2 + 4Ae^{-\frac{\pi n(1-2\Delta)R^2}{4A}} - 4A}{\pi n(1-2\Delta)R^2}$$

4. CONCLUSIONS

Topology control in wireless ad hoc networks introduces many challenges due to multiple conflicting performance optimization criteria. Solutions optimizing one performance criteria are detrimental to others. This work tackles this issue by providing an algorithm that generates a class of topologies and provides the flexibility to network designers to choose the appropriate topology with desired performance characteristics.