Toward a Better Understanding of Complexity

Definitions of Complexity, Cellular Automata as Models of Complexity, Random Boolean Networks

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Complexity

Definitions
Definitions of Complexity

- **Information**
  - The capability of a system to “surprise” an observer, i.e. to provide information.

- **Time-Computational Complexity**
  - The time an algorithm needs to solve a problem.

- **Space-Computational Complexity**
  - The amount of memory an algorithm needs to solve a problem.
Definitions of Complexity (2)

- **Effective Complexity**
  - The degree of order (instead of randomness) of a system.

- **Entropy**
  - The complexity of a system is equal to the thermodynamical measure of its disorder.
Definitions of Complexity (3)

✦ Fractal Dimension
  – The “fuzziness” of a system, measuring the degrees of details a system reveals on arbitrary scales.

✦ Hierarchical Complexity
  – The diversity of different layers, which a hierarchical system is composed of.

✦ Mutual Information
  – The degree to which a part of a system has information about other involved system constituents.
Definitions of Complexity (4)

- **Information Distance**
  - The differences among parts of a system.

- **Grammatical Complexity**
  - The degree of universality a language must have to describe a system (regular, context-free, context-sensitive, universal Turing machine)

- …
Cellular Automata

Global Effects from Local Rules
Cellular Automata

- The CA space is a lattice of cells with a particular geometry.
- Each cell contains a variable from a limited range (e.g., 0 and 1).
- All cells update synchronously.
- All cells use the same updating rule, depending only on local relations.
- Time advances in discrete steps.
One-dimensional finite CA architecture

- $K = 5$ local connections per cell
- Synchronous update in discrete time steps

Cellular Automata: Local Rules — Global Effects
Time Evolution of the $i$th Cell

\[ C_i^{(t+1)} = f(C_i^{(t)}_{i-[K/2]}, \ldots, C_i^{(t)}, C_i^{(t)}_{i+1}, \ldots, C_i^{(t)}_{i+[K/2]}) \]

With periodic boundary conditions:

\[ x < 1: C_x = C_{N+x} \]
\[ x > N: C_x = C_x \boxminus N \]
Value Range and Update Rules

- For $V$ different states (= values) per cell there are $V^K$ permutations of values in a neighbourhood of size $K$.
- The update function $f$ can be implemented as a lookup table with $V^K$ entries, giving $V^{V^K}$ possible rules.
Example Update Rule

- $V = 2, K = 3$
- The rule table for rule 30:

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CA Demos

*Evolvica CA Notebooks*
Cellular Automata

Models of Complexity
(Stephen Wolfram Approach)
Analysis of Cellular Automata

✧ … as Discrete Dynamical Systems

– Discrete idealization of partial differential equations
– Set of possible (infinite) CA configurations forms a Cantor set
– CA evolution may be viewed as a continuous mapping on this Cantor set
– Entropies, fractal dimensions, Lyapunov exponents
Analysis of Cellular Automata

✧ ... as Information-Processing Systems

- Parallel-processing computer with a simple grid architecture.
- Initial configuration is processed by the evolution of the cellular automaton.
- What types of formal languages are generated?
Four Classes of Patterns

- Wolfram classifies CAs according to the patterns they evolve:
  - 1. Pattern disappears with time.
  - 2. Pattern evolves to a fixed finite size.
  - 3. Pattern grows indefinitely at a fixed speed.
  - 4. Pattern grows and contracts irregularly.

   - 3/text.html: Fig. 1
Four Qualitative Classes

- 1. Spatially homogeneous state
- 2. Sequence of simple stable or periodic structures
- 3. Chaotic aperiodic behaviour
- 4. Complicated localized structures, some propagating

- 85-cellular/7/text.html: Fig. 3 (first row)
Classes from an **Information Propagation** Perspective

- ✦ 1. No change in final state
- ✦ 2. Changes only in a finite region
- ✦ 3. Changes over an ever-increasing region
- ✦ 4. Irregular changes
Degrees of Predictability for the Outcome of CA Evolution

- 1. Entirely predictable, independent of initial state
- 2. Local behavior predictable from local initial state
- 3. Behavior depends on an ever-increasing initial region
- 4. Behavior effectively unpredictable
Suggested Explorations:

- Natural CA-like Phenomena (CBN, 15.5)
- Stephen Wolfram’s vs. Chris Langton’s CA Classification (CBN, 15.2, 15.3)
- Conway’s Game of Life (CBN, 15.4)
- Self-Similarity and Fractal Geometry (CBN, Ch. 5)
- L-Systems and Fractal Growth (CBN, Ch. 6)
Suggested Explorations: (2)

✧ Fractals (CBN, Ch. 9):
  – Simplicity and Complexity

✧ Nonlinear Dynamics in Simple Maps
  – Logistic map, stability vs. instability, bifurcations, chaos (CBN, Ch. 10)

✧ Strange Attractors
  – Hénon-Lorenz attractor (CBN, Ch. 11)
Modeling Excitable Media

- Slime mold growth,
- star formation in spiral disk galaxies,
- cardiac tissue contraction,
- diffusion-reaction chemical systems and
- infectious disease epidemics

– would seem to be quite dissimilar systems.
– Yet, in each of these cases, various spatially distributed patterns, such as concentric and spiral wave patterns, are spontaneously formed.
Modeling Excitable Media

• The underlying cause of the formation of these self-organized, self-propagating structures is that these are excitable media, consisting of spatially distributed elements which can
  – become excited as a result of interacting with neighboring elements,
  – subsequently returning incrementally to the quiescent state
  – in which they are again receptive to being excited.

• The excited-refractory-receptive cycle that characterizes these systems can be modelled using multi-state cellular automata.
2-D CA: Emergent Pattern Formation in Excitable Media

Neuron excitation

Neuron excitation (relaxed)

Hodgepodge
Cellular Automata

Swarm Systems

Random Boolean Networks

Classifier Systems
Complex Systems

Emergent Behaviours and Patterns from Local Interactions
Stevens et al., 1988
What to Learn from Ant Colonies as Complex Systems

- Fairly simple units generate complicated global behaviour.

- “If we knew how an ant colony works, we might understand more about how all such systems work, from brains to ecosystems.” (Gordon, 1999)
Emergence in Complex Systems

✦ How do neurons respond to each other in a way that produces thoughts?
✦ How do cells respond to each other in a way that produces the distinct tissues of a growing embryo?
✦ How do species interact to produce predictable changes, over time, in ecological communities?
✦ ...
Complexity through Emergence
Examples of Swarm Systems ...
References