

# Lecture #1: Mathematics Review

## Assumptions

- Preliminary material for this lecture — including the supplemental “mathematics review” — has been read.
- You can refer to the supplemental “mathematics review” during this presentation as needed.

## Questions for Review

You should be able to answer the following questions.

1. Consider *mathematical proofs*.
  - (a) What is a *mathematical proof*?
  - (b) What is an *axiom*?
  - (c) Give an example of an axiom.
  - (d) What is a *theorem*?
  - (e) What is a *proof technique*?
  - (f) Give an example of a proof technique.
  - (g) Give another example of a proof technique.
  - (h) Why are mathematical proofs important?
  - (i) Describe at least five *mistakes* that you should watch for and avoid when writing mathematical proofs.

2. Consider **mathematical induction**.

- (a) What is the **standard form** of the **principle of mathematical induction**?
- (b) Describe the **structure** of a proof that uses the standard form of mathematical induction.
  - (i) What are you proving in the **basis**?
  - (ii) What is the **inductive hypothesis**?
  - (iii) What is the **inductive claim**?
  - (iv) What, precisely, are you establishing in the **inductive step**?
- (c) What is the **strong form** of the **principle of mathematical induction**?
- (d) Describe the **structure** of a proof that uses the strong form of mathematical induction.
  - (i) What are you proving in the **basis**?
  - (ii) What is the **inductive hypothesis**?
  - (iii) What is the **inductive claim**?
  - (iv) What, precisely, are you establishing in the **inductive step**?

## The Problem To Be Solved

Our goal is to prove the following.

*Claim:* The **Dumbledore numbers** are a sequence  $D_0, D_1, D_2, D_3, \dots$  of numbers studied at the Hogwarts School of Witchcraft and Wizardry. They are defined as follows: For every integer  $n$  such that  $n \geq 0$ ,

$$D_n = \begin{cases} 0 & \text{if } n = 0, \\ 1 & \text{if } n = 1, \\ 2 \times D_{n-1} - D_{n-2} & \text{if } n \geq 2. \end{cases}$$

Let  $n$  be an integer such that  $n \geq 0$ . Then  $D_n = n$ .

## **Setting a Plan: Identifying a Proof Technique**

*Proof Technique:*

*Setup — Preliminary Information:*

*What We are Inducting On:*

*The Property of This Value Being Established:*

**Setting a Plan:  
Using the Proof Technique to Develop an Outline — and  
Filling in the Details**

**Components of Proof To Be Written:**

**Basis:**

*What We Need to Show:*

*Figuring Out Why This is True:*

*What the Proof Should Actually Include:*

## **Inductive Hypothesis**

*How This Part of the Proof Should Start:*

Suppose that

It is necessary and sufficient to use the following

Inductive Hypothesis:

to prove the following

Inductive Claim:

*Figuring Out Why This is True:*

*What the Rest of the Proof Should Actually Include:*

## **What Happens If We Get Stuck or Things Don't Work Out?**

# What Follows

## Breakout Session

If time allows at the end of the scheduled lecture time then you will be invited to move to breakout rooms, in small groups, so that you can get to know a few other students in this course.

Of course, you may discuss the course material if you wish to! If you would prefer to discuss something else, then the following questions might be considered for discussion.

1. Compare and contrast **comic book superheroes** and **Disney princesses**. How are the same? How are they different?
2. Which **comic book superhero** would you prefer to be?
3. Which **Disney princess** would you prefer to be?
4. Is **Wreck-It Ralph** a Disney princess?<sup>1</sup>

As an aid, information about a select group of comic book superheroes and Disney princesses will be made available on the course web site.

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<sup>1</sup>A CPSC 331 student asked this question using “chat” on Zoom during the first CPSC 331 lecture in Fall 2020. I do not recall that a completely satisfactory answer was given.