Lecture #7: Regular Operations and Closure Properties of Regular Languages

Assumptions

- Preliminary material for this lecture has been reviewed.

Questions for Review

1. Name the ***regular operations***.
2. Suppose that Σ is a finite and nonempty alphabet and let $L_1, L_2 \subseteq \Sigma^\star$. Give the formal definition of the ***union*** of L_1 and L_2. Explain in your own words what this language is.
3. Suppose that Σ, L_1 and L_2 are as above. Give the formal definition of the ***concatenation*** of L_1 and L_2. Then explain in your own words what this language is.
4. Suppose that Σ and L_1 are as above. Give the formal definition of the ***star*** of L_1. Then explain in your own words what this language is.
5. What is a ***closure property***? List the closure properties that were stated and proved (at least, informally) in this lecture.
6. Why are closure properties useful? (Note that they were used to prove something about several languages as part of this lecture.)

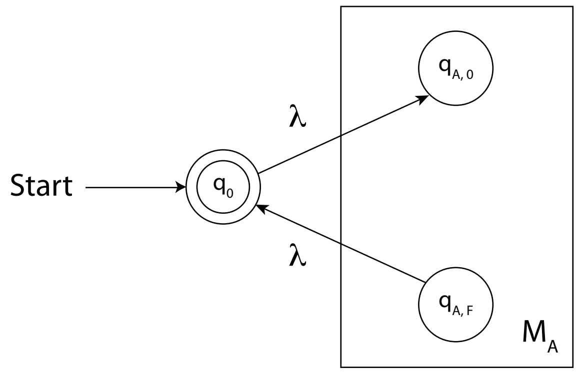

Figure 1: Nondeterministic Finite Automaton with Language $A^\star$

Objective

The proof of the following claim will be developed in more detail.

Claim #4: If Σ is a finite and nonempty alphabet and $A \subseteq \Sigma^\star$ is a regular language, then $A^\star$ is also a regular language.

Proof: Suppose A is a regular language. Then there exists a nondeterministic finite automaton

$$M_A = \{Q_A, \Sigma, \delta_A, q_{A,0}, F_A\}$$

such that $L(M_A) = A$ and this NFA has all the properties described in Claim #1 from the lecture notes — so that, in particular, $F_A = \{q_{A,F}\}$ for some state $q_{A,F} \in Q_A$.

Renaming states as needed we may assume that $q_0 \notin Q_A$.

Now consider an NFA that looks like the one in Figure 1.

This is more formally defined as follows: The nondeterministic finite automaton, shown here, is the NFA

$$M = (Q, \Sigma, \delta, q_0, F)$$

where

- $Q = \{q_0\} \cup Q_A$;
- For all $\sigma \in \Sigma_\lambda$,

$$\delta(q_0, \sigma) = \begin{cases} \{q_{A,0}\} & \text{if } \sigma = \lambda, \\ \emptyset & \text{if } \sigma \in \Sigma. \end{cases}$$

- For all $q \in Q_A \setminus \{q_{A,F}\}$ $\sigma \in \Sigma_\lambda$, $\delta(q, \sigma) = \delta_A(q, \sigma)$.
- For all $\sigma \in \Sigma_\lambda$,

$$\delta(q_{A,F}, \sigma) = \begin{cases} \{q_0\} & \text{if } \sigma = \lambda, \\ \emptyset & \text{if } \sigma \in \Sigma. \end{cases}$$

- $F = \{q_0\}$.

The following subclaims will now be proved.

Subclaim #4(a): Let the nondeterministic finite automaton M be as given above. Then $A^\star \subseteq L(M)$.

Subclaim #4(b): Let the nondeterministic finite automaton M be as given above. Then $L(M) \subseteq A^\star$.

The result follows directly from these subclaims. $\square$.

It therefore remains only to prove each of these subclaims.

Subclaim #4(a): Let the nondeterministic finite automaton M be as given above. Then $A^\star \subseteq L(M)$.

Method of Proof:

Details of Proof:

Subclaim #4(b): Let the nondeterministic finite automator M be as given above. Then $L(M) \subseteq A^\star$.

Method of Proof:

Details of Proof:

Breakout Session

Please consider the following question about the long-running British television series, *Doctor Who*: If you could be a *companion* of one of the first four Doctors, then whose companion would you be?

(a) First Doctor

(b) Second Doctor

(c) Third Doctor

(d) Fourth Doctor