

# Proving Termination and Analyzing the Running Time of a Simple Algorithm with a While Loop

## A Suggested Exercise

### About This Exercise

This exercise is intended to help you to practice and improve your skills in proving termination and analyzing the running time of a simple algorithm with a `while` loop.

### Problems To Be Discussed in This Tutorial

Consider the following computational problem.

#### Sum of Array Elements

*Precondition:* An integer array  $A$  with some positive length  $n$  is given as input.

*Postcondition:* The value

$$\sum_{i=0}^{n-1} A[i]$$

is returned as output.

Consider, as well, the following algorithm (as an algorithm that can be used to solve the above problem).

```
integer arraySum ( integer[] A ) {  
  1. integer sum := A[0]  
  2. integer i := 0  
  3. while (i < A.length - 1) {  
  4.   i := i + 1  
  5.   sum := sum + A[i]  
  }  
  6. return sum  
}
```

1. State a **bound function** for the `while` loop and prove that your answer is correct.
2. Use this to prove that this algorithm terminates whenever it is executed when the precondition for the “Sum of Array Elements” problem is initially satisfied. Recall that it now follows (since its partial correctness has also been established) that this algorithm correctly solves this problem.
3. Use this to state the number of steps executed by this algorithm, when it is executed with an input array `A` with positive length  $n$ . Your answer should be a function of  $n$  and you should state this as precisely you can.

Furthermore, while your intermediate work might involve one or more summations, your final answer should not include any.