Cellular Automata

and beyond ... 

The World of Simple Programs

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Cellular Automata

Lindenmayer Systems

Random Boolean Networks

Classifier Systems
Cellular Automata

Global Effects from Local Rules
Cellular Automata

• The CA space is a lattice of cells (usually 1D, 2D, 3D) with a particular geometry.
• Each cell contains a variable from a limited range of values (e.g., 0 and 1).
• All cells update synchronously.
• All cells use the same updating rule (in uniform CA), depending only on local relations.
• Time advances in discrete steps.
One-dimensional Finite CA Architecture

- Neighbourhood size: 
  \[ K = 5 \]

  local connections per cell

- Synchronous update in discrete time steps

Time Evolution of Cell i with K-Neighbourhood

\[ C_i^{(t+1)} = f(C_i^{(t)} - \left\lfloor \frac{K}{2} \right\rfloor, ..., C_{i-1}^{(t)}, C_i^{(t)}, C_{i+1}^{(t)}, ..., C_{i+\left\lfloor \frac{K}{2} \right\rfloor}^{(t)}) \]

With periodic boundary conditions:

\[ x < 1 : C_x = C_{N+x} \]

\[ x > N : C_x = C_x - N \]
Value Range and Update Rules

- For \( V \) different states (= values) per cell there are \( V^K \) permutations of values in a neighbourhood of size \( K \).

- The update function \( f \) can be implemented as a lookup table with \( V^K \) entries, giving \( V^{V^K} \) possible rules.

\[
\begin{align*}
V^K &= \begin{cases}
00000: 1 & \ldots V \\
00001: \_ & \\
00010: \_ & \\
\vdots & \\
11110: \_ & \\
11111: \_ & 
\end{cases}
\end{align*}
\]

<table>
<thead>
<tr>
<th>( v )</th>
<th>( K )</th>
<th>( v^K )</th>
<th>( V^{v^K} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
<td>8</td>
<td>256</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>32</td>
<td>( 4.3 \times 10^9 )</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>128</td>
<td>( 3.4 \times 10^{38} )</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
<td>512</td>
<td>( 1.3 \times 10^{154} )</td>
</tr>
</tbody>
</table>
Cellular Automata: Local Rules — Global Effects

Demos
History of Cellular Automata

• Alternative names:
  – Tessellation automata
  – Cellular spaces
  – Iterative automata
  – Homogeneous structures
  – Universal spaces

• John von Neumann (1947)
  – Tries to develop abstract model of self-reproduction in biology (from investigations in cybernetics; Norbert Wiener)

• J. von Neumann & Stanislaw Ulam (1951)
  – 2D self-reproducing cellular automaton
  – 29 states per cell
  – Complicated rules
  – 200,000 cell configuration
  – (Details filled in by Arthur Burks in 1960s.)
History of Cellular Automata (2)

• Threads emerging from J. von Neumann’s work:
  – Self-reproducing automata (spacecraft!)
  – Mathematical studies of the essence of
    • Self-reproduction and
    • Universal computation.

• CAs as Parallel Computers (end of 1950s / 1960s)
  – Theorems about CAs (analogies to Turing machines) and their formal computational capabilities
  – Connecting CAs to mathematical discussions of dynamical systems (e.g., fluid dynamics, gases, multi-particle systems)

• 1D and 2D CAs used in electronic devices (1950s)
  – Digital image processing (with so-called cellular logic systems)
  – Optical character recognition
  – Microscopic particle counting
  – Noise removal
History of Cellular Automata (3)

• Stansilaw Ulam at Los Alamos Laboratories
  – 2D cellular automata to produce recursively defined geometrical objects (evolution from a single black cell)
  – Explorations of simple growth rules

• Specific types of Cas (1950s/60s)
  – 1D: optimization of circuits for arithmetic and other operations
  – 2D:
    • Neural networks with neuron cells arranged on a grid
    • Active media: reaction-diffusion processes

• John Horton Conway (1970s)
  – Game of Life (on a 2D grid)
  – Popularized by Martin Gardner: *Scientific American*
Stephen Wolfram’s World of CAs
Stephen Wolfram’s World of CAs

<table>
<thead>
<tr>
<th>1981 - 1983: Discoveries about cellular automata...</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Images of cellular automata and related papers](image1, image2, image3)</td>
</tr>
</tbody>
</table>
Stephen Wolfram's World of CAs
Stephen Wolfram’s World of CAs

2002 - NKS and Beyond...

TO BE CONTINUED...
Example Update Rule

- \( V = 2, K = 3 \)

- The rule table for rule 30:

<table>
<thead>
<tr>
<th></th>
<th>111</th>
<th>110</th>
<th>101</th>
<th>100</th>
<th>011</th>
<th>010</th>
<th>001</th>
<th>000</th>
</tr>
</thead>
<tbody>
<tr>
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</tr>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

\[
128 \quad 64 \quad 32 \quad 16 \quad 8 \quad 4 \quad 2 \quad 1
\]

\[
16 + 8 + 4 + 2 = 30
\]

See examples ...
CA Demos

- Evolvica CA Notebooks
Four Wolfram Classes of CA

- **Class 1:**
  A fixed, homogeneous, state is eventually reached (e.g., rules 0, 8, 128, 136, 160, 168).
Four Wolfram Classes of CA

- **Class 2:**
  
  A pattern consisting of separated periodic regions is produced (e.g., rules 4, 37, 56, 73).

**Images:**

- Rule 4: 
  - Even rows have a 1, odd rows have a 0.
- Rule 37: 
  - Periodic regions with varying complexity.
- Rule 56: 
  - Diagonal patterns with alternating colors.
- Rule 73: 
  - Complex patterns with vertical and horizontal bands.
Four Wolfram Classes of CA

- **Class 3:**
  A chaotic, aperiodic, pattern is produced (e.g., rules 18, 45, 105, 126).
Four Wolfram Classes of CA

- **Class 4:**
  Complex, localized structures are generated (e.g., rules 30, 110).
Class 4: Rule 30
Class 4: Rule 110
Further Classifications of CA Evolution

• Wolfram classifies CAs according to the patterns they evolve:
  – 1. Pattern disappears with time.
  – 2. Pattern evolves to a fixed finite size.
  – 3. Pattern grows indefinitely at a fixed speed.
  – 4. Pattern grows and contracts irregularly.

• Qualitative Classes
  – 1. Spatially homogeneous state
  – 2. Sequence of simple stable or periodic structures
  – 3. Chaotic aperiodic behaviour
  – 4. Complicated localized structures, some propagating
Further Classifications of CA Evolution (2)

- Classes from an Information Propagation Perspective
  - 1. No change in final state
  - 2. Changes only in a finite region
  - 3. Changes over an ever-increasing region
  - 4. Irregular changes

- Degrees of Predictability for the Outcome of the CA Evolution
  - 1. Entirely predictable, independent of initial state
  - 2. Local behavior predictable from local initial state
  - 3. Behavior depends on an ever-increasing initial region
  - 4. Behavior effectively unpredictable
Random Boolean Networks

Generalized Cellular Automata
Crystallization of Connected Webs

[S. Kauffman: At Home in the Universe]
Random Nets Demo
Random Network Architecture

Network at time $t$

Network at time $t+1$

wiring scheme
pseudo neighbourhood

Network at time $t$
Time Evolution of the i-th Cell

- Cell $i$ is connected to $K$ cells $w_{i1}, w_{i2}, \ldots, w_{iK}$; with $w_{ij}$ from $\{1, \ldots, N\}$.
- $N^K$ possible alternative wiring options.
- Update rule for cell $i$:

$$C^{(t+1)}_i = f_i(C^{(t)}_{w_{i1}}, C^{(t)}_{w_{i2}}, \ldots, C^{(t)}_{w_{iK}})$$
Wiring/Rule Schemes

• A random network of size $N$ with neighbourhood size $K$ can be assigned

$$S = (N^K)^N \times (V^{V^K})^N$$

alternative wiring and rule schemes.

• Example:
  $V = 2$, $N = 16$, $K = 15$; $S = 2^{832}$. 
States and Cycles

[S. Kauffman: Leben am Rande des Chaos]
Kauffman’s Random Boolean Networks

Boolean functions represented by shades of green. Stuart Kauffman used this network to investigate the interaction of proteins within living systems.

Binary values that have changed are white. Unchanged values are blue.

These networks settle very quickly into an oscillatory state.

http://members.rogers.com/fmobrien/experiments/boolean_net/BooleanNetworkApplet_both.html
Attractor Cycles

[A. Wuensche, Discrete Dynamics Lab]
Basin of Attraction Field

Nodes: $n = 13$; Connectivity: $k = 3$; States: $2^{13} = 8192$

[A. Wuensche, Discrete Dynamics Lab]
Basin of Attraction Field

Nodes: n = 13; Connectivity: k = 3; States: $2^{13} = 8192$

[A. Wuensche, Discrete Dynamics Lab]
Mutations on Random Boolean Networks

Figure 21: The basin of attraction field of (a) The RBN ($n=6$, $k=3$) as defined in the table (above), and (b) the RBN following a 1 bit mutation to one of its rules. Some differences in the fields are evident. The result of a 1 bit perturbation to a reference state of all 1s (rs) is indicated by its 1 bit mutants (m).

[A. Wuensche 98]
Attractor = Cell Type?

- From the set of all possible gene activation patterns, the regulatory network selects a specific sequence of activations over time.

- A differentiaed cell doesn’t change its type any more.
  - Hence, only a constrained set of genes is active
  - = state cycle
  - = attractor?
References

- Wuensche, A. Discrete Dynamics Lab: http://www.santafe.edu/~wuensch/ddlab.html