Review Questions for Reading #6

1. Suppose that $f : \mathbb{N} \to \mathbb{N}$ or $f : \mathbb{R} \to \mathbb{R}$. What does it mean for $f$ to be asymptotically positive?

Suppose, for the remainder of these questions, that either $f, g : \mathbb{R} \to \mathbb{R}$ or $f, g : \mathbb{N} \to \mathbb{N}$ and that $f$ and $g$ are asymptotically positive.

2. State the definition of “$f \in O(g)$”.

3. Describe, more informally, what it means about the relationship between $f$ and $g$ when $f \in O(g)$.
4. Describe a sequence of steps that you can follow in order to use the above definition to prove that \( f \in O(g) \) for a given pair of functions \( f \) and \( g \).

5. State the **Limit Test for \( O(g) \)**.

6. Why is it (at least arguably) important that you know how to use the definition of \( O(g) \), instead of just the limit test, to prove that \( f \in O(g) \)?
7. State *l'Hôpital's Rule*.

8. Why is it (sometimes) useful to know the above rule when you are trying to use the limit test to prove that $f \in O(g)$ for a given pair of functions $f$ and $g$?

9. State the definition of $f \in \Omega(g)$. Then say, less formally, what this means about the relationship between $f$ and $g$. 
10. Describe **three** ways to prove that \( f \in \Omega(g) \).

When doing this you will need to name two theorems that were given, in the notes for this reading, concerning this. (Note that you should also be able to **state** these theorems!)

11. State the definition of \( f \in \Theta(g) \). Then say, less formally, what this means about the relationship between \( f \) and \( g \).

12. Describe a process that can be followed to prove that \( f \in \Theta(g) \) for a given pair of functions \( f \) and \( g \).
13. State the definition of \( f \in o(g) \). Then say, less formally, what this means about the relationship between \( f \) and \( g \).

14. Describe two methods that can be used to prove that \( f \in o(g) \) for a pair of functions \( f \) and \( g \). You will need to name a theorem, stated in the notes for this reading, to do this — but you should also be able to state this theorem.
15. State the definition of \( f \in \omega(g) \). Then say, less formally, what this means about the relationship between \( f \) and \( g \).

16. Describe three methods that can be used to prove that \( f \in \omega(g) \) for a pair of functions \( f \) and \( g \). You will need to name two theorems, stated in the notes for this reading, to do this — but you should also be able to state these theorems.
17. Define a polynomial function with degree $d$.

18. Give three useful properties about the rates of growth of polynomial functions with various degrees.

19. Define an exponential function.
20. Give six useful properties about the rates of growth of polynomial functions and exponential functions.

21. Give the definition of a logarithmic function.

22. Give two useful properties about the rates of growth of polynomial functions and logarithmic functions.