On the Performance of Successive Interference Cancellation in Random Access Networks

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***Abstract*—Successive Interference Cancellation (SIC) is a physical-layer technique that enables reception of multiple overlapping transmissions. While SIC has the potential to boost the network throughput, if the MAC protocol employed in the network is agnostic to such a capability at the physical-layer, the full potential of SIC can not be utilized in the network. There have been a number of studies to design new SIC-aware MAC protocols or adjust the existing protocols to exploit SIC. Despite that, the exact effect of MAC protocols on the throughput of SIC-enabled networks is unknown. In this paper, we propose a novel SIC-aware MAC protocol based on the disparity of user channels in a wireless network and analyze its performance. We consider a simple random access protocol with no SIC as the base configuration and compare it with three other configurations with different levels of SIC-awareness. We show that while a SIC-enabled physical layer without a SIC-aware MAC protocol can increase the throughput of the network by 1.5x, a specifically designed MAC protocol is far more efficient achieving up to 3.3x improvement in throughput. Our SIC-aware MAC protocol is fully distributed and hence subject to selfish behavior of users. Thus, we also consider the case where the users behave selfishly. We model our proposed protocol as a one-shot simultaneous move game and derive a mixed strategy Nash equilibrium. We also show that we can set the cost of packet transmission in such a way that we get the optimal system throughput at the Nash equilibrium.**

I. INTRODUCTION

In traditional *multiple access control (MAC)* protocols, the underlying assumption is that concurrent transmission of more than one packet at a time results in a collision, and all of the packets need to be re-transmitted. However, advancements in wireless networks have introduced new *multipacket reception (MPR)* techniques which enable reception of multiple overlapping packets transmitted concurrently. Due to the advent of MPR capability in wireless networks, traditional wireless MAC protocols with the *single-user-at-a-time* assumption are no longer optimal. This calls for the design of new wireless MAC protocols which are tailored for the MPR capability. In this paper, we specifically focus on random access MAC protocols designed for a special MPR technique known as successive interference cancellation.

Successive interference cancellation (SIC) is an MPR technique that uses the structured nature of interference to decode multiple overlapping transmissions [1], [2]. Assume a composite signal resulting from concurrent transmission of some packets is received at a receiver. Employing SIC, one of the overlapping packets (*e.g.*, the one corresponding to the strongest signal) is decoded first, considering the rest of the signals as noise. After the first packet is decoded, the receiver reconstructs the corresponding analog signal and removes it from the original composite signal. At this stage, the remainder of the composite signal is free from the interference of the first overlapping signal. The same technique is applied successively to decode the remaining signals. Since at each stage, the remaining signals are treated as noise, the maximum rate achievable by a user depends not only on its received signal power but also on the *order* in which the signals are decoded. For the overlapping signals to be decodable at the receiver, other than the order of the decoding, the transmission *rate* should also be carefully controlled [3].

SIC-enabled receivers are much simpler than other MPR-enabled hardwares [1]. This is because they use the same decoder to decode the composite received signal at different stages. Therefore, using SIC, neither a complicated decoder nor multiple antennas is required to enable MPR capability [3], [4]. It also makes SIC more practical than other MPR techniques such as joint detection [5]. In addition, it is known that other multiple access techniques such as CDMA and OFDMA are no more efficient than SIC [2, Ch. 6]. As a consequence, SIC has been recently considered in commercial wireless systems as a way to increase system throughput [6], [7].

There have been a number of studies in the area of SIC-aware MAC protocols. It has been shown that SIC can achieve near Shannon capacity in cellular networks in which communications are closed-loop and accurate channel estimations exist (see [1] and references therein). It is also known that SIC can improve the throughput of wireless LANs. For instance, Halperin *et al.* [4] experimentally studied the effect of SIC in unmanaged wireless networks with carrier sensing. They concluded that SIC can effectively improve the bandwidth utilization in wireless networks. In contrast, Sen *et al.* [8] reported that SIC may not be a promising approach to improve the throughput of wireless networks. This arises the question "By how much can SIC improve the throughput of wireless networks?". The question motivates us to perform a careful analytical comparison between different configurations of MAC and the physical layer, with and without SIC, to find out the effect of SIC on the performance of wireless networks.

In this paper, we consider a SIC-enabled receiver at the physical layer. We study a case in which the users in the network are split into two groups; one group with a high received power and the other group with a low received power.

We assume the decoder is capable of decoding at most two concurrent transmissions[1], one from a low-power user and the other one from a high-power user. However, transmission of more than one packet from each group will result in a failure in decoding all packets (see subsection III-B for more details). Based on this model, we propose a random access MAC protocol, called RAS-MAC (Random Access SIC-enabled MAC). We analyze the protocol in two different cases. In the first case we assume the users have perfect information about the number of users in each group (*i.e.*, high-power and low-power users). In this case, we show that the derived throughput formula has a unique optimal solution and the point is analytically computable. In the second case we assume the exact number of the users in each group is not known but they follow a Poisson distribution with a known average.

Our goal is to focus on the throughput improvements achieved by utilizing SIC in a network. Therefore, rather than considering a complicated MAC protocol, we opt for a simple random access protocol in order to develop a tractable analytical model that captures the effects of SIC rather than the complex dynamics of MAC/PHY interaction. Furthermore, we note that random access MAC protocols offer a simple and fully decentralized method to control access to a shared wireless channel. In these protocols each node transmits a packet with a specific probability in each time slot. Collisions are detected by missed acknowledges [9], [10].

With some simplifying assumptions, we compare the effect of different MAC protocols on the throughput of the network. We consider a simple random access MAC protocol with no SIC at the physical layer as the base configuration and compare with it three other cases. We show: (I) Having the same MAC protocol as the base model and replacing the physical layer with a SIC-enabled one increases the throughput by 1.5x. (II) Having a SIC-enabled physical layer and tuning the base model's MAC protocol gives a 1.6x increase in the network throughput. (III) Using our proposed protocol RAS-MAC on top of a SIC-enabled physical layer increases the throughput by 3.23x to 3.30x in compare to the base model.

Furthermore, we analyze the RAS-MAC protocol from a game theoretic perspective. We model the protocol as a one-shot simultaneous move game and find a symmetric mixed strategy Nash equilibrium for the game. We also show that one can adjust the costs of transmissions such that the equilibrium happens at the optimal system throughput. This is extremely important because it shows that we can specify the costs so that selfish behavior of the users result in the optimal system throughput.

The rest of the paper is organized as follows. Section II explains some of the related work. Section III presents the system model and describes the proposed MAC protocol. Section IV describes the effect of SIC-aware MAC protocol on the network throughput by comparing a number of configurations. Section V analyses the proposed protocol for the case that the exact number of users is not known. A game theoretic analysis of the proposed MAC protocol is explained in section VI. Section VII concludes the paper.

[1]Note that many practices such as [4], [8] consider only two levels of decoding for SIC.

II. Related Work

A large body of research in this area studies the MAC protocol design in the presence of *capture effect*. Capture effect is essentially the ability to decode the most powerful signal in the case of multiple concurrent transmissions. For instances of works studying the impact of the capture effect on the random access protocols, see [11]–[14]. Metzner first observed that splitting the users into two groups of high-power users and low-power users and using the capture effect can increase the maximum utilization of the slotted ALOHA protocol from 36.8 to about 53 percent.

Ghez *et al.*'s seminal work [15] is considered a landmark in the MPR research area. They studied the stability of slotted ALOHA based on a model known as the reception matrix. The reception matrix specifies the probabilities of k successful receptions out of n transmitted packets. Many of the later studies are based on the reception matrix model. For instance, Zhao and Tong's multiqueue [16] and dynamic queue [17] protocols use the same model to design a MAC protocol with MPR capability. While the reception matrix is a powerful model to abstract the high-level behavior of an MPR-enabled physical layer, it does not accurately capture the dynamics of a SIC-enabled physical layer such as variable rate transmission and power control. Thus, in this paper, we directly use the received signal-to-interference-plus-noise ratio to accurately capture the effect of SIC at the physical layer.

In [18], Wang and Li proposed the hybrid ALOHA protocol. In this protocol, different users transmit their training sequences at non-overlapping manner. The training sequences are then used to obtain a good estimation of the channel to decode the overlapping bits of the packets.

There are only a few works specifically considering SIC at the MAC layer. Other than the papers already mentioned in section I, in [7] Hou *et al.* described how to employ SIC in the EV-DO Rev A reverse link to achieve the multiple access channel sum rate capacity. They showed how interference cancellation can readily be applied to commercial EV-DO base stations without modifications to user terminals or standards. In [19] Yu and Giannakis proposed an algorithm called SICTA which uses SIC in a tree algorithm. SICTA retains the received signal vector resulting from a collision in a time slot. It then uses the received signal vector of the subsequently decoded packets to cancel the interference in the collided signal and decode it.

In the area of game theoretic modeling of random access networks, MacKenzie and Wicker's work [20] is the closest one to our paper. They first modeled the multipacket slotted ALOHA protocol and obtained the Nash equilibrium and stability region in the presence of selfish users. However, they used a reception matrix as their underlying model, which makes their work different from ours.

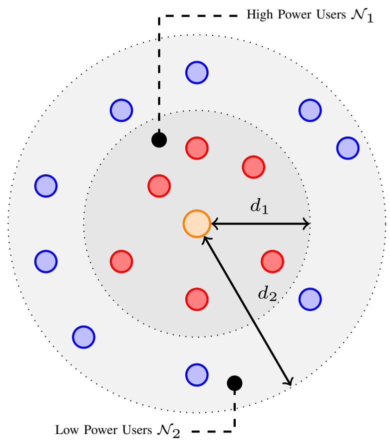


Fig. 1: The access point is placed at the center. Users in the range $(0, d_1)$ are considered high-power users. Users in the range (d_1, d_2) are considered low-power users.

III. System Model and Assumptions

A. Network Model and Notations

We consider a network consisting of an access point and n users scattered around it. The time is slotted and the size of the packets are selected so that the packet transmission time fits into a time slot. As depicted in Fig. 1, users are split into two groups; the set of high-power users $\mathcal{N}_1$ (within distance d_1) and the set of low-power users $\mathcal{N}_2$ (within distance d_2 but not d_1). In subsection III-B, we justify the rational behind splitting the users into two groups and further describe the details. Let $\mathcal{N}$ denote the set of all users, *i.e.*, $\mathcal{N} = \mathcal{N}_1 \cup \mathcal{N}_2$. The number of users in $\mathcal{N}$, $\mathcal{N}_1$ and $\mathcal{N}_2$ are represented by n, n_1 and n_2 respectively (*i.e.*, $n = n_1 + n_2$). We assume all nodes have infinite backlog. Therefore, in each time slot there are n users ready to transmit. We also denote by p_1, and p_2 the probability that a node in $\mathcal{N}_1$ and $\mathcal{N}_2$ transmits a packet. Note that we only consider symmetric cases in which all the nodes in a group transmit with equal probability. Let $\lambda_1 = p_1 n_1$ and $\lambda_2 = p_2 n_2$ show the expected number of transmissions from each group in each time slot. We use the '*' mark to denote the optimal value (in terms of system throughput) of a variable. For instance, while λ denotes the average number of packets in a time slot, λ^* indicates the optimal average number of packets which results in the optimal system throughput.

B. SIC Model

To use the full advantage of a SIC receiver, the *rates* at which the packets are transmitted and the *order* in which the signals are decoded as well as the *received powers* must be carefully specified [21]. In a random access network, in addition to the aforementioned parameters, the *probability* of transmission should also be determined. We first describe the relation between rate, order and received power in the presence of SIC. Then we explain our proposed protocol called RAS-MAC.

For simplicity, we limit SIC to only two levels of decoding. Note that due to SIC limitations such as error propagation and linear decoding time [2, Ch. 6], in practice, only a few levels of decoding are used [4], [8]. However, with some modifications, our model can be extended to more than two levels of decoding.

The achievable rate of a user depends on its received power as well as the decoding order. Assume the composite signal $S = S_1 + S_2 + Z$ is received by the access point wherein S_1, S_2 are overlapping signals received from two distinct users and Z is the noise signal at the access point. Let P_1, P_2, and N_0 denote the powers of S_1, S_2, and Z respectively. Suppose S_1 is decoded first. Therefore, by the Shannon capacity theorem, the achievable rate of the high-power user, denoted by r_1, is bounded by the following inequality,

$$r_1 \leq \log\left(1 + \frac{P_1}{P_2 + N_0}\right) \text{ bits/s/Hz}. \tag{1}$$

Assuming perfect removal of S_1 from S, the achievable rate of the low-power user, denoted by r_2, is bounded by,

$$r_2 \leq \log\left(1 + \frac{P_2}{N_0}\right) \text{ bits/s/Hz}. \tag{2}$$

From (1) and (2) it is obvious that swapping the order of the decoding changes the achievable rates. In addition, modification of P_1 and P_2 alters the rates too.

To overcome the complicated order/rate/power/probability selection problem, we could allow the users to choose arbitrary rates, powers, and transmission probabilities and try to decode the signals in an opportunistic manner from the most powerful to the least powerful signal (as was done in [3], [4]). However, as long as the order of decoding is not specified, the users are forced to choose conservatively low rates, otherwise their packets may not be decodable at all.

To put SIC into practice, RAS-MAC enforces a set of constraints on transmissions. It always decodes the stronger signal first and the weaker signal next. Optimal r_1 and r_2 are selected based on the available rates, and received powers P_1 and P_2 are controlled so that (1) and (2) are satisfied. Note that as long as (1) and (2) hold, our protocol works fine. Thus, r_1 and r_2 may be selected so that an exact power control is not required. In addition, we assume the following inequality holds,

$$\log\left(1 + \frac{P_1}{P + P_2 + N_0}\right) < r_1 \text{ bits/s/Hz}, \tag{3}$$

for all $P > \epsilon$ where $0 < \epsilon < min(P_1, P_2)$ is a constant. Inequality (3) means that the concurrent transmission of any additional signal other that S_2 results in failure to decode S_1. For any given P_1 and P_2, this ensures the rates are as high as possible. Additionally, failure to decode S_1 leads to failure to decode S_2 because in this case S_1 cannot be subtracted from the composite signal S. In other words, with the given conditions, we assume the received signal is decodable if and only if at most two concurrent transmissions, one from $\mathcal{N}_1$ and the other one from $\mathcal{N}_2$ have occurred.

Note that in practice, most of the transceivers support only a few different data rates [22]. Thus, choosing the best r_1 and

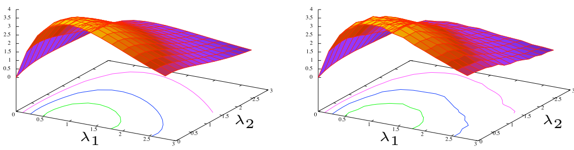


Fig. 2: Comparison of the throughput plot, equation (4) (left) vs Monte-Carlo simulation (right) for $r_1 = 10$, $r_2 = 1$ and $0 \leq \lambda_1, \lambda_2 \leq 3$. The model is very close to the simulation result.

r_2 can be simply done by enumerating over the available rates.

The probability of transmission in each group is chosen so that the sum throughput of the system is maximized. We study this problem in subsection IV-A.

IV. Effect of SIC-Aware MAC on Throughput

We compare a number of different PHY/MAC combinations to obtain some insight into the effect of a properly designed MAC protocol on the throughput of the network for a SIC-enabled physical layer.

A. Analysis of RAS-MAC

In this subsection we analyze the RAS-MAC protocol described in subsection III-B. Recall that we assume one node from $\mathcal{N}_1$ and one node from $\mathcal{N}_2$ can send their packets simultaneously by implementing SIC at the receiver. However, if more than one node from each group sends a packet, we cannot decode any of the packets from any of the groups. In addition, we assume packet transmissions in $\mathcal{N}_1$ and $\mathcal{N}_2$ follow a Poisson distribution[2] with averages λ_1 and λ_2. Considering this model, there are only three cases that give us a non-zero throughput:

1) One packet from $\mathcal{N}_1$ and one packet from $\mathcal{N}_2$ are sent. The probability of occurrence of this case is $\lambda_1 e^{-\lambda_1} \lambda_2 e^{-\lambda_2}$ and the achievable throughput is $r_1 + r_2$.
2) One packet from $\mathcal{N}_1$ and zero packet from $\mathcal{N}_2$ is sent. In this case the probability of occurrence is $\lambda_1 e^{-\lambda_1} e^{-\lambda_2}$ and achievable throughput equals r_1.
3) Zero packet from $\mathcal{N}_1$ and one packet from $\mathcal{N}_2$ is sent. In this case the probability of occurrence is $\lambda_2 e^{-\lambda_1} e^{-\lambda_2}$ and achievable throughput equals r_2.

Summing up the three cases and replacing λ_1 by $n_1 p_1$ and λ_2 by $n_2 p_2$, we can write the expected throughput of the network as follows,

$$\tau_{\text{RAS-MAC}}(p_1, p_2) = e^{-n_1 p_1 - n_2 p_2} \Big((r_1 + r_2)\, n_1 p_1 n_2 p_2 + r_1 n_1 p_1 + r_2 n_2 p_2 \Big) . \tag{4}$$

Equation (4) has a single optimal point which is (see Appendix 1 for the proof),

[2]This is a technical assumption to simplify the analysis. Other distributions can be incorporated albeit with more mathematical difficulties. We note that in the regime of large number of users, Poisson assumption provides a reasonable approximation for the average behavior of the system, *i.e.*, a mean-value analysis.

TABLE I: List of configurations used for comparison

	SIC-enabled	
Config. Name	**PHY**	**MAC**
PHY$^-$/MAC$^-$	No	No
PHY$^+$/MAC$^-$	Yes	No
PHY$^+$/MAC^{+1p}	Yes	Yes, uses the same probability p^* for all users
PHY$^+$/MAC^{+2p}	Yes	Yes, uses different probabilities p_1^* and p_2^* for users in high-power and low-power groups

$$\langle p_1^*, p_2^* \rangle = \langle \frac{1}{n_1} \frac{-(r_2 - r_1) + \sqrt{\Delta}}{2(r_1 + r_2)}, \frac{1}{n_2} \frac{(r_2 - r_1) + \sqrt{\Delta}}{2(r_1 + r_2)} \rangle , \tag{5}$$

where, $\Delta = (r_2 - r_1)^2 + 8 r_1 r_2$.

At this point, the value of the throughput function is given by,

$$\tau_{\text{RAS-MAC}}(p_1^*, p_2^*) = \frac{1}{2} e^{\frac{-\sqrt{\Delta}}{2(r_1+r_2)}} \left((r_1 + r_2) + \sqrt{\Delta} \right) . \tag{6}$$

In addition, for $r_1 \geq r_2$, it can be proved that $\lambda_2^* \leq \lambda_1^* \leq 1$ (see the appendix). Fig. 2 compares the throughput of the network for both Monte-Carlo simulation and equation (4) for $r_1 = 10$, $r_2 = 1$ and $0 \leq \lambda_1, \lambda_2 \leq 3$. It is observed that both plots are very close to each other, meaning that (4) indeed closely matches the real world scenario. We have performed extensive simulation experiments which consistently confirm this conclusion.

B. Considered System Configurations

In this subsection, we describe the MAC protocol plus physical layer configurations considered for comparison. We look at 4 different configurations of PHY/MAC. A simple random access protocol with no SIC at the physical layer is considered as the base configuration. The three other configurations are compared to the base one; they are all SIC-enabled at the physical layer, but they have different levels of SIC utilization at the MAC layer. Table I shows the list of configurations used for comparison. Detailed description of the configurations follows.

1) PHY$^-$/MAC$^-$: This is the simplest configuration used for comparison. In this configuration, the physical layer is not SIC-enabled; therefore, the MAC layer knows nothing about SIC as well (*i.e.*, it uses the classical *single-user-at-a-time* semantic). We use a simple ALOHA like [9] protocol at the MAC layer. However, in ALOHA, no rate differentiation is considered among the users, while our model considers two different rates. Given $n = n_1 + n_2$ and two rates r_1 and r_2, the expected network throughput using PHY$^-$/MAC$^-$ is given by $\frac{n_1 r_1 + n_2 r_2}{n_1 + n_2} np\, e^{-np}$, where $\frac{n_1 r_1 + n_2 r_2}{n_1 + n_2}$ is the average throughput conditioned on exactly one transmission in a time slot and, $np\, e^{-np}$ is the probability of transmission of exactly one packet in a time slot. The optimal throughput happens at $p^* = \frac{1}{n}$ and is equal to,

$$\tau_{\text{PHY}^-/\text{MAC}^-} = \frac{n_1 r_1 + n_2 r_2}{n_1 + n_2} \frac{1}{e} . \tag{7}$$

2) PHY$^+$/MAC$^-$: This configuration considers exactly the same MAC protocol as the one used in PHY$^-$/MAC$^-$, *i.e.*, a simple random access protocol with $p = \frac{1}{n}$. However, here we use a SIC-enabled physical layer. Since the MAC protocol is not SIC-aware, it can only use the SIC capability provided by the physical layer "opportunistically". Since $p = \frac{1}{n}$, and the physical layer is SIC-enabled, the throughput of PHY$^+$/MAC$^-$ is given by,

$$\tau_{\text{PHY}^+\text{/MAC}^-} = \tau_{\text{RAS-MAC}}(\frac{1}{n}, \frac{1}{n}), \tag{8}$$

where $\tau_{\text{RAS-MAC}}(.)$ is specified by (4).

3) PHY$^+$/MAC^{+1p}: We consider a SIC-enabled physical layer for this configuration. In addition, we assume a random access MAC protocol which is SIC-aware but assigns the same probability p to all users. Although PHY$^+$/MAC^{+1p} selects the best p, in general, it is not the optimal configuration. This is because a configuration which assigns different probabilities to the two groups of users, in the worst case, reduces to this configuration. The throughput of PHY$^+$/MAC^{+1p} is given by,

$$\tau_{\text{PHY}^+\text{/MAC}^{+1p}} = \tau_{\text{RAS-MAC}}(p^*, p^*), \tag{9}$$

where $\tau_{\text{RAS-MAC}}(.)$ is specified by (4). To obtain the value of p^* for (9), we need to find the positive root of the following equation,

$$\frac{\mathbf{d}}{\mathbf{d}p}\tau_{\text{RAS-MAC}}(p, p) = 0. \tag{10}$$

While the above equation can be solved analytically, the solution is omitted due to space limitations.

4) PHY$^+$/MAC^{+2p}: A SIC-enabled physical layer plus the RAS-MAC protocol described in subsection III-B is used in this configuration. Therefore the throughput of this configuration is given by,

$$\tau_{\text{PHY}^+\text{/MAC}^{+2p}} = \tau_{\text{RAS-MAC}}(p_1^*, p_2^*). \tag{11}$$

C. Efficiency of the Configurations

While RAS-MAC differentiates between the two groups of users, the rest of the MAC protocols considered for the comparison assume the same probability for all users. This makes the sum throughput of those protocols dependent on n_1 and n_2 while the sum throughput of RAS-MAC is not dependent on n_1 and n_2. Because of that, for the comparison, first (in subsections IV-C1, IV-C2, and IV-C3) we assume $n_1 = n_2 = n/2$ (*i.e.*, there are equal number of low-power and high-power users). Later, in subsection IV-C4, we consider the case $n_1 \neq n_2$ and comment on the possible throughput gains.

1) Efficiency of PHY$^+$/MAC$^-$ vs PHY$^-$/MAC$^-$: We define the performance gain function $\mathcal{G}_1(.)$ as follows,

$$\mathcal{G}_1(r_1, r_2) = \frac{\tau_{\text{PHY}^+\text{/MAC}^-}}{\tau_{\text{PHY}^-\text{/MAC}^-}}. \tag{12}$$

Using (7) and (8) and letting $n_1 = n_2 = \frac{n}{2}$ for all $r_1, r_2 > 0$ we have,

$$\mathcal{G}_1(r_1, r_2) = \frac{\frac{3}{4}\frac{r_1+r_2}{e}}{\frac{1}{2}\frac{r_1+r_2}{e}} = \frac{3}{2}, n_1 = n_2 = \frac{n}{2}, \tag{13}$$

which means that having a SIC-enabled physical layer without touching the MAC protocol obtains a 1.5x throughput gain.

2) Efficiency of PHY$^+$/MAC^{+1p} vs PHY$^-$/MAC$^-$: We define the performance gain function $\mathcal{G}_2(.)$ as follows,

$$\mathcal{G}_2(r_1, r_2) = \frac{\tau_{\text{PHY}^+\text{/MAC}^{+1p}}}{\tau_{\text{PHY}^-\text{/MAC}^-}}. \tag{14}$$

Using (9) and letting $n_1 = n_2 = \frac{n}{2}$ we obtain the optimal value $p^* = \frac{\sqrt{2}}{n}$. The value of $\tau_{\text{PHY}^+\text{/MAC}^{+1p}}$ at this point is given by,

$$\tau_{\text{PHY}^+\text{/MAC}^{+1p}} = \tau_{\text{RAS-MAC}}(\frac{\sqrt{2}}{n}, \frac{\sqrt{2}}{n}) = \frac{\left(2+\sqrt{2}\right) e^{-\sqrt{2}} \left(r_1 + r_2\right)}{2\sqrt{2}}. \tag{15}$$

Using (14), (7), and (15) we have,

$$\mathcal{G}_2(r_1, r_2) = \frac{\frac{\left(2+\sqrt{2}\right)(r_1+r_2)}{2\sqrt{2}\; e^{\sqrt{2}}}}{\frac{r_1+r_2}{2e}} = \frac{1+\sqrt{2}}{e^{\sqrt{2}-1}} \approx 1.6. \tag{16}$$

This means that with the same probability for all users, we cannot obtain a throughput gain better than 1.6x in the case of $n_1 = n_2$.

3) Efficiency of PHY$^+$/MAC^{+2p} vs PHY$^-$/MAC$^-$: Let $\mathcal{G}_3(r_1, r_2)$ denote the throughput gain of PHY$^+$/MAC^{+2p} over PHY$^-$/MAC$^-$, *i.e.*,

$$\mathcal{G}_3(r_1, r_2) = \frac{\tau_{\text{PHY}^+\text{/MAC}^{+2p}}}{\tau_{\text{PHY}^-\text{/MAC}^-}}. \tag{17}$$

We derive lower and upper bounds on $\mathcal{G}_3(.)$ for $n_1 = n_2 = n/2$. Using (7) and (11) we have,

$$\mathcal{G}_3(r_1, r_2) = \frac{\frac{1}{2}e^{\frac{-\sqrt{\Delta}}{2(r_1+r_2)}}\left((r_1 + r_2) + \sqrt{\Delta}\right)}{\frac{r_1+r_2}{2e}}, \tag{18}$$

where $\Delta = r_1^2 + r_2^2 + 6r_1r_2$. We show $3.23 \leq \mathcal{G}_3(.) \leq 3.30$, *i.e.*, despite the complex appearance of (18) it is approximately a constant. Let,

$$\alpha = \frac{\sqrt{\Delta}}{(r_1 + r_2)}. \tag{19}$$

We can rewrite (18) as,

$$\mathcal{G}_3(r_1, r_2) = h(\alpha) = e^{1-\alpha/2}(1 + \alpha). \tag{20}$$

For $r_1, r_2 \geq 0$, we have , $r_1^2 + r_2^2 + 6r_1r_2 \leq 2(r_1 + r_2)^2$ and $r_1^2 + r_2^2 + 6r_1r_2 \geq (r_1 + r_2)^2$. Therefore , $1 \leq \alpha \leq \sqrt{2}$. Additionally, for $\alpha \geq 1$ we have,

$$\frac{\mathbf{d}}{\mathbf{d}\alpha}h(\alpha) = -\frac{1}{2}e^{1-\frac{\alpha}{2}}(\alpha - 1) < 0,\ \alpha \geq 1. \tag{21}$$

That is for $\alpha \geq 1$, $h(\alpha)$ is a decreasing function. This means, for $r_1, r_2 \geq 0$, the maximum of $h(\alpha)$ happens at $\alpha = 1$ and the minimum is at $\alpha = 2$. More formally,

$$(1 + \sqrt{2})e^{1-\sqrt{2}/2} \leq \mathcal{G}_3(r_1, r_2) \leq 2\sqrt{e}. \tag{22}$$

It means that the proposed MAC protocol (RAS-MAC) on top of a SIC-enabled physical layer, obtains at least 3.23x and at most 3.30x throughput gain over PHY$^-$/MAC$^-$ when $n_1 = n_2$.

4) Efficiency of PHY$^+$/MAC^{+1p} vs PHY$^+$/MAC^{+2p} for $n_1 \neq n_2$: In subsections IV-C1, IV-C2, and IV-C3, we showed that for $n_1 = n_2$ the performance of PHY$^+$/MAC^{+2p} is superior to the other configurations. We also showed that PHY$^+$/MAC^{+1p} stands at the second place.

It is obvious that PHY$^+$/MAC^{+1p} cannot outperform PHY$^+$/MAC^{+2p} in terms of throughput since the former is a special case of the latter configuration. The best case of PHY$^+$/MAC^{+1p}, in comparison to PHY$^+$/MAC^{+2p}, happens at the point $p^* = p_1^* = p_2^*$. As a result, both of the configurations perform the same in this case.

Now, consider a case where n_1 is a small positive integer and n_2 is a very large number. Also assume $r_1 > r_2$. In this case, since PHY$^+$/MAC^{+1p} assigns the same probability to the users, we can neglect the very small throughput of $\mathcal{N}_1$ in comparison to the large throughput of users in $\mathcal{N}_2$. Therefore, in this case, PHY$^+$/MAC^{+1p} achieves almost the same throughput as that of PHY$^+$/MAC$^-$.

D. Further Discussion

One may argue that due to concurrent transmission of packets in PHY$^+$/MAC^{+2p}, the rates might be lower than the rates in PHY$^-$/MAC$^-$. It can be shown that for any $\alpha > 0$, replacing r_1 and r_2 by αr_1 and αr_2 in $\tau_{\text{RAS-MAC}}(p_1^*, p_2^*)$, changes the throughput by a factor of α. Let assume that $\alpha = 0.5$, *i.e.,* the concurrent transmission of two packets halves the rates. It is still beneficial to use RAS-MAC since based on the result of subsection IV-C3, we still get at least $0.5 \times 3.23 \approx 1.61$x throughput improvement.

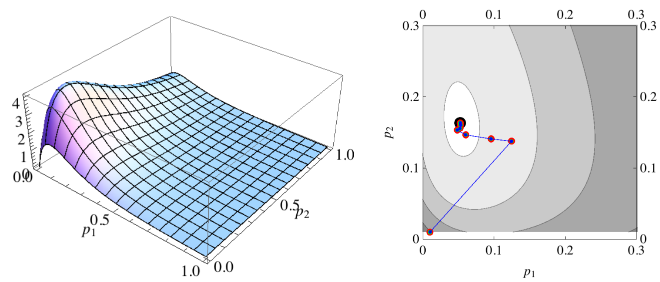
V. Probabilistic Analysis of RAS-MAC

In previous sections, we assumed the exact values of n_1 and n_2 are known to the users. In this case, users can compute the optimal transmission probabilities as described in subsection IV-A. In this section, we extend the analysis to the case where exact values of n_1 and n_2 are not known but the distribution of the number of users in each group is known. Specifically, we assume that the number of users in each group follows a Poisson distribution with a known average. We use the notation $\tilde{n_1}$ and $\tilde{n_2}$ for random variables and $\overline{n_1}$ and $\overline{n_2}$ for the average of the variables. We also denote by $\tilde{\tau}_{\text{RAS-MAC}}(.)$ the throughput of the network in the probabilistic case.

Note that sum of two Poisson random variables with parameters $\overline{n_1}$ and $\overline{n_2}$ is a Poisson random variable with parameter $\overline{n_1} + \overline{n_2}$. Therefore, our assumption is the same as to consider the total number of nodes in the network as a Poisson variable $\tilde{n}$ and split them into two groups of size $s\tilde{n}$ and $(1-s)\tilde{n}$ where $s = \frac{\overline{n_1}}{\overline{n_1} + \overline{n_2}}$.

For $i = 1, 2$, let $Pr(\tilde{n_i} = k)$ denote the probability of $\tilde{n_i}$ being equal to k (*i.e.,* probability of exactly k transmissions from the group $\mathcal{N}_i$).

The expected throughput $\tilde{\tau}_{\text{RAS-MAC}}(.)$ is shown in (26). Equation (26) is too complicated to be solved analytically. We conjecture that (26) is a quasi-concave function over $0 < p_1, p_2 \leq 1$ (see Fig. 3(a)). Therefore, we believe it is efficiently solvable using numerical methods. Fig. 3(b) shows the contour plot along with the steps of finding the maximum of (26) using a Quasi-Newton method [23]. Starting from $p_1 = 0.01$ and $p_2 = 0.01$, we were able to find the maximum of the function up to 18 digits after the decimal point in only 11 iterations. Since p_1 and p_2 are probabilities close to zero, very small positive values are good starting values to find the optimal point. Next, we show that for large n_1 and n_2 the analytical solution obtained for the exact case (*i.e.,* $\tau_{\text{RAS-MAC}}(.)$) is a good approximation for the probabilistic case (*i.e.,* $\tilde{\tau}_{\text{RAS-MAC}}(.)$), when using the averages instead of the exact number of users (*i.e.,* $n_1 = \overline{n_1}$ and $n_2 = \overline{n_2}$).



(a) $\tilde{\tau}_{\text{RAS-MAC}}(p_1, p_2)$ is a quasi-concave function over $0 < p_1, p_2 \leq 1$.

(b) Starting from $\langle 0.01, 0.01 \rangle$ using a Quasi-Newton method, the maximum of $\tilde{\tau}_{\text{RAS-MAC}}(.)$ is found in 11 steps.

Fig. 3: The expected throughput of the network from (26) for $r_1 = 5$, $r_2 = 10$, $n_1 = 10$, and $n_2 = 5$.

A. Comparison of $\tau_{\text{RAS-MAC}}(.)$ and $\tilde{\tau}_{\text{RAS-MAC}}(.)$

We use numerical results to show that the difference between exact and probabilistic case of RAS-MAC is relatively small. In addition, the optimal point of each of the two cases (*i.e.,* $\langle p_1^*, p_2^* \rangle$ and $\langle \tilde{p_1}^*, \tilde{p_2}^* \rangle$) happen to be very close to each other. Because of that, the answer given by (5) (*i.e.,* $\langle p_1^*, p_2^* \rangle$), obtained by $n_1 = \overline{n_1}$ and $n_2 = \overline{n_2}$, can be used as a good approximate answer for (26) (*i.e.,* $\langle \tilde{p_1}^*, \tilde{p_2}^* \rangle$).

We define the error function $\mathbb{E}_{\mathbf{1}}(.)$ as the percentage of difference between the throughputs of the probabilistic and exact cases normalized by the optimal throughput of the probabilistic case for $0 \leq p_1, p_2 \leq 1$, *i.e.,*

$$\mathbb{E}_{\mathbf{1}}(p_1, p_2) = 100 \times \frac{\tilde{\tau}_{\text{RAS-MAC}}(p_1, p_2) - \tau_{\text{RAS-MAC}}(p_1, p_2)}{\tilde{\tau}_{\text{RAS-MAC}}(p_1^*, p_2^*)}. \quad (23)$$

Fig. 4(a) shows the plot of $\mathbb{E}_{\mathbf{1}}(.)$ for $n_1 = 20$, $n_2 = 20$, $r_1 = 10$, and $r_2 = 5$. The value of the index is within 2% for all $0 \leq p_1, p_2 \leq 1$. It shows that (4) and (26) are very close to each other.

Fig. 4(b) shows the contour plot of (26) along with the optimal points of $\tau_{\text{RAS-MAC}}(.)$ and $\tilde{\tau}_{\text{RAS-MAC}}(.)$. It is observed that even for small n_1 and n_2's the optimal point of the two cases are very close to each other.

The next plot shows that when n_1 and n_2 are large, the optimal point of (4) (which is computable analytically using (5)) may be used, with a good accuracy, as the answer of (26). Let's define $\mathbb{E}_{\mathbf{2}}(.)$ as,

$$\tilde{\tau}_{\text{RAS-MAC}}(p_1,p_2) = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} Pr(\tilde{n_1}=i)Pr(\tilde{n_2}=j)\tau_{\text{RAS-MAC}}(p_1,p_2)$$
$$= \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} \frac{\overline{n_1}^{\,i} e^{-\overline{n_1}}}{i!}\frac{\overline{n_2}^{\,j} e^{-\overline{n_2}}}{j!} e^{-ip_1-jp_2}((r_1+r_2)ip_1jp_2 + r_1ip_1 + r_2jp_2) \tag{26}$$
$$= e^{\overline{n_1}\,(e^{-p_1}-1)+\overline{n_2}\,(e^{-p_2}-1)-p_1-p_2}\left(\overline{n_2}\,p_2r_2e^{p_1} + \overline{n_1}\,p_1(r_1(\overline{n_2}\,p_2 + e^{p_2}) + \overline{n_2}\,p_2r_2)\right).$$

$$\mathbb{E}_{\mathbf{2}}(n_1,n_2) = 100 \times \frac{\tilde{\tau}_{\text{RAS-MAC}}(\tilde{p_1}^*,\tilde{p_2}^*) - \tilde{\tau}_{\text{RAS-MAC}}(p_1^*,p_2^*)}{\tilde{\tau}_{\text{RAS-MAC}}(\tilde{p_1}^*,\tilde{p_2}^*)}, \tag{25}$$

where $\langle p_1^*, p_2^*\rangle$ is the optimal answer to (4) and $\langle \tilde{p_1}^*, \tilde{p_2}^*\rangle$ is the optimal answer to (26). Fig. 4(c) shows the value of $\mathbb{E}_{\mathbf{2}}(.)$ for $r_1 = 10$, $r_2 = 5$, and $1 \le n_1, n_2 \le 20$. For $n_1, n_2 \ge 5$, the value of the index is almost zero. This verifies our claim that for large n_1 and n_2, the optimal point of $\tau_{\text{RAS-MAC}}(.)$ can be used as a good approximation of the optimal point of $\tilde{\tau}_{\text{RAS-MAC}}(.)$.

VI. Game Theoretic Analysis of RAS-MAC

In previous sections, we looked at the RAS-MAC protocol from a system designer perspective and obtained the optimal system throughput. We already assume that users follow the rules specified by the system designer, *i.e.*, they send the packets with the optimal system probability given by (5). What happens if the uses behave selfishly and send their packets with a higher probability to obtain a higher rate? In this case, the system might not be able to reach its optimal throughput. In other words, the optimal throughput for the users does not necessarily result in the optimal system throughput.

In this section, we present a game theoretic analysis of RAS-MAC with selfish behavior of users. We model the protocol as a one-shot simultaneous move game and derive a mixed strategy Nash equilibrium for the game (see [24] for the definitions). We also show that the system designer can "charge" the users so that the Nash equilibrium coincides with the optimal system throughput.

Consider the set of nodes $\mathcal{N} = \mathcal{N}_1 \cup \mathcal{N}_2$ as the set of players. The action set of each player is to either *send* a packet or *wait*. Without loss of generality, we assume that sending a packet will cost $0 < c_i < 1$ for a node in $\mathcal{N}_i$ ($i \in \{1,2\}$) and a successful transmission will have a benefit of 1 for any node.

Note that we are only interested in a symmetric equilibrium, *i.e.*, an equilibrium in which all the nodes in $\mathcal{N}_i$ follow the same strategy. It is clear that a symmetric pure Nash equilibrium does not exist for the game since an always-send or an always-wait strategy will not result in equilibrium[3]. Therefore, we look for a mixed strategy equilibrium.

Let a node in $\mathcal{N}_i$ choose to send with probability p_i and wait with probability $1-p_i$. Since all the nodes in $\mathcal{N}_i$ choose the same strategy, the rate of packet transmission in $\mathcal{N}_i$ would be $\lambda_i = p_i n_i$. Therefore, the utility functions are defined as,

$$\begin{cases} u_1(\lambda_1,\lambda_2) = \dfrac{1}{n_1}\left(\lambda_1 e^{-\lambda_1}((1+\lambda_2)e^{-\lambda_2}) - \lambda_1 c_1\right) & \text{(26a)}\\ u_2(\lambda_1,\lambda_2) = \dfrac{1}{n_2}\left(\lambda_2 e^{-\lambda_2}((1+\lambda_1)e^{-\lambda_1}) - \lambda_2 c_2\right) & \text{(26b)}\end{cases}$$

Where $u_1(.)$ and $u_2(.)$ are the utility functions of the group $\mathcal{N}_1$ and $\mathcal{N}_2$, respectively. Note that we assume a Poisson distribution for the packet transmissions in each group. Based on this assumption, the probability of transmitting zero and one packet from the group $\mathcal{N}_i$ is $e^{-\lambda_i}$ and $\lambda_i e^{-\lambda_i}$ respectively. Thus, $\lambda_1 e^{-\lambda_1}((1+\lambda_2)e^{-\lambda_2})$ and $\lambda_1 c_1$ are the expected benefit and expected cost for all of the nodes in $\mathcal{N}_1$. The same argument applies to $\mathcal{N}_2$ too. At the equilibrium point, the following equations should be satisfied,

$$\begin{cases} \dfrac{\partial u_1}{\partial \lambda_1} = \dfrac{1}{n_1}\left((1-\lambda_1)e^{-\lambda_1}(1+\lambda_2)e^{-\lambda_2} - c_1\right) = 0 & \text{(27a)}\\ \dfrac{\partial u_2}{\partial \lambda_2} = \dfrac{1}{n_2}\left((1-\lambda_2)e^{-\lambda_2}(1+\lambda_1)e^{-\lambda_1} - c_2\right) = 0 & \text{(27b)}\end{cases}$$

In fact, the Nash equilibrium is at the intersection of $\frac{\partial u_1}{\partial \lambda_1} = 0$ and $\frac{\partial u_2}{\partial \lambda_2} = 0$ (see Fig. 5). By simplifying (27), we obtain that,

$$\begin{cases} \lambda_1 = 1 - W_0\left(\dfrac{ec_1}{(1+\lambda_2)e^{-\lambda_2}}\right) & \text{(28a)}\\ \lambda_2 = 1 - W_0\left(\dfrac{ec_2}{(1+\lambda_1)e^{-\lambda_1}}\right), & \text{(28b)}\end{cases}$$

where $W_0(.)$ is the upper branch of Lambert's W_n function. The Lambert's W_n is a set of functions (*i.e.*, W_0 and W_1) consisting of the branches of the inverse relation of $f(w) = we^w$. The upper branch (*i.e.*, W_0) is in the domain $(-1,+\infty)$, while the lower branch (*i.e.*, W_1) is in the domain $(-\infty,-1)$. Since W_0 cannot be expressed in terms of elementary functions, it is unlikely that one can solve the system (27) analytically.

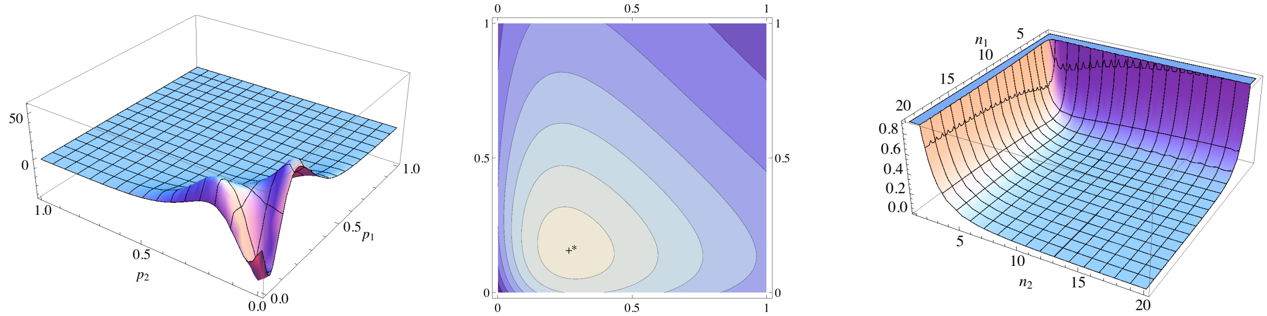
A. Algorithm for Computing the Equilibrium

Fortunately, the W_0 function can be approximated using Newton's method by the following formula at some point z,

$$w_{j+1} = w_j - \frac{w_j e^{w_j} - z}{e^{w_j} + w_j e^{w_j}}, \tag{29}$$

where w_j is the estimated value of the function at iteration j. Based on (29), we propose a simple algorithm (see Algorithm 1) to numerically solve the system of equations (28).

[3]The only pure Nash equilibrium is the one in which one node from $\mathcal{N}_1$ and one node from $\mathcal{N}_2$ send and the rest of the nodes wait. Clearly, this is not a symmetric equilibrium.



(a) Plot of $\mathbb{E}_{\mathbf{1}}(.)$ (see (23)) for $n_1 = 20$, $n_2 = 20$, $r_1 = 10$, $r_2 = 5$, and $0 \leq p_1, p_2 \leq 1$. The value is within 2%.

(b) The optimal point of (26) (the '+' sign) vs. the optimal point of (4) (the '*' sign) for $n_1 = 3$, $n_2 = 3$, $r_1 = 10$, $r_2 = 5$. The Euclidean distance of the points is less than 0.025.

(c) Plot of $\mathbb{E}_{\mathbf{2}}(.)$ for $r_1 = 10$, $r_2 = 5$, and $1 \leq n_1, n_2 \leq 20$. For $n_1, n_2 \geq 5$ the value is very close to zero.

Fig. 4: Comparison of the throughput functions $\tau_{\text{RAS-MAC}}(.)$ and $\tilde{\tau}_{\text{RAS-MAC}}(.)$. Note that to draw the plots whenever required we used the values n_1 and n_2 as the average of the random variables (*i.e.*, $\overline{n_1}$ and $\overline{n_2}$) in (26).

The algorithm starts with an initial value $\langle \lambda_1^{init}, \lambda_2^{init} \rangle$. At each iteration, first it fixes λ_2 and computes λ_1 using (29). Then, it fixes λ_1 and computes the value of λ_2 using (29). The algorithm repeats these steps until it converges with respect to the desired numerical precision. Fig. 5 shows the steps of Algorithm 1 for a sample run. Based on our numerical simulations, only a few (less than 5) iterations of Algorithm 1 result in solutions that are accurate to within 10 decimal point.

The steps involved in computing the Nash equilibrium for the probabilistic case is similar to the steps described for the exact case of the protocol, and hence omitted for brevity. To derive the utility functions, the same technique as the one used in (26) can be applied. To find the Nash equilibrium having the utility functions, one may use an algorithm similar to Algorithm 1.

Algorithm 1 Estimating the solution of the system (28)

$\lambda_1^0 = \lambda_1^{init}$
$\lambda_2^0 = \lambda_2^{init}$
$\mathcal{E}_0 = \infty$
$i = 0$
while $\mathcal{E}_i > \epsilon$ **do**
  $\lambda_1^{i+1} = 1 - W_0(\frac{ec_1}{(1+\lambda_2^i)e^{-\lambda_2^i}})$ ▷ using (29)
  $\lambda_2^{i+1} = 1 - W_0(\frac{ec_2}{(1+\lambda_1^{i+1})e^{-\lambda_1^{i+1}}})$ ▷ using (29)
  $i = i + 1$
  $\mathcal{E}_i = \sqrt{(\lambda_1^i - \lambda_1^{i-1})^2 + (\lambda_2^i - \lambda_2^{i-1})^2}$
end while

B. Setting a Cost for Transmission

Using (27), we obtain the following relation at the equilibrium point,

$$\begin{cases} c_1 = (1-\lambda_1)e^{-\lambda_1}(1+\lambda_2)e^{-\lambda_2} & \text{(30a)} \\ c_2 = (1-\lambda_2)e^{-\lambda_2}(1+\lambda_1)e^{-\lambda_1} & \text{(30b)} \end{cases}$$

That is, for any given rates $0 < \lambda_1, \lambda_2 < 1$, we can find c_1 and c_2 so that the equilibrium happens at that rate.

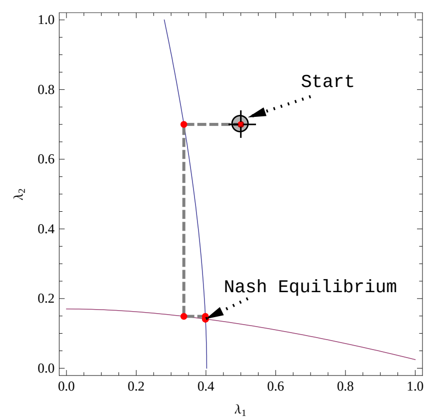


Fig. 5: A sample run of Algorithm 1 for $c_1 = 0.4$ and $c_2 = 0.7$, starting from $\langle 0.5, 0.7 \rangle$. The horizontal curve shows the plot of $\frac{\partial u_1}{\partial \lambda_1} = 0$. The vertical curve shows the plot of $\frac{\partial u_2}{\partial \lambda_2} = 0$. The dotted line shows the path from the starting point $\langle \lambda_1^{init}, \lambda_2^{init} \rangle$ to the Nash equilibrium.

Furthermore, it can be shown that $0 < c_1, c_2 < 1$ at that point. That is, at the optimal system throughput the costs are always non-negative and less than the benefit ($= 1$). Note that a portion of the costs is inherent in the packet transmission, *e.g.*, we need to consume energy to send a packet. However, the designer of the network may charge the nodes with extra cost to force nodes to transmit at the desired probability. To charge the users, a lightweight payment method such as the micropayment of [25] may be used.

VII. Conclusion

We considered a SIC-enabled wireless network and proposed a new SIC-aware MAC protocol called RAS-MAC for the network. We compared our MAC protocol to a number of other protocols in terms of throughput. The result of comparison is that while opportunistic usage of SIC or adjustment of the existing protocols can increase the throughput of the network, designing new MAC protocols tailored for SIC is far more effective. In addition, when network users are selfish, we modeled RAS-MAC as a one-shot simultaneous move game and derived a mixed strategy Nash equilibrium for the game.

We also showed that for any given pair of rates for the high-power and low-power users, we can charge the users so that at the equilibrium point we have the optimal system throughput.

Appendix 1

We replace n_1p_1 by λ_1 and n_2p_2 by λ_2 in (4). At the optimal point of (4), we have (31a) and (31b),

$$\begin{cases} \dfrac{\partial \tau_{\text{RAS-MAC}}}{\partial \lambda_1} = -e^{-\lambda_1-\lambda_2}\Big((r_1+r_2)\lambda_1\lambda_2 + r_1\lambda_1 \\ \qquad +r_2\lambda_2) + e^{-\lambda_1-\lambda_2}((r_1+r_2)\lambda_2 + r_1\Big) = 0 \quad \text{(31a)} \\ \dfrac{\partial \tau_{\text{RAS-MAC}}}{\partial \lambda_2} = -e^{-\lambda_1-\lambda_2}\Big((r_1+r_2)\lambda_1\lambda_2 + r_1\lambda_1 \\ \qquad +r_2\lambda_2) + e^{-\lambda_1-\lambda_2}((r_1+r_2)\lambda_1 + r_2\Big) = 0 \quad \text{(31b)} \end{cases}$$

Since for bounded λ_1 and λ_2, we have $e^{-\lambda_1-\lambda_2} \neq 0$, it is obtained that,

$$\lambda_2 = \lambda_1 - \frac{r_1 - r_2}{r_1 + r_2}\,. \tag{32}$$

Using (32) and (31b) we have,

$$(r_1+r_2)\,\lambda_1^2 + (r_2-r_1)\,\lambda_1 + \frac{-2r_1r_2}{r_1+r_2} = 0\,, \tag{33}$$

which is a quadratic equation in λ_1. Since $\Delta = (r_2-r_1)^2 + 8r_1r_2 > 0$, we obtain two distinct solutions for λ_1, which are,

$$\lambda_1^{*1}, \lambda_1^{*2} = \frac{-(r_2-r_1) \pm \sqrt{(r_2-r_1)^2 + 8r_1r_2}}{2\,(r_1+r_2)}\,. \tag{34}$$

However, since $|(r_2-r_1)| < \left|\sqrt{(r_2-r_1)^2 + 8r_1r_2}\right|$,

$$\lambda_1^* = \frac{-(r_2-r_1) + \sqrt{(r_2-r_1)^2 + 8r_1r_2}}{2\,(r_1+r_2)}\,, \tag{35}$$

is the only positive answer for λ_1. Using (32) and (35), we obtain the following value for λ_2, which is apparently positive,

$$\lambda_2^* = \frac{(r_2-r_1) + \sqrt{(r_2-r_1)^2 + 8r_1r_2}}{2\,(r_1+r_2)}\,. \tag{36}$$

For $r_1 \geq r_2$, we show that $\lambda_2 \leq \lambda_1 \leq 1$. Rewrite (35) as,

$$\lambda_1^* = \frac{\sqrt{(r_1+r_2)^2 - 4r_1r_2} + \sqrt{(r_1+r_2)^2 + 4r_1r_2}}{2\,(r_1+r_2)}\,. \tag{37}$$

then,

$$\begin{aligned} (\lambda_1^*)^2 = \frac{2(r_1+r_2)^2 + 2\sqrt{(r_1+r_2)^4 - (4r_1r_2)^2}}{4\,(r_1+r_2)^2} \leq \\ \frac{2(r_1+r_2)^2 + 2\sqrt{(r_1+r_2)^4}}{4\,(r_1+r_2)^2} = 1 \end{aligned} \tag{38}$$

I is clear that $\lambda_2 \leq \lambda_1$, thus the proof is complete.

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