CPSC 313 — Tutorial Exercise #8
Nonregular Languages

1 About This Exercise

The following lecture concerns material found in Section 1.3 of *Introduction to the Theory of Computation* and presented in the following lectures.

- Lecture #10: Nonregular Languages (Part One)
- Lecture #11: Nonregular Languages (Part Two)

This exercise will be discussed on Thursday, February 10. Please try to solve the problems in this exercise ***before*** attending this tutorial, so that you can participate in discussions of solutions for these problems with other students and the teaching assistants.

There will certainly not be time for solutions for all of these problems to be presented! However, there should certainly be enough time for the solution of one of these problems to be discussed in detail.

Problems To Be Solved

You should assume that each of the following languages is being considered as a subset of $\Sigma^\star$ for the alphabet $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$.

1. Prove that the language
$$L = \{\mathtt{a}^n\mathtt{b}^n\mathtt{c}^n \mid n \geq 0\}$$
   is not a regular language.

   Hint: This problem can be solved by studying, understanding, and then modifying one of the example proofs in the lectures about this.

The next two problems are a *little bit* trickier: They can be solved using the Pumping Lemma for Regular Languages, but you cannot just set i to be equal to 0 or 2 in order to solve them.

2. Prove that the language $L = \{a^n b^m \mid m \geq n\}$ is not regular.

3. (This one needs you to be careful when choosing the string s): Prove that the language $L = \{a^n b^m \mid m < n\}$ is not regular.

   On the other hand, you can use closure properties along with the result of the second question, after studying, understanding, and modifying another example from class.

The first problem, below, can also be solved using the Pumping Lemma for Regular Languages — but you have to somewhat clever in choosing the value of the integer "i" used to get a contradiction here. The second problem can then be solved using closure properties.

4. Recall that a positive integer i is ***prime*** if $i \geq 2$ and $i = j \times k$, for positive integers j and k, only if $j = 1$ (and $k = i$) or $j = i$ (and $k = 1$). Thus 2, 3, 5, and 7 are the first four primes.

   Prove that the language
   $$L = \{\text{a}^n \mid n \text{ is prime}\}$$
   is not a regular language.

   Note: You may use the following result without proving it: There are infinitely many prime numbers so that, in particular, there are ***arbitrarily large*** prime numbers.

5. On the other hand, a positive integer i is ***composite*** if ≥ 2 and i is not prime — so that there exist integers j and k such that neither j not k is equal to 1 and $i = j \times k$.

   Prove that the language
   $$L = \{\text{a}^n \mid n \text{ is composite}\}$$
   is not a regular language, either.

The remaining problems do not resemble problems that are likely to be included on an assignment or test in this course. However, it is probably worthwhile to think about them, because they concern ***mistakes that students sometimes make***, when they try to prove that a language is not regular, that you should watch for and avoid.

6. What is wrong with the following proof? (Note that it certainly ***is*** incorrect, since the language given here is a regular language.)

Claim: The language

$$L = \{\mathsf{a}^n \mid \text{n is even}\}$$

is not a regular language.

Proof: The Pumping Lemma for Regular Languages will be used to prove this.
With that noted, let p be any arbitrarily chosen integer such that $p \geq 1$.
Consider the string $s = \mathsf{a}^{2p}$.

- $s \in L$, since $2p$ is an even number.
- $|s| = 2p \geq p$.

Now let $x = \lambda$, $y = \mathsf{a}$, and $z = \mathsf{a}^{2p-1}$. Then $|y| = 1 > 0$, so that $y \neq \lambda$, and $|xy| = |\mathsf{a}| = 1 \leq p$. However, if $i = 0$ then

$$xy^iz = xz = \mathsf{a}^{2p-1} \notin L,$$

since $2p - 1$ is an odd number. Thus Properties 1, 2 and 3 in the statement of the Pumping Lemma are not satisfied.
Therefore L is not a regular language.

Hint: Compare this to the examples in the notes not he Pumping Lemma and the “Summary of a Process” to be followed when using this result. Where does this deviate from this process, and how?

7. What is wrong with the following proof? (Note that it certainly ***is*** incorrect, since the language given here is a regular language.)

   Claim: The language $L = \emptyset$ is not a regular language.

   Proof: We now know that the languages

   $$L_1 = \{\mathsf{a}^n\mathsf{b}^n \mid n \geq 0\} \qquad \text{and} \qquad L_2 = \{\mathsf{a}^p \mid p \text{ is prime}\}$$

   are both nonregular languages. Since $L_1 \cap L_2 = \emptyset$, $\emptyset$ is also a nonregular language, as claimed.

   Hint: We only proved that the set of ***regular*** languages were closed under a bunch of operations...