Outline for CPSC617: Category Theory for Computer Science (2013)

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Introduction

This course is intended as an introduction to Category Theory useful to Computer Scientists, Mathematicians, Logicians, and Philosophers. Category theory is a foundational subject for much of modern mathematics, for semantics in computer science, and even for theoretical physics.

Category theory is an extraordinarily diverse structural mathematical theory which touches almost all areas of modern mathematics. For this reason it is often referred to as "abstract nonsense". Indeed Category Theory is *the* chosen mathematical tool in which to state abstract results. Its abstractness means that its ideas can be applied in many places ... this is both a strength and a weakness. It is a weakness as often such an application will not *immediately* produce results which are regarded as being highly significant in a particular field! It is a strength as it allows the transfer of ideas from one field to another – which *can* produce significant insights.

A significant value of category theory is that it makes the basic results of many subjects more accessible and allows one to distinguishes sharply between results which hold for general reasons and results which are peculiar to an area.

Category theory has a close relationship to logic. Indeed almost all modern proof theory is of a very categorical nature. This is particularly so for the investigation of non-standard settings (e.g. the semantics of concurrency, linear logic, semantics of quantum computing) where the usual logical tools and intuitions tend to fail.

The course

This course aims to provide the core results of category theory. Category theory is now, after 70 years of development, a huge subject. Here is a list of some basic topics which – time allowing – may get covered:

- The definition of a category;
- Basic examples of categories;
- Maps: epic, monic, section, retractions, isomorphism, factorization;

- Functors and natural transformations;
- The Yoneda lemma;
- Adjoints and universal properties;
- Limits and colimits: products, equalizers, pullbacks;
- Monads: the Kleisli category and the Eilenberg-Moore category;
- Cartesian closed categories;
- Data types: initial and final algebras;
- Fibrations.

Assessment

There will be four assignments (60%) and a term paper (40%) which will be an "in depth" study of a chosen area which you must present. For the assignments I encourage you to work together although you must eventually write out the solutions individually: you are expected to understand the solution!