Eliciting Entailment

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ABSTRACT

Interactive construct elicitation systems may be seen on the one hand as tools for self-realization and construct development and on the other as dialectic database systems in which the entire data structure is built up through a process of question, answer and discussion. Current computer-based construct elicitation programs use distance-based measures to feed back construct similarities and to perform cluster analysis. This paper describes new techniques for inferring the entailment structure between constructs. It gives an example of the application of this technique embodied in the computer program ENTAIL. It throws new light upon the role of the opppposite, rather than the negation, in construct theory, logic and dialectics. It is one in a series of papers on fuzzy set semantics for personal constructs.

1 INTRODUCTION

It is now twenty five years since Kelly (1955) published his seminal book on personal construct theory. It provides a remarkably far-reaching and well-structured foundation for epistemology. His work is anchored very firmly both in its close correspondence to the actual behaviour of people and in its coherent and consistent philosophy. This is not to say that Kelly fully worked out a logically, philosophically and psychologically complete model of knowledge acquisition. His attempts to link his work to other philosophical studies of epistemology, his attempt to present it axiomatically, and his embodiment of it as an empirical tool through the repertory grid, are all incomplete. They need much further development and modification to take them to levels of scholarship, science and technology which would allow them to stand critical comparison with related work.

However, there are now many who would endorse Kelly's intuition for what he proposed as a starting position: his model of the personal scientist acquiring a personal model of his world; and his idea of personal constructs as filters through which we perceive events:

> "Man looks at his world through transparent templets which he creates and then attempts to fit over the realities of which the world is composed." (Kelly 1955 pp.8-9)

He continually emphasizes the epistemological status of these constructs in predicting and controlling the world and their ontological status as personal conjectures rather than reality-derived absolutes:

> "Constructs are used for predictions of things to come, and the world keeps on rolling on and revealing these predictions to be either correct or misleading. This fact provides the basis for the revision of constructs and, eventually, of whole construct systems." (Kelly 1955 p.14)

When it came to the formal and practical representation of constructs Kelly took them to be binary in nature such that each event construed was classified as belonging to one "pole" of a construct, or the other. In essence Kelly placed the same fundamental emphasis as did Spencer Brown in his seminal work, Laws of Form, on the human, creative operation of "making a distinction":

> "The theme of this book is that a universe comes into being when a space is severed or taken apart... By tracing the way we represent such a severance, we can begin to reconstruct, with an accuracy and coverage that appear almost uncanny, the basic forms underlying linguistic, mathematical, physical and biological science, and can begin to see how the familiar laws of our own experience follow inexorably from the original act of severance." (Spencer Brown 1969 p.v)

It casts an interesting light on the further development of Kelly's work that Spencer Brown goes on to use the notion of a distinction to develop a logical "calculus of distinctions" with fewer primitives than the classical propositional calculus which he claims avoids the paradoxes of previous approaches. In his own practical development of a personal construct technology through the "repertory grid" and the extraction of "factors" from it Kelly treats constructs as if they gave a vector of measurements of the event rather than a logical representation of it. This approach seems to have been followed also by all

			ELEMENTS											
	X	O	1	2	3	4	5	6	7	8	9	10	11	12
C1	Intensity. They both are interested in other people. Concerned with world problems. Ambitious. Slightly detached.	Humorous. Creative. Unconventional approach to work & relationships. Exciteable.	X	X	O	O	X	O	O	X	O	X	O	O
C2	Individualistic. Musical. Calm(exteriorally). Unconventional. Non-aggressive. Loyal. Interested in myth & fantasy. Homely. Land-loving. Tending toward introversion. Unusual humour.	Self aware. Controlled. Sporting. Experienced in relationships. Attracted to sophistication & the exotic. Extroverted. Light hearted.	O	O	O	O	X	X	O	O	O	O	X	O
C3	Generous. Interested in history. Slow living. Perfectionist in work. Unusual relationships.	Direct. Political. Super active. Strong integrity. Committed.	O	O	O	O	X	X	X	O	X	O	X	O
C4	Ambitious. Questioning. Quick minds. Confident. Interested in "societies ills."	Artistic. Capable. Gentle. Romantic. Exploratory.	X	X	X	X	O	O	O	X	O	X	O	X
C5	Outdoor enthusiasts. Anxious to succeed. Anxious about success with other sex. Active. Enigmatic. Need mental stimulation.	Creative. Enjoys comfort. Relaxed.	X	X	X	X	O	O	O	X	O	O	O	X
C6	Enjoy intellectual discussion. Difficult to understand initially. City livers. Seek challenges. Insecure backgrounds.	Affectionate. Humble. Sensitive. Musical. Involved with those immediately around. Compassionate. Philosophical.	O	X	X	X	O	O	X	X	X	X	O	X
C7	Energetic. Sociable. Politically concerned interests. Dynamic. Restless. Factual approach as opposed to interest in fantasy world.	Thorough. Care for detail. Extremely creative. Not concerned with social success. Gentle. Perceptive.	X	X	X	X	O	O	O	X	X	O	O	X
C8	Both need company. Gregarious. Prepared to compromise. Factual approach. Enjoy discussion.	Musical. Scientific but also keen on the "unreal" world. Fantastical.	X	X	X	X	O	O	O	X	X	X	O	X

Figure 1 Jane's Grid on Acquaintances

later workers on the analysis of the repertory grid through a variety of methods such as principal components analysis. In this paper we show that the analysis of construct systems as logical structures both encompasses many of the advantages of such methods and also leads to interesting new directions of analysis.

2 GRID ANALYSIS

Figure 1 is a repertory grid from Shaw (1980 p.79) showing Jane's allocation of twelve acquaintances to the poles of eight constructs. It is a particularly good illustrative example because Jane has given far more background explanation to the poles of her constructs than is usually available and this makes it easier to assess the prima facie meaningfulness of any analysis. The only difference between Figure 1 and Figure 6.4 in the book is that Shaw uses the letter "X" for the assignment to the left-hand pole and the letter "O" for the assignment to the right-hand pole, whereas we have used the numbers "1" and "0" respectively. This change to numerals is deliberate because we wish to examine how the values in the grid may be viewed in two ways: firstly as numerical values; and then as logical values.

We will concentrate initially on the relations between the constructs in a grid such as that shown in Figure 1. For any given construct we may regard the numbers in the grid as a vector of values giving the assignment of each element in turn to one or other of the poles of the construct. From this point of view each construct becomes represented as a point in a multi-dimensional space whose dimension is the number of elements involved. The natural relation to examine between constructs is then the distance between them in this space. Two constructs which are zero distance apart are such that all elements are construed in the same way in relation to them and hence we might infer that they are being used in the same way - in some way they are equivalent constructs. Gaines and Shaw (1980) show how such distance measures are used to produce a principal components analysis of the construct space using INGRID (Slater, 1977); also a Q-analysis of the structure behind such data (Atkin, 1974); and a cluster analysis of the constructs using FOCUS (Shaw, 1980). All of the techniques based on such a numerical spatial view of construct structures depend on the notion of constructs being equivalent if they are represented by the same point in space and somehow nearly equivalent if they are represented by points close to one another.

These various distance-based analyses of grids provide related methods of clustering elements and constructs in such a way that one can provide feedback on possible structures underlying the construing. They have two factors in common that restrict their application in some contexts. Firstly, the structure exhibited is limited in its semantics to a symmetric relation of 'neighbourness' between the items clustered. Secondly, the analyses produce results about distances, components, connections, geometrical relationships, and so on, which represent a different way of looking at the data. This may be valuable in itself and may be expressed through basic notions of similarity. However, for some applications such as interactive discussion in conversational grid elicitation it would be preferable to have an analysis that expresses relations in the data in terms more immediately meaningful and direct related to the data itself. It was these considerations that led us to the logical data analysis of grid data.

An alternative way of looking at the grid of Figure 1 views it not as a set of vectors in a space but instead as an assignment of truth-values to logical predicates. We may take the left-hand pole of each construct in Figure 1 to be a logical predicate that may be applied to a person and take the assignment of the value to a particular element in the grid to mean that the predicate is true for that element. Conversely we may take the value of 0 assigned to an element for a construct to mean that the predicate represented by the left-hand pole of that construct is false for that element. It is convenient to use the abbreviation LHPm for the predicate that corresponds to the left-hand pole of construct m. Thus LHP5 is the predicate for the left-hand pole of construct 5. If we then also adopt the convention that En stands for the n'th element then the notation LHPm En may be used to denote the truth value of the predicate corresponding to the left-hand pole of construct m when applied to the logical constant corresponding to the n'th element. A repertory grid, such as that of Figure 1, is then the matrix of such truth values for the m constructs and n elements involved.

Because of the inverse relation between assignments to the opposite poles of a construct in a conventional repertory grid, the predicate corresponding to the right-hand pole is logically related to that corresponding to the left-hand pole. We normally require that an element be assigned to one, and only one pole, so that if LHP E is true then RHP E must be false, and vice versa. Hence, LHP E is essentially the logical negation of RHP E. For the current discussion we shall accept that this relation exists as a constraint between the two predicates corresponding to the two poles. However, it is not an essential one for the theory and we shall discuss later the possibility of relaxing it and the consequences of doing so. For this reason we shall carry out most of the discussion in terms of the left-hand poles and associated predicates primarily, noting occasionally the corresponding phenomena for right-hand poles.

First let us examine the previous relation of equivalence between constructs in logical terms. We can define two logical propositions to be equal if their truth-values are the same, and this also corresponds to their numerical truth-values being equal, e.g.

$$\text{LHPm E} = \text{LHPn E} \qquad (1)$$

We can define two logical propositions involving the same free variable as being equivalent if they are equal for all values of that free variable, e.g.

$$\forall \text{ E} \quad \text{LHPm E} = \text{LHPn E} \qquad (2)$$

and it is then convenient to drop the variable and write:

$$\text{LHPm} \equiv \text{LHPn} \qquad (3)$$

Now this equivalence between the poles of constructs clearly coincides with our previously discussed equivalence in terms of distance. If two propositions are logically equivalent in this way then the vectors of truth-values against elements are the same and hence they are at zero distance apart. The converse may also be shown for any proper distance measures.

However, in terms of logical relations equality is only one of many possible relations. There are six binary logical operators between propositions that establish relations between them. Two of these relations are symmetrical and correspond to the two propositions being equal, or to one being equal to the negation of the other. This corresponds to the reversal or reflection of constructs discussed above. The other four operators are forms of implication between propositions, that one proposition being true implies that the other is also true. The four forms arise because of the possibilities of negation, that one being true implies the other is not and so on. They may all be derived from the one operator, $\supset$, where:

$$\text{LHPm E} \supset \text{LHPn E} \tag{4}$$

means that the assignment of element E to the left-hand pole of construct m implies that it is also assigned to the left-hand pole of construct n.

In contrast to the equality relation, the implication relation is asymmetric. If we assert the implication given in (4) then we are only constraining the truth-value of LHPn E if LHPm E is true. If this is not so and element E is not assigned to the left-hand pole of construct m then we are saying nothing about its assignment to the left-hand pole of construct n. This contrasts to the equality relation asserted in (1) where the proposition LHPm E being false also leads to LHPn E being false in order to satisfy the equality.

One important property of the implication relation is its transitivity. From the way in which we have defined it we can see that if, as well as (4), we have:

$$\text{LHPn E} \supset \text{LHPo E} \tag{5}$$

then we can derive:

$$\text{LHPm E} \supset \text{LHPo E} \tag{6}$$

This is the normal transitivity of an implication relation in a logical calculus.

Asserting mutual implication between two propositions allows us to derive their equality. Thus adding the converse asymmetric assertion:

$$\text{LHPn E} \supset \text{LHPm E} \tag{7}$$

to that of (4) does enable us to derive (1). From this we can see that the relation of implication is a weaker one than that of equality but closely related to it in that if we know the four implication relations between two propositons we may infer the two equivalence relations between them. These results from elementary propositional logic show that it is of interest to consider the implication relation in repertory grid analysis since the equality and equivalence relations normally analysed may be derived from it but not vice versa.

In the same way that we moved from the relation of equality between individual propositions in (1) to that of universal equivalence between them in (2), we may say that one proposition involving a free variable <u>entails</u> another proposition involving the same variable if it has an implication relation with it for all values of the free variable, e.g.:

$$\forall \text{ E} \quad \text{LHPm E} \supset \text{LHPn E} \tag{8}$$

and it is then convenient to drop the variable and write:

$$\text{LHPm} \rightarrow \text{LHPn} \tag{9}$$

We will read this as "the left hand pole of construct m entails the left hand pole of construct n". Clearly entailment, being derived from implication, is also asymmetric, and mutual entailment gives us equivalence in the same way as mutual implication gives us equality. Thus adding the converse entailment to (9):

$$\text{LHPn} \rightarrow \text{LHPm} \tag{10}$$

to (9) itself allows us to derive the equivalence of (3). Note similarly that the entailment relation is transitive like the implication relation so that from (9) and:

$$\text{LHPn} \rightarrow \text{LHPo} \tag{11}$$

we may derive:

$$\text{LHPm} \rightarrow \text{LHPo} \tag{12}$$

We have linked the discussion of this section to personal construct theory. However we note that most of our definitions come direct from classical logic and are independent of personal construct theory. The formal mechanisms for defining entailment are rather more complex than those used here because the logic of entailment in concerned to avoid certain paradoxical results (Anderson & Belnap 1975). The nature of these paradoxes does have some interest in personal construct theory because they are to do with <u>relevance</u> in entailment - does one proposition entail another in a relevant way or just through an artefact of the logical calculus ? Similar, but deeper questions arise when we consider the derivation of entailment from repertory grid data - is one construct relevant to another in the way in which it entails it or is the derived relation a fortuitous one ?

It is also worth noting that our definitions of equivalence and entailment are also related to those in <u>modal</u> logics (Snyder 1972). We can regard (2) and (5) as being definitions of <u>necessary</u> equality and <u>necessary</u> implication in a quantification model of a modal logic. In the context of personal constructs we can see this best by noting that two verbal interpretations of (5) are acceptable: "when you assign an element to the left-hand pole of construct m you <u>always</u> also assign it to the left-hand pole of construct m", or "when you assign an element to the left-hand pole of construct n you <u>necessarily</u> assign it to the left-hand pole of construct m". These links may be formalized through a <u>possible worlds</u> (Bradley & Swartz 1979) model of modal expressions by noting that each element provides a possible world for construing. Entailments according to our definition then become logical implications that are true for all possible worlds currently under consideration. This is a useful and evocative viewpoint because it links personal construct theory with the linguistic semantics of counterfactuals and presuppositions (Lewis 1973) which is very relevant to Kelly's concept of constructs being "used for predictions of things to come". It also provides useful technical links into the formal mechanisms for treating the topological structure of possible worlds and its role in logic and semantics which seem equally applicable to personal construct theory.

To conclude the rather abstract discussion of this section and lead into the more concrete operational implementation of the next it is worth considering a specific example of what we mean by entailment, its asymmetry, and the derivation of equivalence from entailment but not vice versa. The poles of two constructs may be quite distinct in terms of equivalence yet closely related in terms of entailment. For example suppose that in construing people someone uses the two constructs m:runs---doesn't run and n:energetic---passive, then we might well expect to find that LHPm entails LHPn but that LHPn does not entail LHPm, that is that being a runner entails being energetic but being energetic does not entail being a runner. If we analyse such a construct structure in terms of distance measures and hence of equivalence only then we shall not derive such asymmetrical relations between constructs even though they are clearly of equal validity and interest.

3 ENTAIL: A PROGRAM TO DERIVE ENTAILMENTS BETWEEN CONSTRUCTS

It is simple to derive the entailment structure between the poles of constructs. We only have to check the truth of the four possible implications for all elements. Thus LHPm entails LHPn is checked by noting whether whenever an element is assigned to the left-hand pole of m it is also assigned to the left-hand pole of n. If so, then the entailment relation holds true, otherwise it is false. Clearly, as we noted above, it would also suffice to check that whenever an element is not assigned to the left-hand pole of n it is also not assigned to the left-hand pole of m. We call the program that performs this analysis ENTAIL (Entailment Nets Through Analysing Implicational Links). Note that the inference from a particular set of elements that one pole of construct m entails one pole of construct n is an inductive one if we assume that it applies to other elements in addition to those used in its derivation.

Figure 2 shows the entailments between the poles of the constructs derived by ENTAIL from the grid of Figure 1 and drawn out as a directed graph. There are effectively two main sub-graphs which are mirror images of one another plus two isolated poles. One of the graphs shows the entailments for one set of poles, and the other the entailments for the opposite poles. Because of the essential bipolarity assumed in the elicitation of the grid the two graphs are essentially the same with the arrows and poles reversed in one relative to the other. In section 5 we discuss extensions to the form of grids which would result in such pairs of graphs not necessarily having such a simple relation.

Note that we have taken advantage of the transitivity of the entailment relation not to draw in all the arrows strictly necessary. Thus we have not drawn an arrow from LHP2 to RHP7, RHP5, RHP4, and LHP3 because there is an arrow from LHP2 to RHP8 and then one from RHP8 to RHP7, RHP7 to RHP5, and so on. We can see from the figure that LHP2 entails RHP8, RHP7, RHP5, RHP4 and LHP3 by tracing through the graph. Note that the equivalence between LHP3 and RHP4 now shows up as mutual entailment.

However, there is additional information in Figure 2 that goes beyond that available from other methods of analysis. This comes from the directed nature of the entailment links shown. There is equivalence only between LHP3 and RHP4 - all the other relations are one way only. The asymmetry of the entailment relation may be seen by considering that from RHP4 to RHP5 for example. We see from the descriptions of the poles that Jane is saying that any one of her acquaintances who is <u>artistic</u> and so on is also <u>creative</u> and so on. However, the converse does not hold.

From the element data in Figure 2 we can see the reason for this asymmetry. For example, from the elements assigned to RHP4 and RHP5 we can see that the entailment between them not being mutual is due to Jane's acquaintance element 10 being termed <u>creative</u> but not <u>artistic</u>. In this case only one element breaks down the equivalence. If we consider the entailment from LHP8 to LHP5, that her acquaintances who are <u>musical</u> are also <u>artistic</u> then the converse <u>is not</u> true of two acquaintances, elements 9 and 10. And if we consider the entailment from LHP2 to RHP5, that being <u>individualistic</u> entails being <u>creative</u> then the converse is not true for elements 7, 9 and 10.

When we evaluate a graph of entailments such as that shown in Figure 2, we are noting not only the arrows which are present but also those which are absent. There is an entailment from LHP5 to LHP4 but not one from LHP4 to LHP5. Therefore LHP4 is not equivalent to LHP5. There is an aymmetric relation between the two predicates which may be due to a variety of interesting phenomena. We are beginning to <u>interpret</u> the grid through the analysis produced by ENTAIL. However, how sure are we that entailments not shown are actually missing ? How 'near' to being equivalent are the two predicates ? Gaines and Shaw (1980) examine an approach to answering such questions through interaction with the person from whom the grid was elicited. Here we consider only the mathematical analysis of the actual grid data.

One possible approach to the 'strength' of entailment is to relate it to <u>conditional probability</u> measures. Such a measure is useful in giving more detail to the entailment analysis. However it does not satisfy our criterion of providing an analysis intepretable at the same level as the data - the measure itself introduces a new construction which will not be inherently meaningful to the person who generated the grid. An alternative approach to the grading of entailment has been given in Shaw & Gaines (1980)

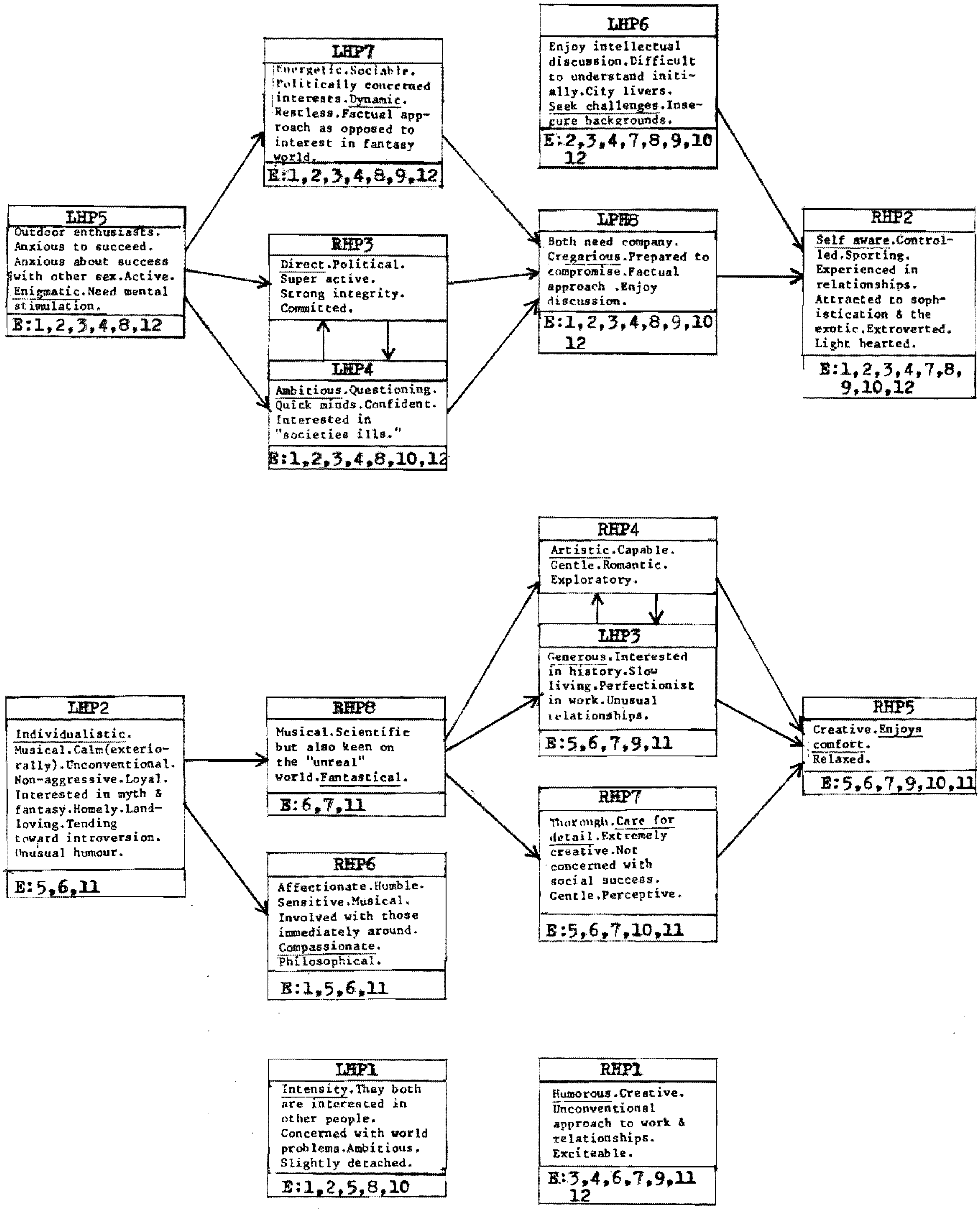

Figure 2 ENTAIL Analysis of Jane's Grid

where we introduce the predicate usually in the analysis performed by ENTAIL. This predicate is a quantifier similar in nature to the "for all" used in defining entailment in (8), but qualified to allow for some disconfirming instances so that it may be read as "for all but N cases" where N is some small number such as 1 or 2. Such a quantifier allows a natural grading of entailment in terms that are immediately meaningful to the originator of the grid: "when you say someone runs you always also say they are energetic and when you say someone is energetic you usually also say they run". Use of the quantifier usually to give a graded analysis gives a structure similar to the connectivity levels coming from Atkin's (1974) Q-Analysis. It is also readily extended to the multilevel case where rating scales rather than binary assignments are used in eliciting a grid.

ENTAIL has facilities for calculating entailments under the quantifier usually. If we apply it to Jane's grid then it condenses the construct structures shown in Figure 2 into just: an equivalence between LHP2, LHP3, RHP4, RHP5, RHP6, RHP7 and RHP8; a similar equivalence between the opposite poles to these; LHP1; and RHP1. With more complex grids, however, we have found the use of graded entailment through such a predicate an important feature of the analysis.

4 EXTENDING ENTAILMENT TO RATING SCALES - FUZZY SEMANTICS

So far in this paper we have analysed grids with binary assignments of elements to poles using a classical logic with two truth values. In this section we show how the logical analysis extends to the multivalued logics (Rescher 1969) with which one can analyse grids based on rating scales. Kelly (1955) presented constructs as binary categories and based his own methodology for eliciting constructs on this. However, other workers found the need for 'shades of grey' between the two poles of a construct and in a later work Kelly notes that this is consistent with his notion of a construct:

> "The construct, of itself, is the kind of contrast one perceives... while constructs do not represent or symbolize events, they do enable us to cope with events, which is a statement of a quite different order...They also enable us to put events into arrays or scales, if we wish." (Kelly 1970, pp.13-14)

It is common in many practical applications of repertory grids to use an N-point scale with 1 being an assignmment to the left-hand pole and N being an assignment to the right-hand pole, and intermediate numbers representing some form of 'intermediate' assignment. N is usually odd, 5 or 7, to allow a 'neutral' mid-point to the scale.

The semantics of such rating scales presents a number of problems in their own right. Kelly's original binary assignments may be interpreted as the truth or falsity of predicates. Intermediate points on a rating scale are not so readily, or uniquely, interpretable. For example, the 'neutral' point 3 on a 1 to 5 scale say may be interpreted as "this element lies half way between the poles", or as "this element should be assigned to both poles", or as "neither pole is appliable to this element", or "sometimes this element comes under one pole and sometimes another", or "I am not sure what pole to put this under", or "I do not wish to construe this element in this way", and so on. In logical terms we are attempting to use a single truth value to encompass many different modalities (White 1975).

The extension of binary distinctions to multi-valued ones may be treated at a fundamental level. We have already noted in section 1 the close relation of Kelly's constructive alternativism to Spencer Brown's "calculus of distinctions". Varela (1979) has shown how Brown's calculus may be extended to the multi-valued case. Within a basic bipolar distinction may be interpolated others through logical operations that correspond to expressions that generate paradoxes of self-reference in classical logic. Varela (1975) shows that an essentially three-valued logical calculus arises from the use of a single self-referential form in Brown's calculus of distinctions. Gaines (1976) shows how such "primitive paradoxes" may be iterated to give an indefinite number of distinctions between the poles of the distinction originally made, and hence how the truth value of an arbitrary proposition may be approximated to any accuracy on a continuous scale through a Dedekind section.

This move from a binary basis for making distinctions to a multivalued one raises problems of a semantic nature even at a fundamental level, particularly those of interpreting intermediate "truth-values" (Haack 1979). However, the need for rating scales in practice, and an appropriate underlying theory, does seem an essential one in terms of the human construct systems and their logic. In the physical sciences the expected and preferred source system in which to represent data is quantitative. We use a source system of physical quantities and their precise measurement. However, the underlying constructs of physics have been derived and refined over a very long period and are themselves of a peculiar, and perhaps unique, nature. The existence of continuous and limitless scales for physical variables of length, time, mass, charge and so on, is an important phenomenon that marks out the constructs involved as being different from those in many other sciences.

The existence of refined measuring schemes for some constructs should not blind us to their close relationships to other constructs for which no such physical measurement exists, for example, the concepts of 'tallness' and 'beauty' (Gaines 1976). The concept, the perception, of 'tallness' exists in a more primitive sense than does the measurement of 'height'. We are able to generate and follow arguments involving 'tallness' without having any concept of inches, centimetres, or any other metric scales. Whilst a 'scientific' analysis might conclude that there is a wide and ill-defined range of physical phenomena that combine in an extremely complex fashion to produce the subjective impression of 'beauty', in everyday reasoning it is as primitive a term as 'tallness'. We certainly do not distinguish between them in arguments such as:

He likes girls that are tall and beautiful.
Mary is not very tall but very beautiful.
He will probably like Mary.

Such considerations led Zadeh (1965) to develop a theory of fuzzy sets that closely paralleled that of classical set theory but allowed for 'shades of grey' in set membership. He extended the definition of the characteristic function of a set to include not just the binary values 0 and 1 but also the continuous interval between them. In classical set theory the characteristic function of a subset maps the elements of the universal set into 1 if they belong to the subset and into 0 if they do not. Zadeh allowed the elements to take the values in between also and called them degrees of membership to the subset. He showed that it was possible to extend the normal set-theoretic operations such as union, intersection and complementation, in a simple and natural way to fuzzy sets with continuous characteristic functions.

Since Zadeh's original study there has been a massive growth in the literature on fuzzy sets and their application to system theory, control engineering, psychological modelling, linguistics, and so on (Gaines & Kohout 1977). The related logical calculus derived from fuzzy set theory in the same way that the classical predicate calculus may be related to conventional set theory is of particular interest for this paper and has been presented as a system for fuzzy reasoning. This logic turns out to be one already studied by the Polish logic Lukasiewicz (Rescher 1969) and of particular importance since White (1979) has shown recently that it avoids paradoxes such as that of Russell's "barber" (Hughes & Brecht 1976) which arise from the unrestricted use of the axiom of comprehension in naive set theory. Since its inception fuzzy set theory has been used to model human verbal reasoning and concept processing. Goguen (1974) takes a formal axiomatic approach to the notion of a 'concept' in natural and artificial languages and shows within a very general category-theoretic framework that one obtains generalized fuzzy sets.

These considerations led Shaw and Gaines (1979, 1980) to propose a fuzzy set semantics for personal constructs that could deal with the analysis of entailment in repertory grids using rating scales. In this paper the fuzzy sets and logic have been left deliberately until this late section so that they do not confuse the basic discussion of systems of entailment and their derivation from grid data. Suppose in the logical analysis in section 2 one now assumes that the predicates LHP and RHP are not just true or false, but also have the possibility of intermediate degrees of membership to being true (with false intepreted as a degree of membership of 0 to being true). Then the rest of the discussion of that section follows virtually without change but one now has a model of entailment in grids whose values are not binary. The implication and entailment operations are now those of Lukasiewicz multivalued logic and entailment holding between two poles is now not just true or false but can also take intermediate values.

The program ENTAIL described in section 3 has been written to take into account such multivalued data. The discussions of other sections also generalize immediately to multivalued data and logics. Clearly the logic system itself now provides another measure of the 'strength' of an entailment and we can see that what is discussed in section 3 differs from this in measuring the strength to which the entailment is verified as being present. Since Lukasiewicz logic defaults back to the standard propositional and predicate calculi when intermediate values are not used it is actually more convenient to develop the whole of the theory of construct structures and analysis described here directly in terms of fuzzy logic and this would seem appropriate for future studies.

One important feature of Zadeh's work has been its emphasis on the linguistic nature of human reasoning and the use of fuzzy set theory to model the use of hedges such as very and rather in human reasoning. This is similar to the interpretation of the points on a rating scale in terms of such hedges as very, slight and quitely used in semantic differential techniques (Osgood, Suci & Tannenbaum 1957). Thus there are natural verbal interpretations of the rating scale when values are input and these may also be applied to the equivalent values resulting from the ENTAIL anlysis. One may say that there is a "quite strong" or a "very strong" entailment from one pole to another. Our requirement that the terminology and concepts of the analysis be those of the data thus continue to be satisfied in the extension to multivalued logics.

5 NEGATION, OPPOSITES AND RELEVANCE

We have previously noted that the role of the two poles of a construct as opposites has not been adequately treated. In our logical analysis the left-hand pole and the right-hand pole have been treated as distinct predicates of equal status. We have noted that the conventional elicitation of constructs leads to an inverse relation between the poles such that the predicate corresponding to one pole behaves as the logical negation of that corresponding to the other. This should perturb us since it appears to lead to precisely those defects of formal logic that Kelly warns against:

> "Now conventional logic would say that black and white should be treated as separate concepts. Moreover, it would say that the opposite of black can only be stated as not black, and the opposite of white can only be stated as not white. Thus the person whose field we mentioned would have shoes which would be just as much not white as the time of day, and he would write on paper which would be just as not black as the distance to his office." (Kelly 1955 p.106)

Part of the problem that Kelly is discussing here is one of relevance. "Not white" is a predicate relevant to shoes but not to the time of day. The standard predicate calculus fails to distinguish between "not" and "not relevant". We noted in section 1 that it is only in recent years that logics accounting for 'relevance' in a very formal sense have been established (Anderson and Belnap 1975). However, what even such logics do not encompass and Kelly brings out is the psychological role of the concept of opposite which has no logical counterpart - it is related to negation but not identical to it.

This introduction of the importance of modelling the role of opposites in human thinking is not peculiar to Kelly but is a continuing theme in philosophy from early times. The Pythagoreans used a table of opposites in analysing entities with ten constructs such as "limited---unlimited" and "good---evil". Mao Tsetung (1937) in his essay "On Contradiction" emphasizes the essential interdependence of opposites:

> "no contradictory aspect can exist in isolation. Without its opposite aspect, each loses the condition for its existence...Without life, there would be no death; without death there would be no life. Without above there would be no below; without below there would be no above...It is so with all opposites; in given conditions, on the one hand they are opposed to each other, and on the other hand they are interconnected, interpenetrating, interpermeating and interdependent" (Mao 1937 p.61)

Mao also brings in the notion of relevance in defining opposites and uses the notions of contradiction yet identity amongst opposites in his exposition of an epistemology which closely mirrors Kelly's constructive alternativism.

This line of reasoning can be traced back through Lenin (1914) to Hegel whose basic logic of thesis and antithesis leading to a synthesis is founded on what seems to be the most careful distinction between opposite and negation in the philosophical literature. Hegel distinguishes between negation as an absolute difference and opposition as an essential difference, and Bogomolov (1977) singles this out as the foundation of dialectical logic:

> "the investigation of the relation of two objects...begins with establishing the difference between them, expressed in the most general form, with their mutual negation (A and -A). To put it differently the second object acts initially as the simple negation of the first and is naturally expressed in logic by its indefinite negation...Describing this kind of development of the concept, Hegel saw in it the transition from absolute difference to essential difference (variety), and from this to opposition (antithesis), as one of the stages of the general path from identity through difference to contradiction." (Bogomolov 1977 p.137)

Thus we may see that Hegel's dialectics is crucially dependent on the transition from the concept of general negation to that of opposition. An opposite is some basis for there being negation, some reason for it, and it is the underlying construct to which this opposition is relevant that Hegel regards as the "synthesis" of the opposition between thesis and antithesis. Thus there is a close relationship between the epistemology put forward by Kelly and that put forward by previous philosophers concerned with dialectics. However, neither Pythagoreans nor Hegelians justify in logical terms their assertion that opposites are fundamental to reasoning. Kelly does not himself do so except by quotations like that at the beginning of this section which point out by example the difference between the negation of a construct and an opposite to it. Indeed one may argue from the presentation so far of a classical logical analysis of the repertory grid that in its original form it has already lost the possibility of coping with either relevance or the distinction between negation and opposition.

If we start with essentially bipolar constructs such that an element must be assigned to one, and only one, pole then we cannot treat relevance within a uniform framework. Kelly has to introduce it separarately in terms of constructs having a "range of convenience". However, by considering an element to have quite distinct assignments to the two poles of a construct, i.e. to a construct and its "opposite" we can also capture the concept of relevance. A construct is irrelevant to an element if the element is assigned to neither of its poles (or, in the context of fuzzy logic, if its degree of membership to both poles is zero). Thus, in terms of Kelly's example at the beginning of this section the construct "white---black" is irrelevant to the time of day because it is both not white and not black. Those who extended his bipolar notion to allow for multipoint rating scales also failed to allow for relevance when they made the scales a 1-dimensional interpolation between the two poles of a construct. However, the approach taken here is readily extended to the multipoint case by allowing separate ratings on the two poles of the construct. It is clearly debatable still whether this explication of relevance captures all its psychological connotations. We would suggest only that it captures some key ones.

What we have proposed is a very simple extension of Kelly's repertory grid methodology that gives us a logic capable of dealing with relevance and Kelly's notion of a "range of convenience. The mechanism used is crucially dependent of every predicate having an "opposite" so that one can distinguish between the predicate being not true for an element (element assigned to opposite predicate) and its being not relevant for the element (element assigned to neither predicate not opposite). This demonstrates the importance of the concept of an "opposite" emphasized by so many different philosophers and gives a formal model for the utility of opposites. In previous papers we have analysed the semantics of opposite predicates and developed various logical constraints upon them (Shaw & Gaines 1979, 1980). However, in the present context of repertory grid analysis an opposite predicate is just whatever the person from whom the grid is elicited chooses it to be. The ENTAIL analysis will cope with assignments to the two poles of a construct that are completely unconstrained in their mutual relationships.

The possibility of making separate assignments to the two poles of a construct and of analysing such extended forms of the repertory grid seems significant for a number of applications already noted in the literature. Slater (1977 p.46) points out that missing data creates major problems for distance-based grid analysis, and yet it is a common problem. Kelly states:

> "The assumption which is specific to a grid form of the test is that all the figures fall within the range of convenience of the constructs...This may not be a good assumption in all cases; it may be that the client has left a void at a certain intersect simply because the construct does not seem to apply one way or the other." (Kelly 1955 p.271)

Landfield (1976 p.97) gives an example of a grid elicited from a patient which goes beyond this and allows the two additional values "N" for neither pole applicable and "?" for either pole applicable. In terms of our discussion above his "N" corresponds to an assignment of false to both poles and his "?" corresponds to an assignment of true to both poles. Thus the grid he elicits is readily analysed by ENTAIL. Obviously when ENTAIL analyses a particular entailment between a pair of poles under these circumstances it is relative to the elements actually construed in relation to those poles. However, it is possible to provide an analysis which does draw as much as possible out of the data given and does not crucially depend on all elements being assigned to one pole of every construct.

It is interesting to note that the logic being used by ENTAIL to deal with Landfield's four "truth values" is precisely that proposed by Belnap (1976) to deal with the epistemology of database systems. He proposes to deal with both missing and contradictory information in a database by allowing four values: Told True; Told False; Not Told; Told True and Told False. Gaines (1979) shows that such a logical structure also avoids the possibility of paradoxes such as that of Russell's barber arising through the imposition of semantic constraints on a database, and suggests the extension of the logic to continuous values in order to avoid deeper paradoxes. Again in this one can see the significance of the separate treatment of the opposite of a predicate in establishing a logic that is pragmatically sound.

6 CONCLUSION

We have previously argued (Shaw and Gaines, 1979) the importance of computer interaction in the elicitation of personal construct structures; interactive programs for personal construct elicitation add a completely new dimension to the process. By reflecting back to the user a continuous analysis of the structure underlying his construct systems such programs can establish a dialogue with the self. Such a self-reflecting conversation over a subject domain has many important features:

(1) it avoids the inter-personal interactions and artefacts of construct elicitation by people; (2) it can also avoid the possible false inferences generated by non-interactive computer analyses; (3) it enables the user to explore his construct system at many levels; (4) the user control of the whole process can direct the dialogue into specific areas of interest as they become apparent.

This paper has detailed the features required of the next generation of programs for the conversational exploration of personal construct systems, and in particular the techniques for inferring the entailment structure between constructs and validating these by discussion with the user.

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