Lecture #6: Equivalence of Deterministic Finite Automata and Nondeterministic Finite Automata

Assumptions

- Preliminary material for this lecture has been reviewed.

Questions for Review

1. Give a brief proof that if there exists a deterministic finite automaton $M = (Q, \Sigma, \delta, q_0, F)$ whose language is $L \subseteq \Sigma^\star$, then there exists a ***nondeterministic*** finite automaton $\widehat{M}$ whose language is L as well.
2. Suppose, instead, that you have been given a ***nondeterministic*** finite automaton $M = (Q, \Sigma, \delta, q_0, F)$ whose language is $L \subseteq \Sigma^\star$ and that you wish to design a ***deterministic*** finite automaton $\widehat{M} = (\widehat{Q}, \Sigma, \widehat{\delta}, \widehat{q}_0, \widehat{F})$ whose language is L as well.

   (a) What *information* must $\widehat{M}$ remember in order to correctly decide whether a given string belongs to L? That is, what *information* needs to be considered and used in order to define the states in $\widehat{Q}$?

   (b) Describe a process that you can follow in order to use M to define both the set $\widehat{Q}$ of states in $\widehat{M}$ and the transition function $\widehat{\delta}$.

   (c) Which state in $\widehat{Q}$ should be the start state $\widehat{q}_0$? Why?

   (d) How should be set $\widehat{F}$ of accepting states in $\widehat{M}$ be defined?

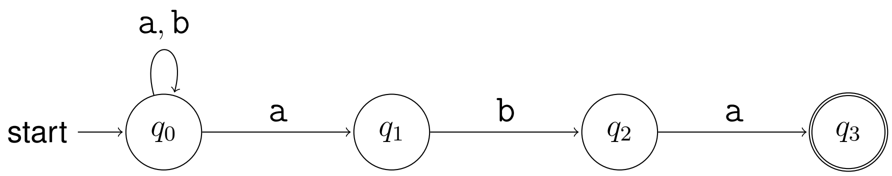
3. Suppose that $|Q| = n$ for a positive integer n. How large might $\widehat{Q}$ be, as a function of n?

Conversion to a Deterministic Finite Automaton — Using the Process from Class

Recall that $\Sigma = \{\mathtt{a}, \mathtt{b}\}$. Let $L \subseteq \Sigma^\star$ be the following language:

$$L = \{w \in \Sigma^\star \mid \omega \text{ ends with } \mathtt{aba}\}.$$

Consider, the following ***nondeterministic*** finite automaton $M = (Q, \Sigma, \delta, q_0, F)$ with the above alphabet Σ and the following transition diagram.



If we used the ***process described in the lecture notes*** to a produce a deterministic finite automaton, with the same language, then the following would result:

Conversion to a Deterministic Finite Automaton — Another Process

Some tests (including *Introduction to the Theory of Computation*) describe with a process in which we begin, right away, by including a state corresponding to a subset of the states in the NFA, for *every* possible subset. If we did this instead, for the above example, then this is what we would get.

Breakout Session

Review the information about the ***Barbie Liberation Organization*** (B. L. O.) that has been supplied. Prepare to say which of the following terms describes the members of this group.

(a) Social justice warriors

(b) Defenders of liberty

(c) Dangerous radicals

(d) All of the above

(e) None of the above