Proving the Correctness of a Simple Recursive Algorithm

Suggested Exercise

About This Exercise

This exercise is intended to help you practice and improve your skills in using mathematical techniques to prove that a simple recursive algorithm is correct.

Problems To Be Discussed in the "Tutorial"

Recall the problem "Maximal Element in Part of an Integer Array", considered in the "lecture" part of this unit. Consider the following recursive algorithm.

```
maxInRange2 ( integer[] A, integer low, integer high ) {
1.  if (low == high) {
2.    return A[low]
    } else {
3.    return max(maxInRange2(A, low, high − 1), A[high])
    }
}
```

1. Sketch (or, better yet, write out in full) a proof that this algorithm also correctly solves the "Maximal Element in Part of an Integer Array" problem.

   What form(s) of mathematical induction can be used to prove this claim?

2. Give a bound function for this recursive algorithm and explain, briefly, why it is correct.

3. Give a set of assertions for this recursive algorithm that can be used to document a proof that it is correct.

4. Give the trace(s) of execution and recursion tree for this algorithm when it is executed on an input array A with length 5 such that

   $$A[0] = 8, \quad A[1] = 10, \quad A[2] = 4, \quad A[3] = 24, \quad \text{and} \quad A[4] = 3,$$

   and with inputs $\texttt{low} = 0$ and $\texttt{high} = 4$.