

CPSC 313 — Tutorial Exercise #1

Review of Mathematical Induction

About This Exercise

This exercise will be discussed during the tutorials on Monday, January 17, Tuesday, January 18 and Wednesday, January 19. It is intended to help you to review mathematical induction and assess your ability to write proofs using this proof technique — but it is *also* intended to introduce the way that tutorials in this course will be used.

A Bit About These Problems and How To Approach Them

When considering the following problems, please note that there are *at least three steps* involved for each problem, here, and that *you need to do all three of them*:

- (a) ***Making Sure That You Understand the Problem:*** Look carefully at the description of the problem to be solved. Make sure that you understand the meaning of any notation and technical terms that are used in the problem statement — reviewing material from this course and from prerequisite courses, as needed, in order to make sure that you understand the problem statement.

If in doubt, ***contact the instructor and ask what you need in order to understand the problem, before the tutorial.*** The teaching assistant will try to answer questions about this during the tutorial as well, but it will probably be too late for you to complete the exercise if you still need to ask about this, at that point.

- (b) ***Figuring Out How To Solve the Problem:*** Sometimes you are asked to use a particular method or strategy when you are given a problem to solve — but, at other times, you are just given the problem to be solved. You should understand that *some* method, discussed in the course, should be applied — but there might be more than one of them to be considered and used.

You should **identify one or more methods that might be used to solve this problem**. Review any course material to make sure that you **understand** these methods — asking questions about them as needed.

Then **try to solve the problem using one of the methods you identified**. If you did not manage to do this then try to understand what went wrong or where (and how) you got stuck. Even if it doesn't seem like it, you have probably learned something important!

At this point, you can try again, working with what you learned from your unsuccessful attempt. Alternatively, if there are other methods that might still be used, then you can try one of those.

If you *did* manage to solve the problem then (perhaps, after taking a break) read over your solution again, looking for any mistakes that you might have made or things that you forgot to do. Try to improve what you have got.

- (c) **Writing the Solution Down Properly:** Your final “solution” should be a written document that can be used by an other person — who you might not know and might never meet — to understand how this problem can be solved. If the problem was to “prove a claim” then this other person should believe that the claim is true (even if that person thought it was false, before) and understand *why* it is true too.

This “other person” can be assumed to be familiar with the material in the prerequisites for this course. If your answer is vague or incomplete then you should probably assume that any guess that the reader makes, in order to understand your work, **will not be the one you want**.

Pretty much nothing else can be assumed about that other person. Your written solution should be organized to meet the needs of **that other person** — not for yourself or a close friend who can ask you about what you wrote, and not the instructor, a teaching assistant, or some other “marker” who can be assumed to know a solution for the problem, already.

There is a **fourth step** that sometimes has to be considered: **Presenting your solution to a group of people**. In general, we won't worry about this in this course.

Tutorial exercises are being given to you, ahead of time, so that you can try to solve problem **before the tutorials** and then use the tutorials to discuss **all of the above** — and not just the “completed solution” that you get at the end — with other students and the tutorial leader(s). Of course, it's hoped that you will learn how to use the course material to solve problems, along the way!

The initial tutorials might seem frustrating because **you are are “learning a learning process”, along with the course material**. If you stick with it then it is possible that later tutorials will be less frustrating, as you become more familiar with “the learning process” and don't have to focus on it, along with everything else.

The Problems To Be Solved

1. Prove that

$$\sum_{i=1}^n i^3 = \frac{n^2 \cdot (n+1)^2}{4}$$

for every integer $n \geq 1$.