An Efficient Framework for Private Function Evaluation

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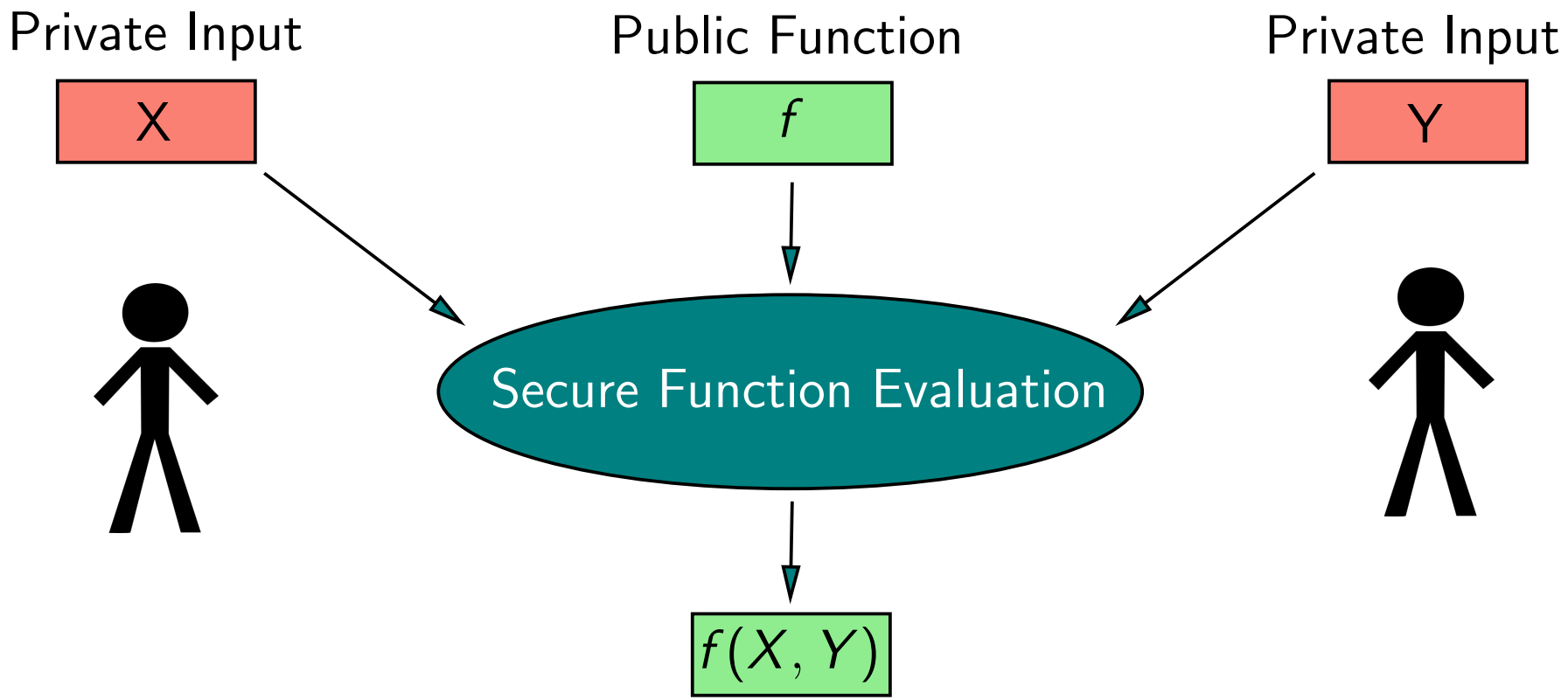
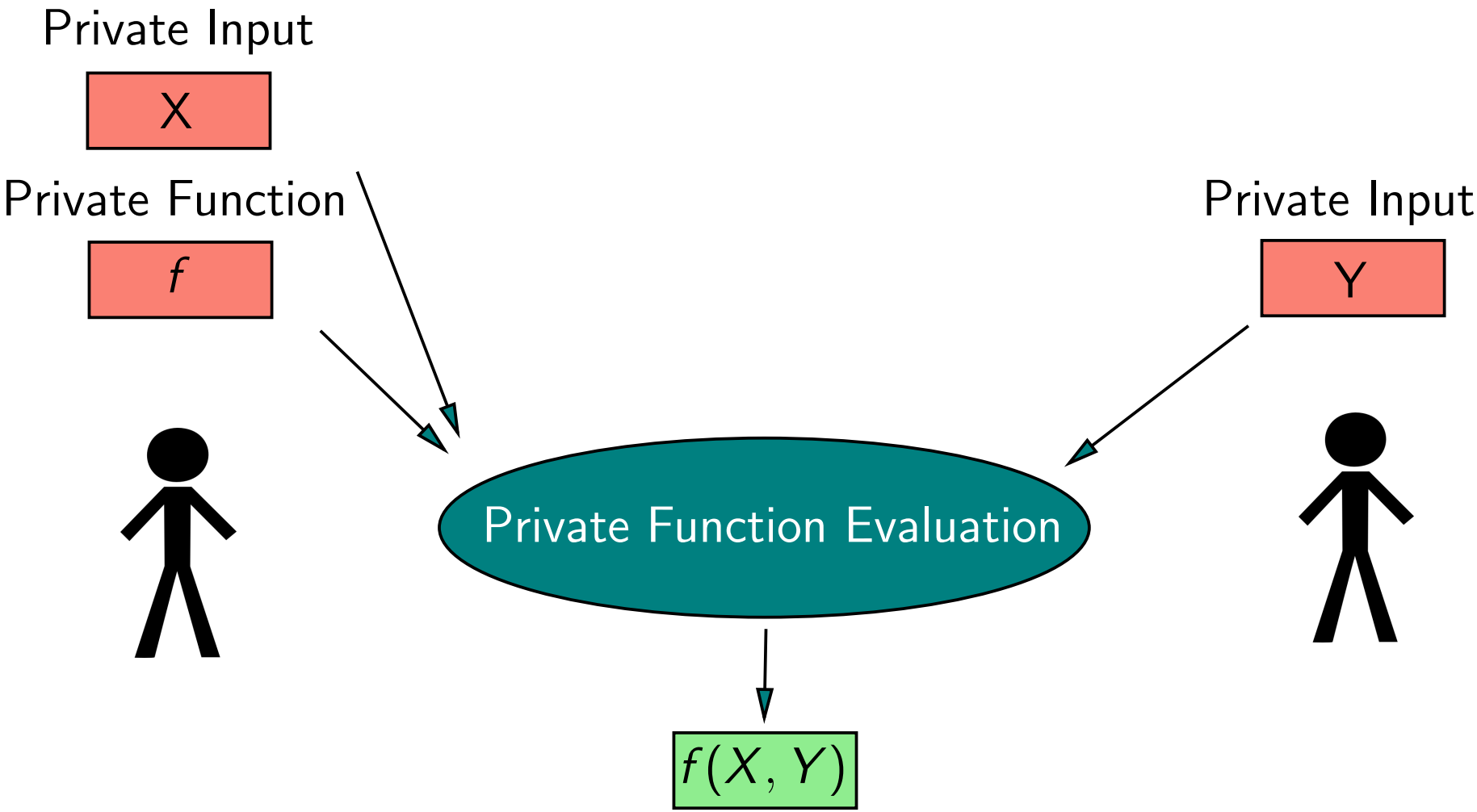
University of Calgary

Eurocrypt 2013

Secure Function Evaluation



Private Function Evaluation



Motivation

- Revealing the function can compromise privacy
- Revealing the function can reveal security vulnerability
- Intellectual property: Genetic tests, credit history check,...
- Leakage from output?

Private Function Evaluation: Universal Circuit approach

- General SFE: Convert function $f \rightarrow$ Circuit C and securely evaluate

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- General SFE: Convert function $f \rightarrow$ Circuit C and securely evaluate
- General PFE: Build Universal circuit handling the circuits with the desired size and use general SFE
- Valiant [76]: universal circuit of size $O(g \log g)$ to handle circuits of size g.

Private Function Evaluation: Questions?

- *Can we achieve better concrete efficiency?*

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- *Can we achieve the same asymptotic efficiency as SFE?*

Our Contribution

- A general framework for designing PFE
  - Modular design, based on abstract functionalities

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  - Modular design, based on abstract functionalities
- Applying our framework to several well-known MPC constructions
- A novel protocol for *oblivious switching network evaluation* (OSN)
  - A generalization of the oblivious shuffling problem
  - Based on OT and non-crypto operations

Current state

Semi-honest

They all leak the circuit size and input length.

- Two-Party PFE: Linear solution of [KM11]
- Multi-party PFE: no customized construction, only universal circuit based approaches with at best $O(m^2 g \log g)$ complexity
- Arithmetic circuit PFE: no customized solution, except for arithmetic universal circuits with best known $O(g^5)$ gates

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Our work:

- Two-Party PFE: More efficient solutions.
- Multi-party PFE: We present the first linear multi-party PFE based on GMW.
- Arithmetic circuit PFE: We present the first linear arithmetic PFE.

Our Framework for PFE

- We need to hide two types of information about the circuit:
  1. *Circuit topology*
  2. *Function of the gates* in the circuit

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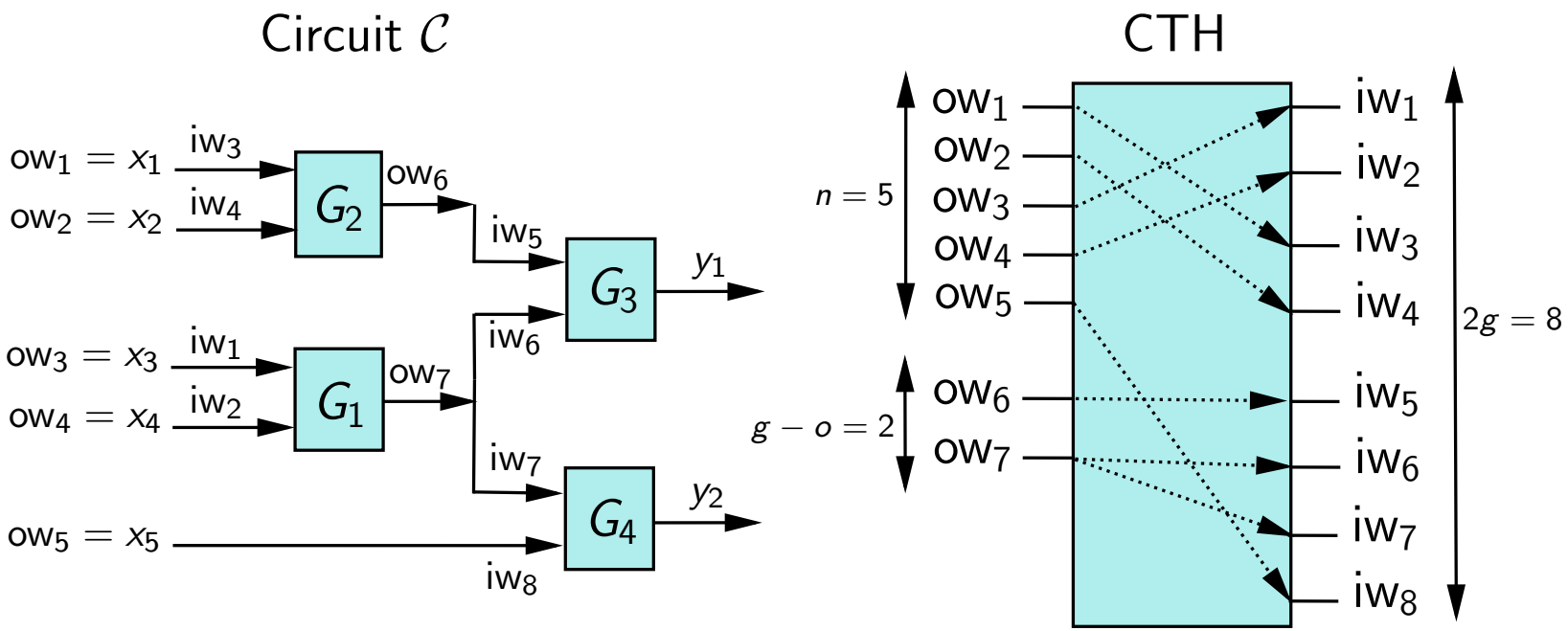
- We need to hide two types of information about the circuit:
  1. *Circuit topology*
  2. *Function of the gates* in the circuit
- We divide the task of private function evaluation into two different functionalities:
  1. The Circuit Topology Hiding (CTH) functionality
  2. The Private Gate Evaluation (PGE) functionality

Circuit Topology Hiding (CTH) functionality

- *Observation:* the topology of a circuit $\mathcal{C}$ can be fully described using a mapping $\pi_{\mathcal{C}} : \{1 \dots |\mathsf{OW}|\} \rightarrow \{1 \dots |\mathsf{IW}|\}$
  - OMAP query
  - Reveal query
  - We refer to this mapping as an *extended permutation* (EP)
- Inputs are shares and outputs are freshly generated shares.



The Private Gate Evaluation (PGE) functionality

- The PGE functionality :
  - PFE for a single gate
  - Shared inputs and output

Framework

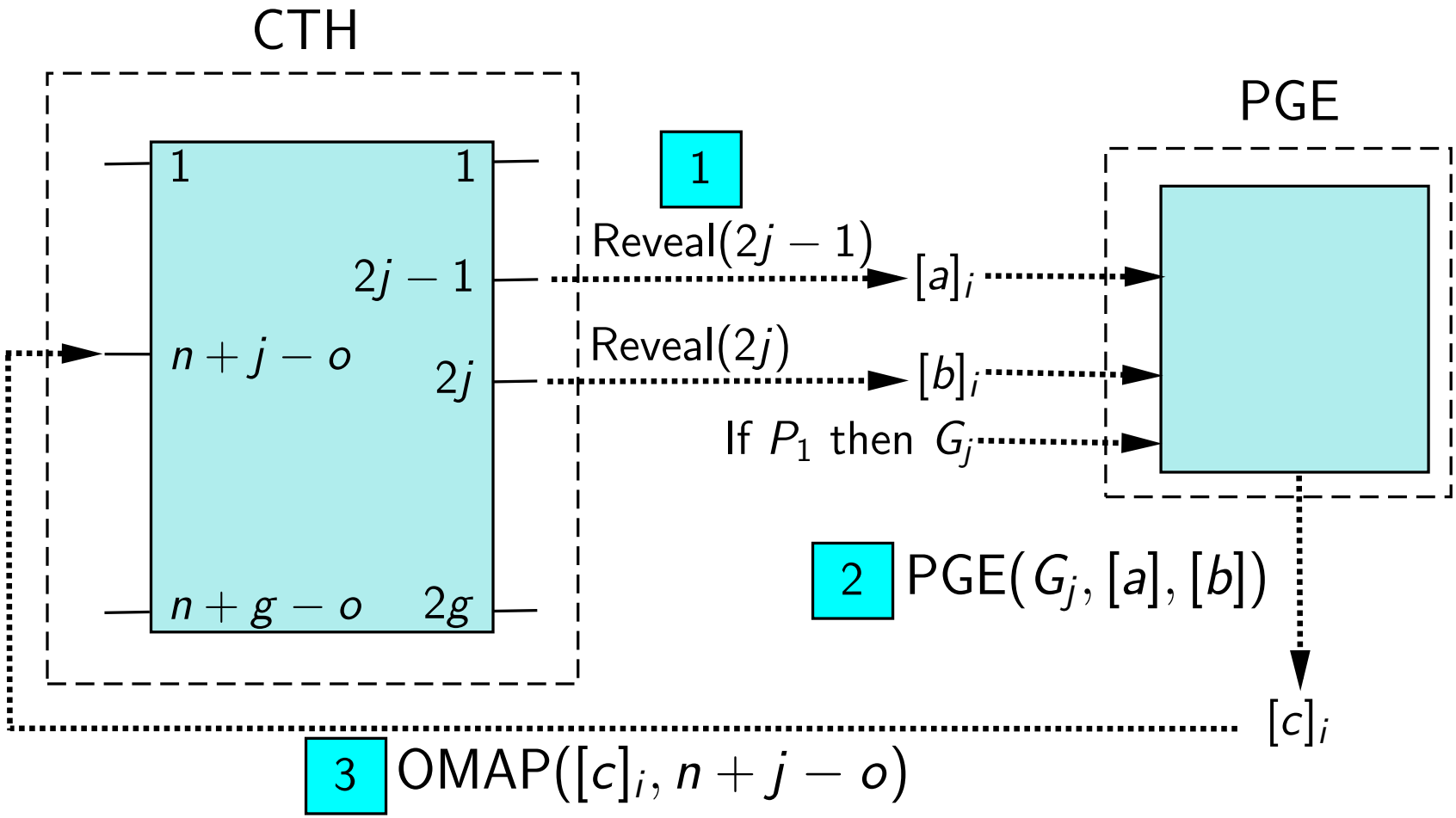
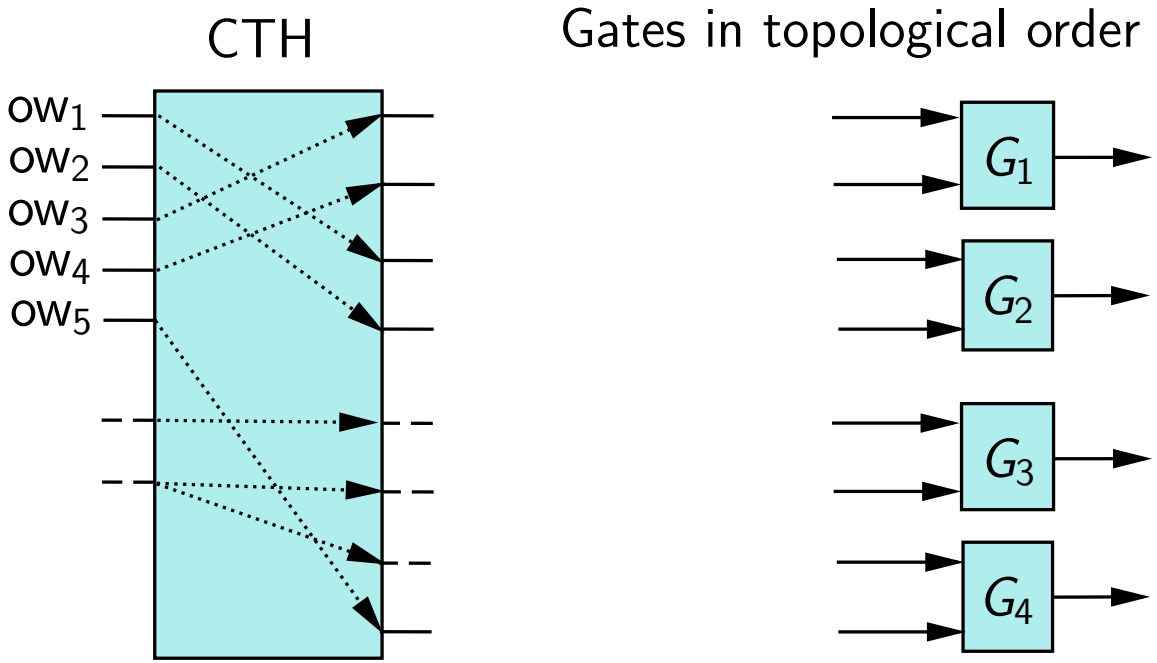


Figure : Steps of framework for party i and the jth gate in a topological order.

Framework: Initialization



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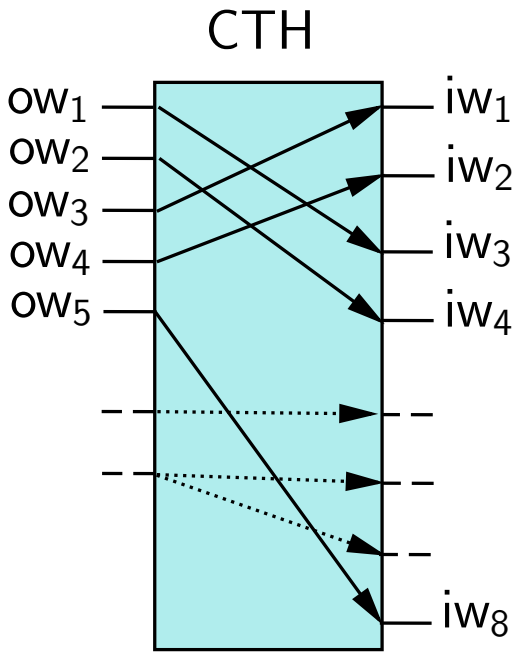
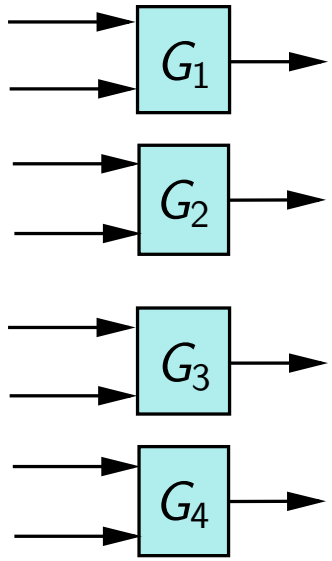
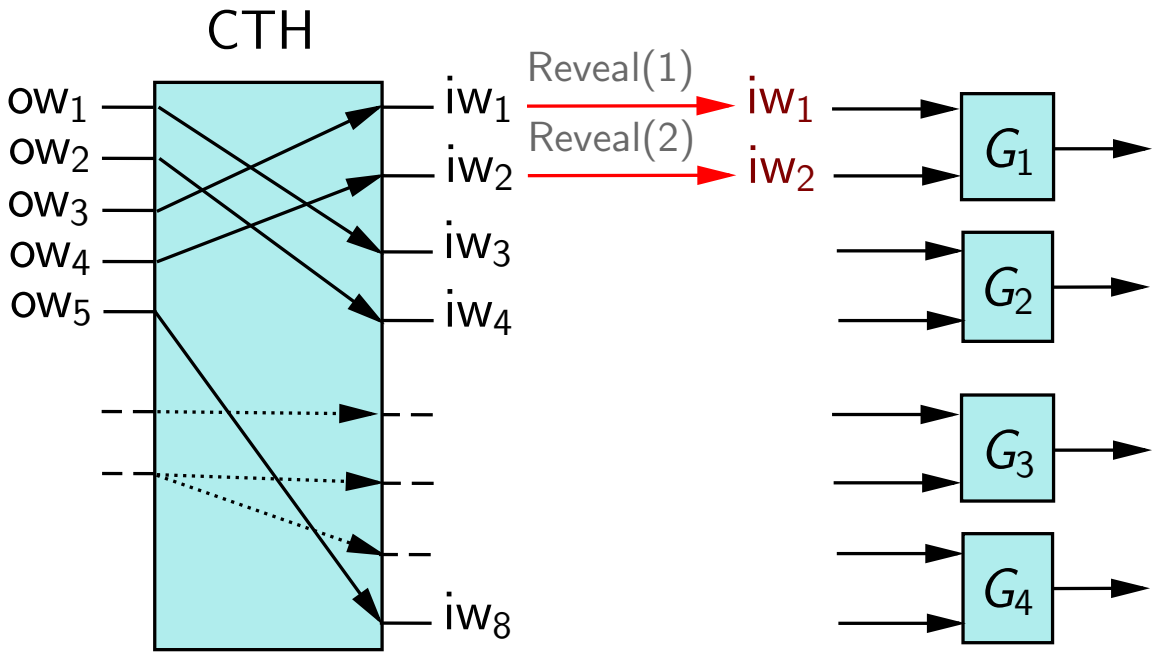




$\mathrm{OMAP}([x_1]_i, 1)$
$\mathrm{OMAP}([x_2]_i, 2)$
$\vdots$
$\mathrm{OMAP}([x_5]_i, 5)$

Framework: Private Function Evaluation

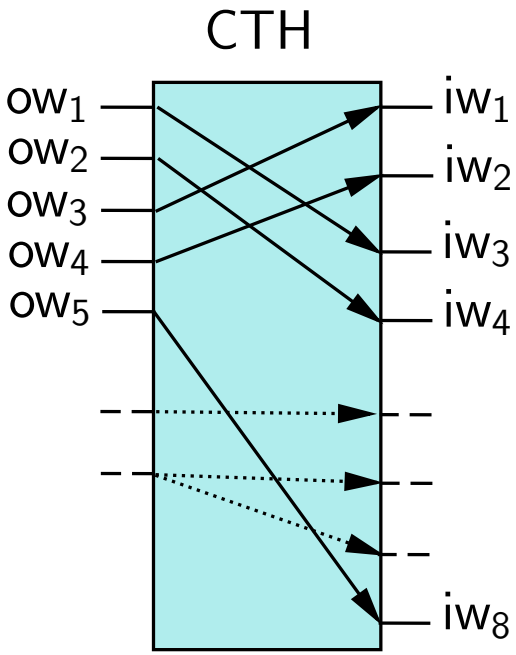


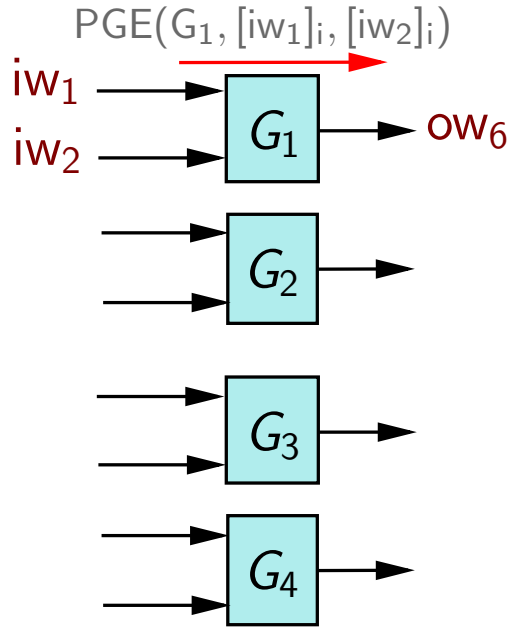
1 Reveal$(2j-1)$

2 PGE$(G_j, [a], [b])$

3 OMAP$([c]_i, n+j-o)$

Framework: Private Function Evaluation





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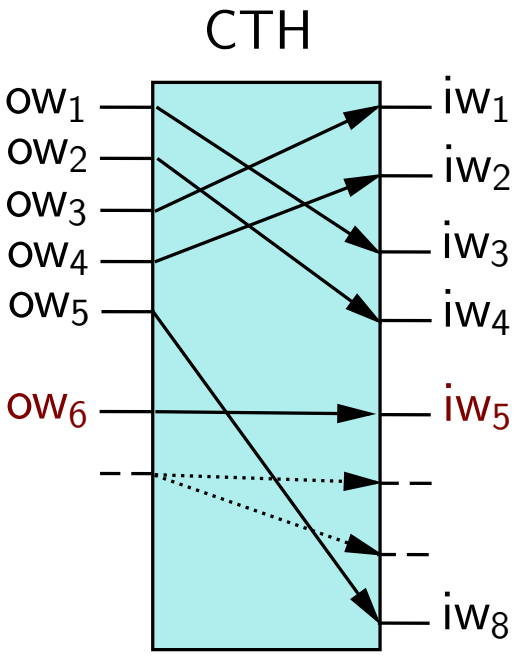

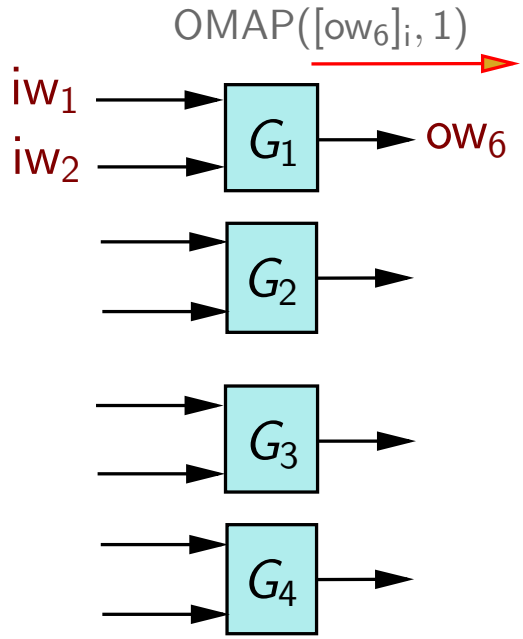

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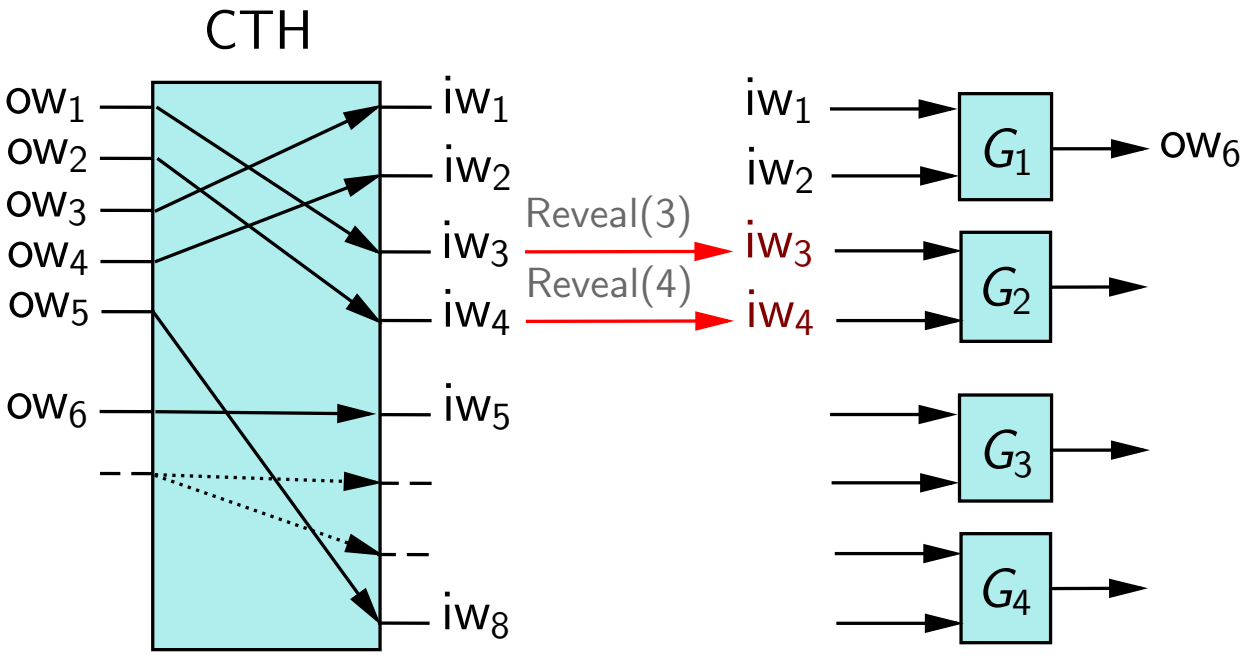

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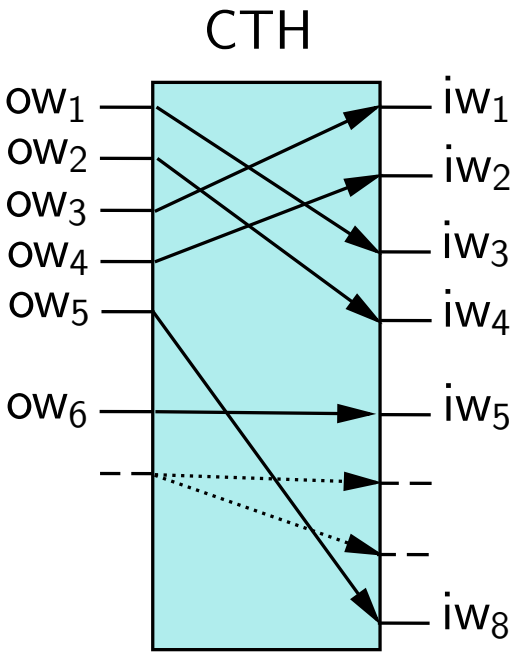

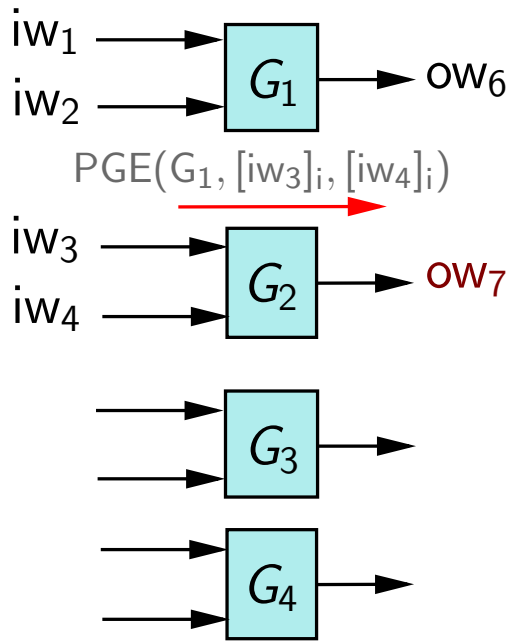

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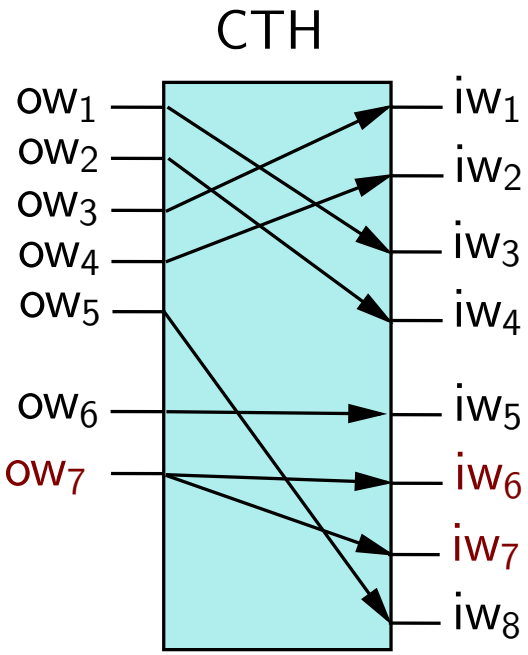

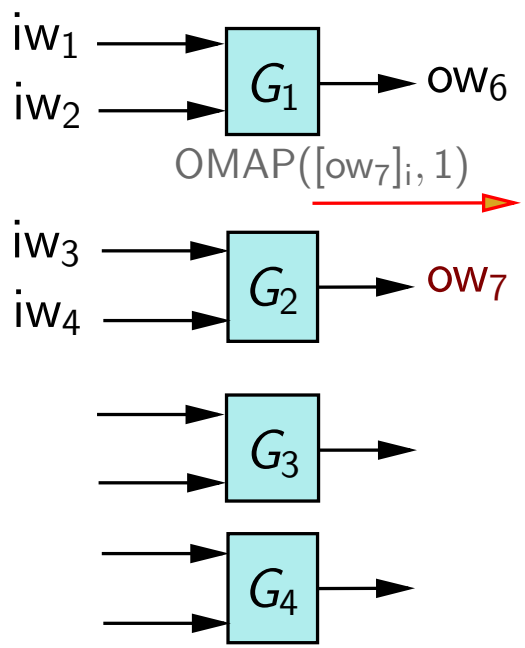

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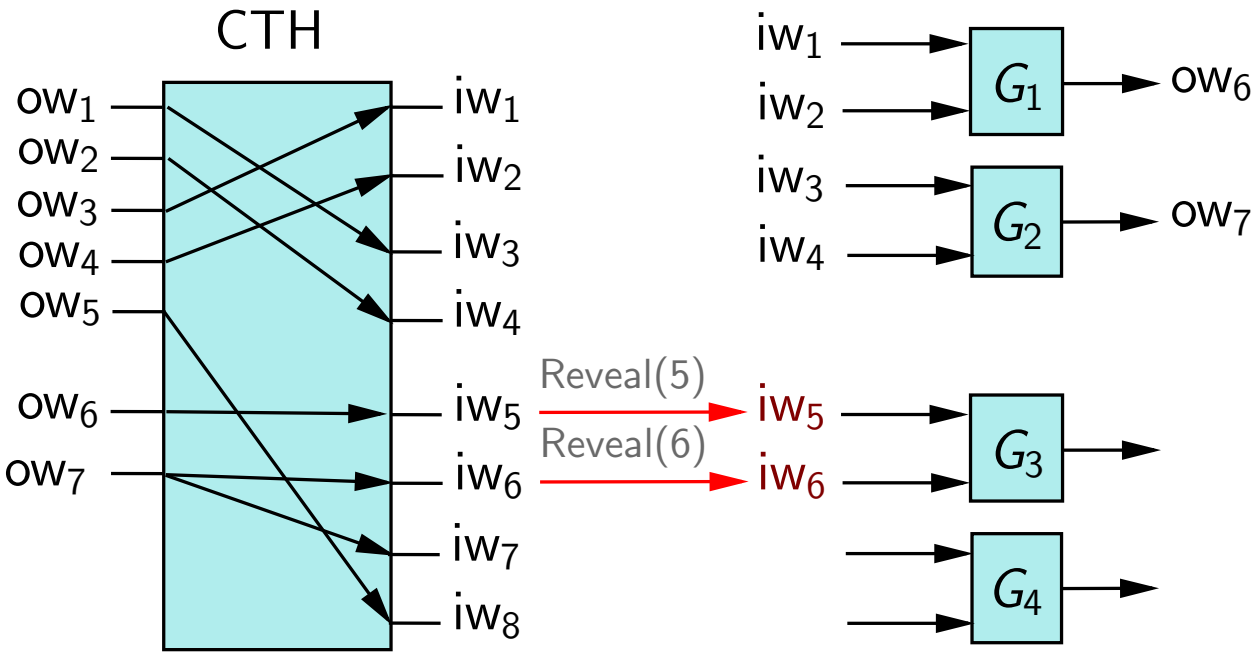

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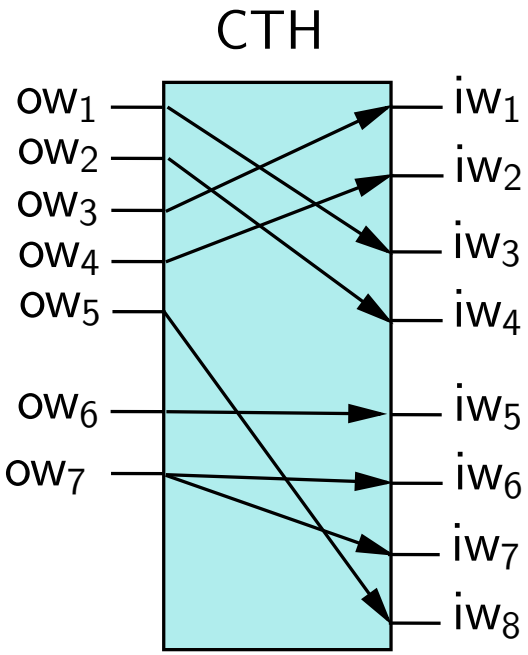

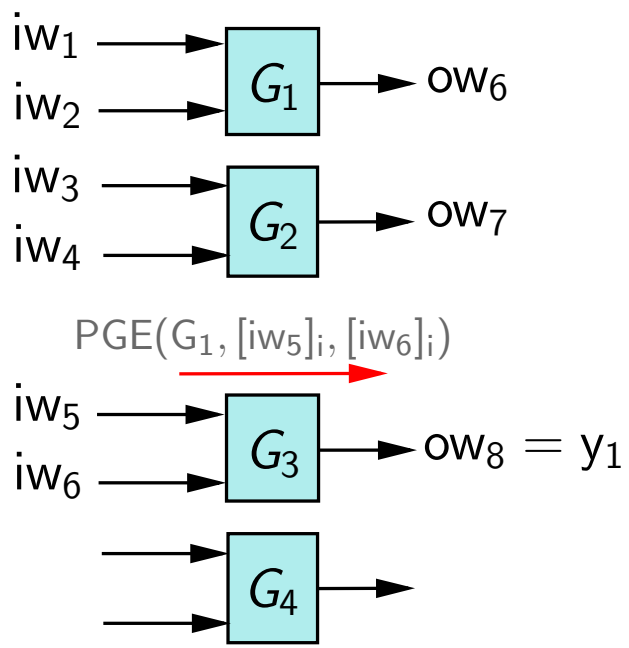

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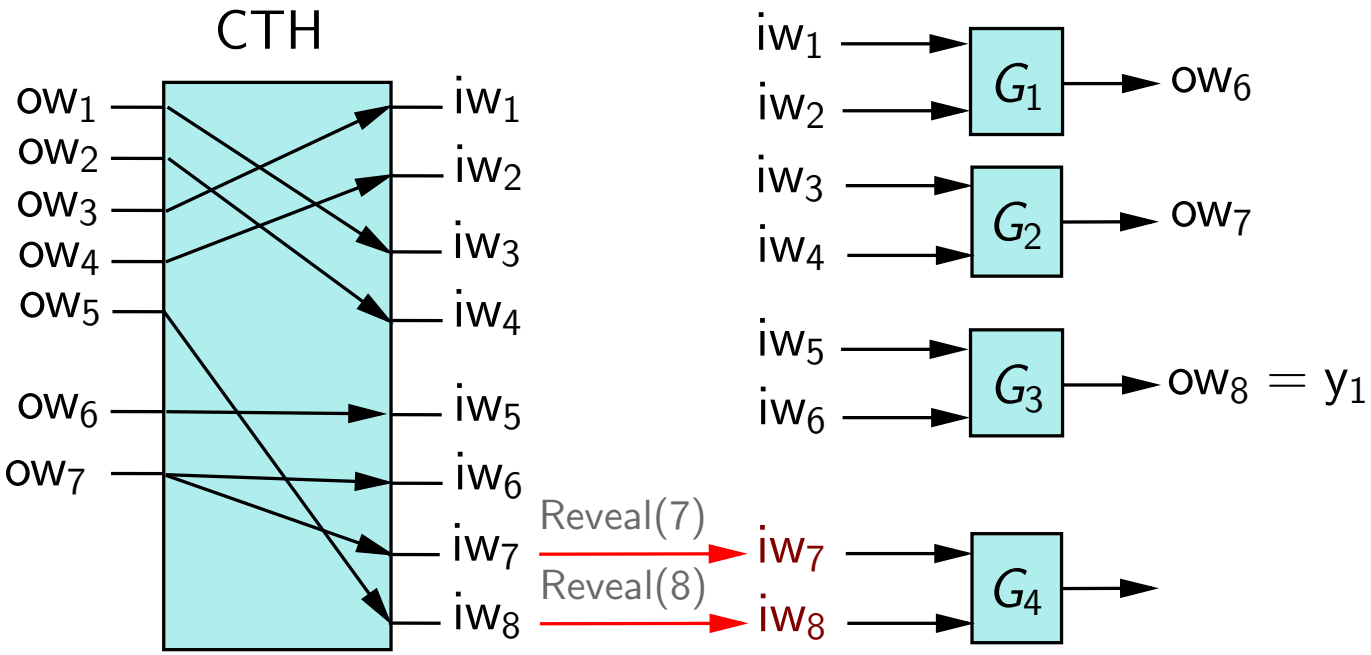

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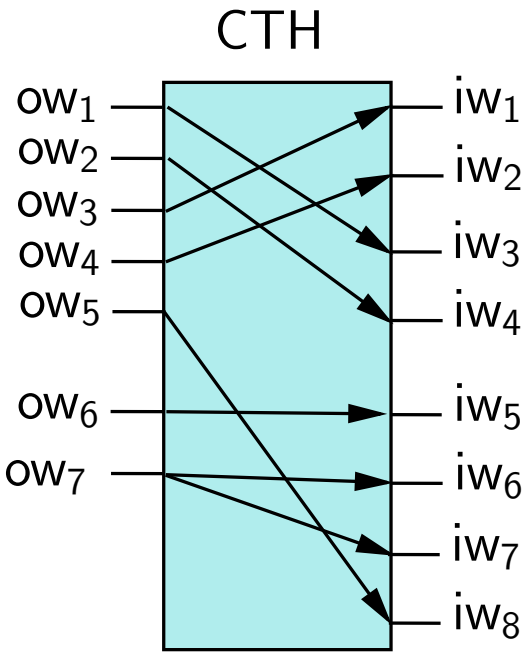

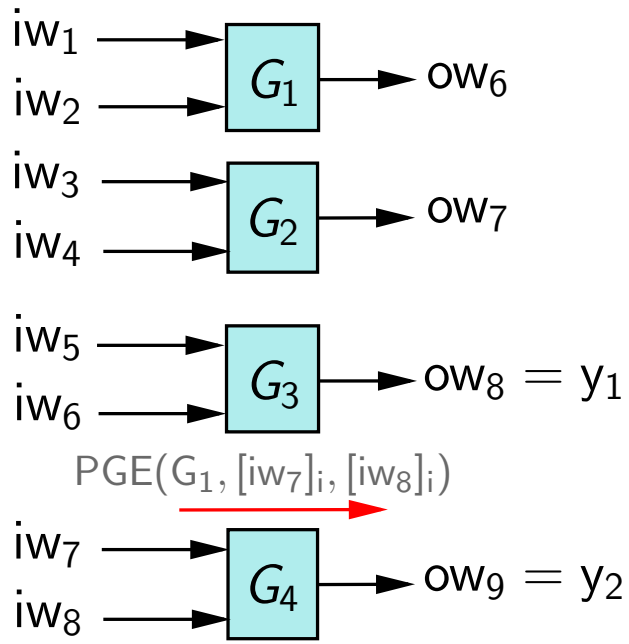

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Realization of CTH functionality

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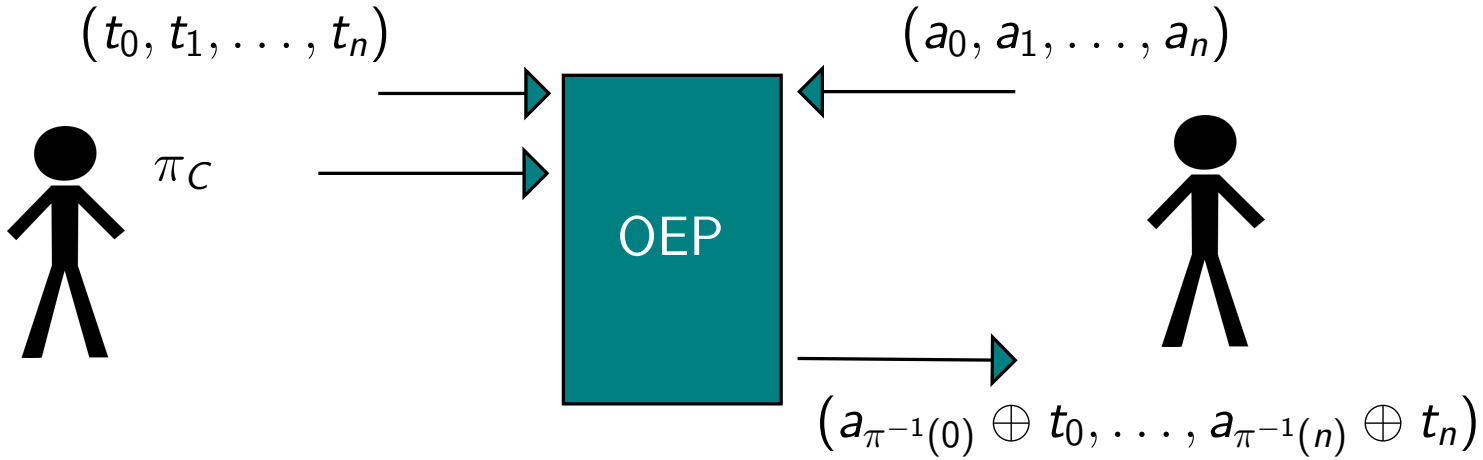
- A main component of our $\mathcal{F}_{\mathcal{CTH}}$ realization is a protocol for *oblivious evaluation* of this extended permutation (OEP)

OEP Protocol



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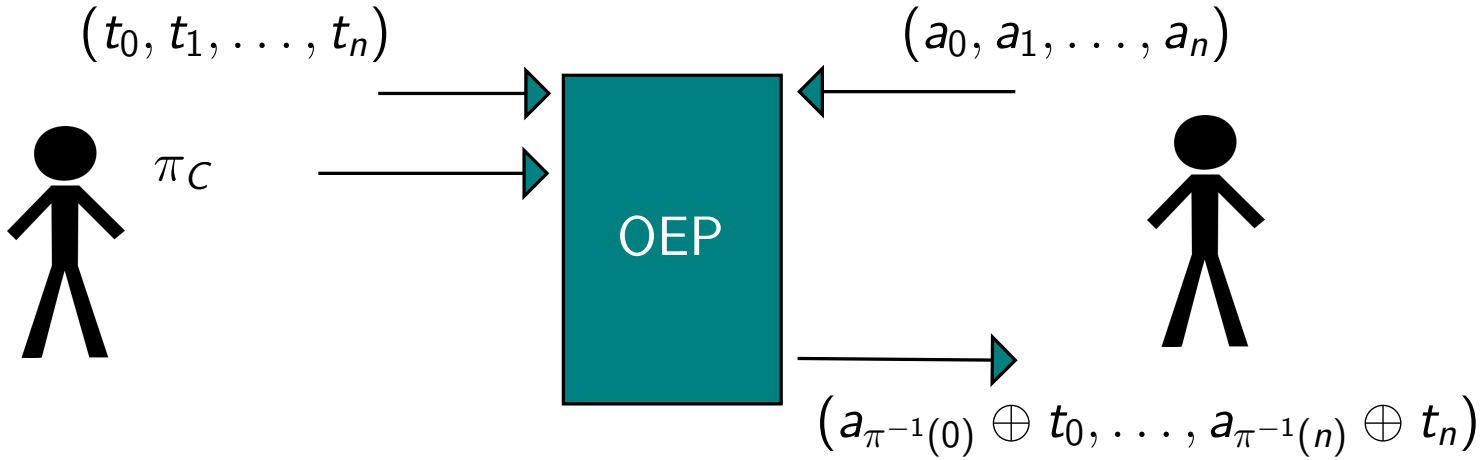

- HE-OEP: Can be realized using a singly homomorphic encryption

SN-OEP: OEP via Generalized Switching Networks

- Implement an extended permutation using a generalized switching network SN

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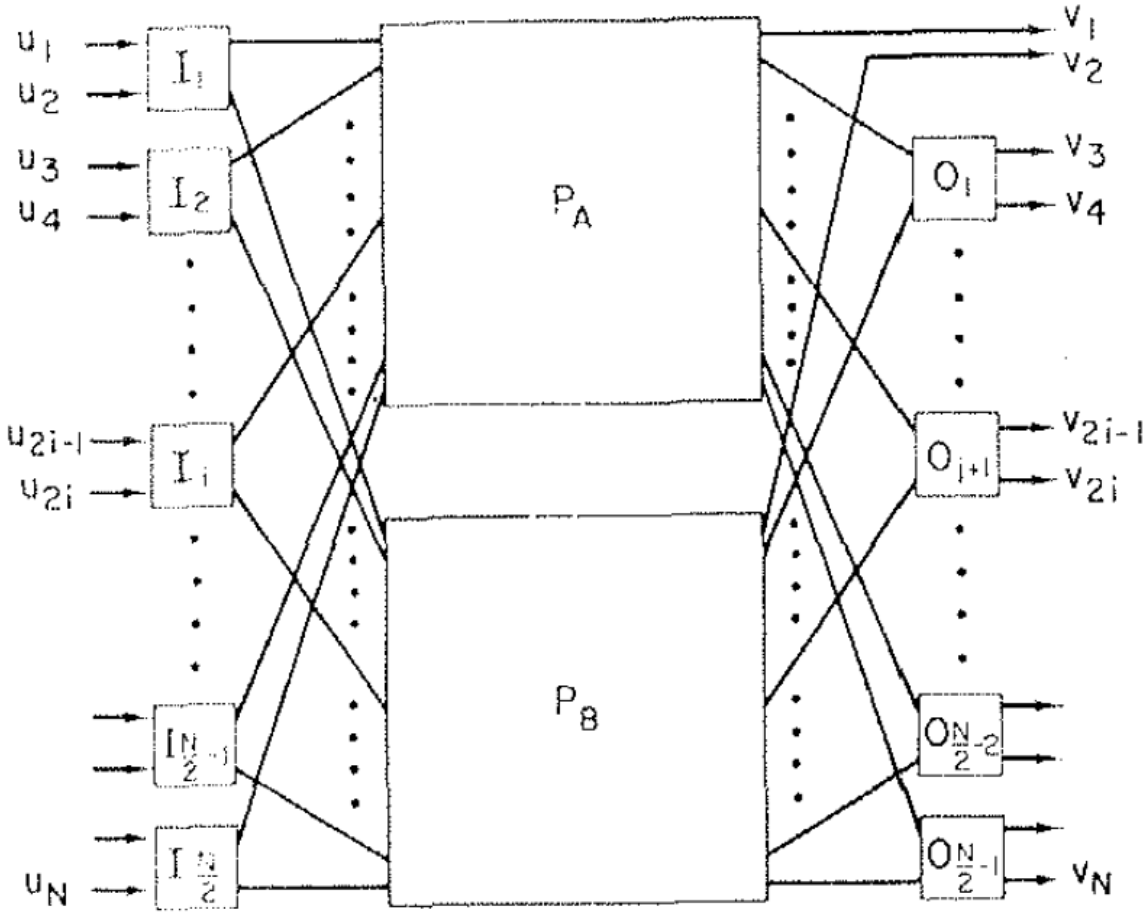
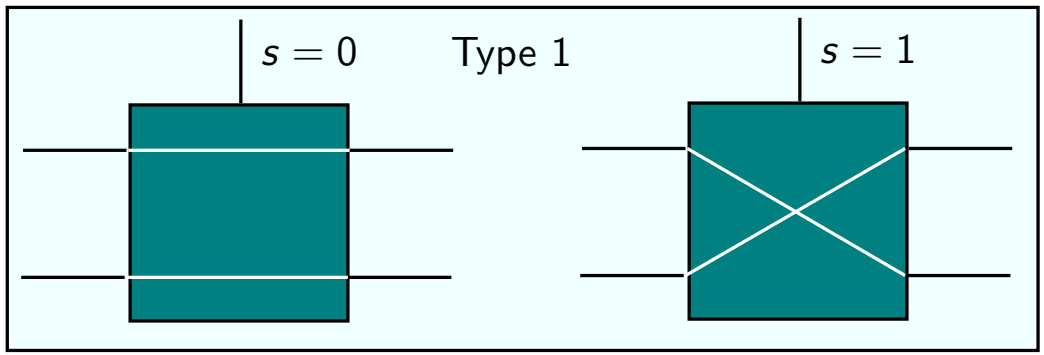
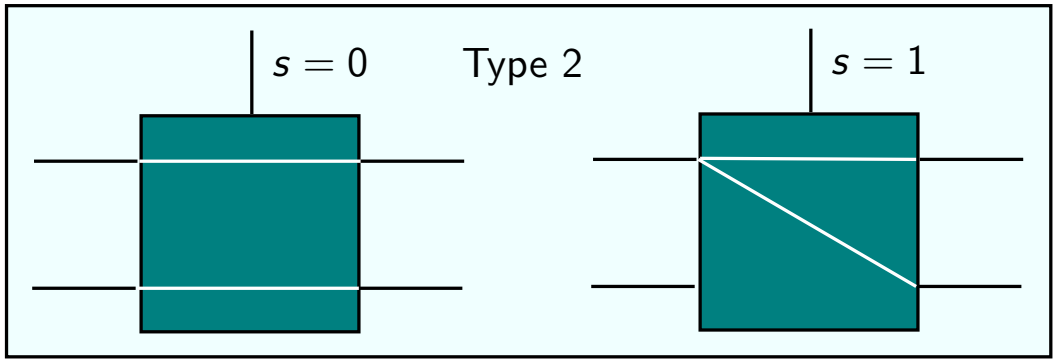
- Implement an extended permutation using a generalized switching network SN
- Obliviously evaluate the generalized switching network



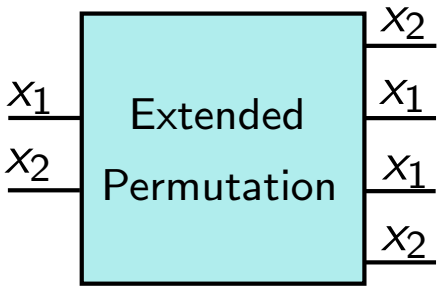
Generalized Switch

- In our generalized notion of a switch, each of the two output strings can take the value of each of the two input strings.
- We use the following types of switches in our EP construction:





Extended Permutation Network



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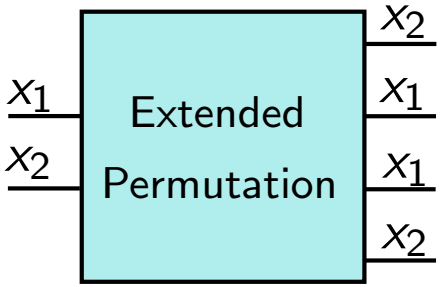

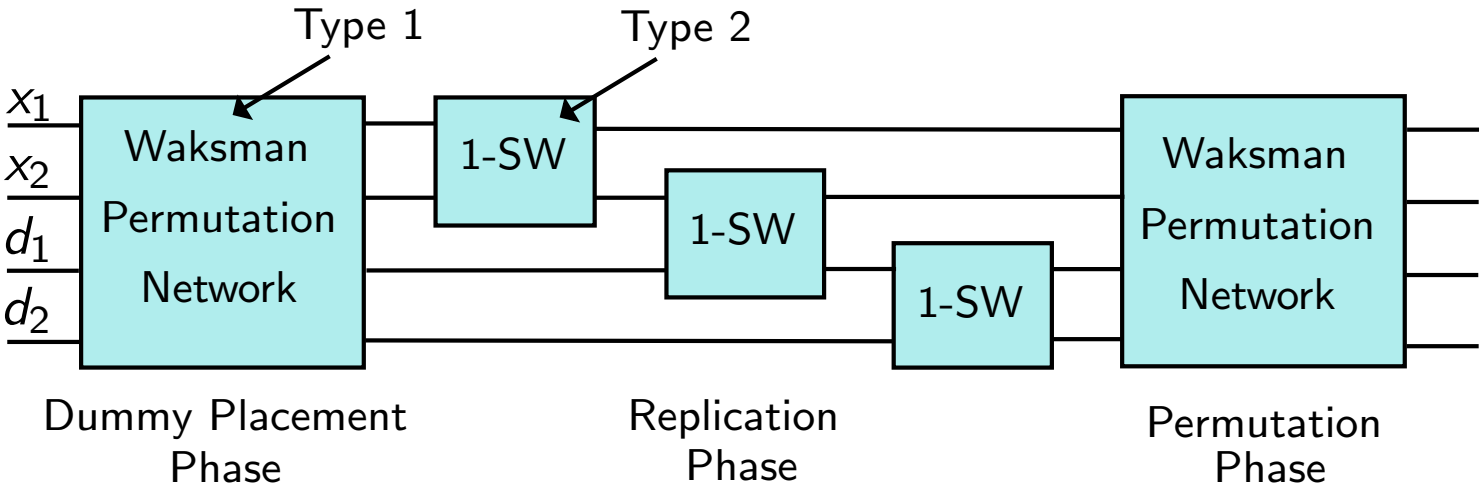

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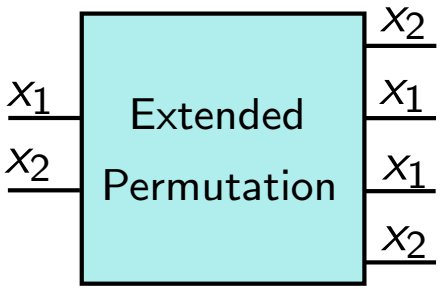

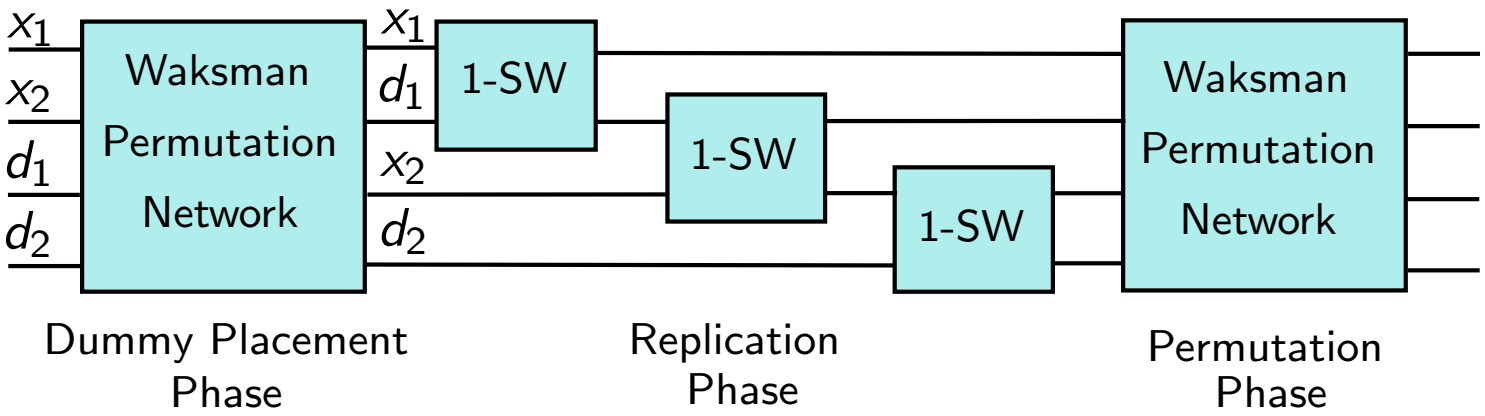

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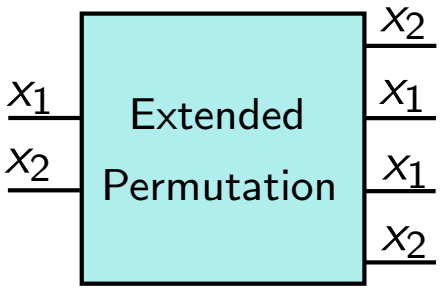

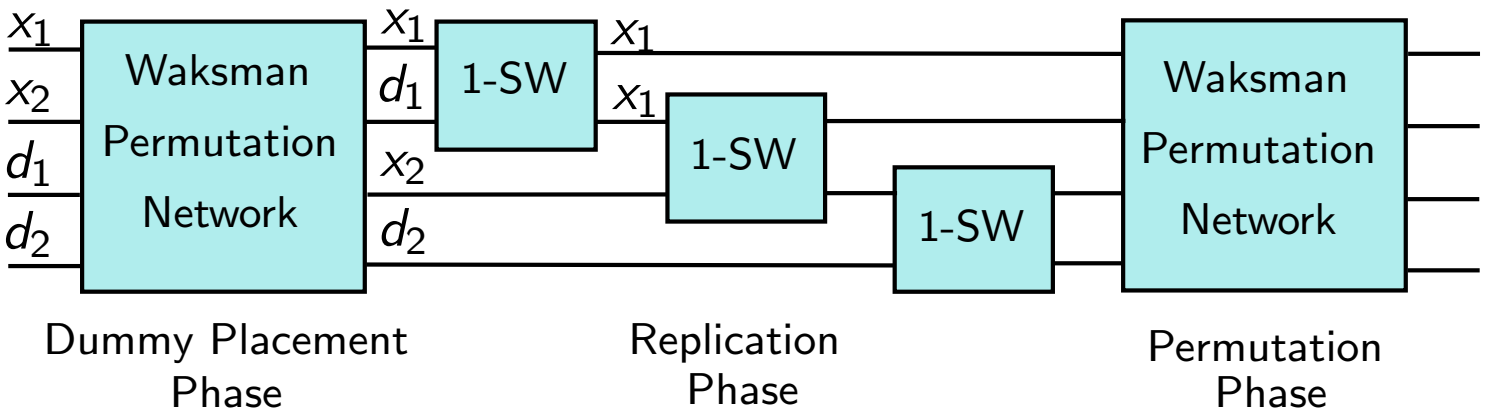

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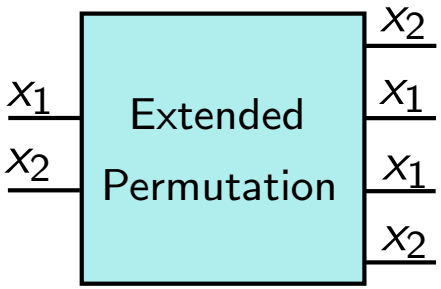

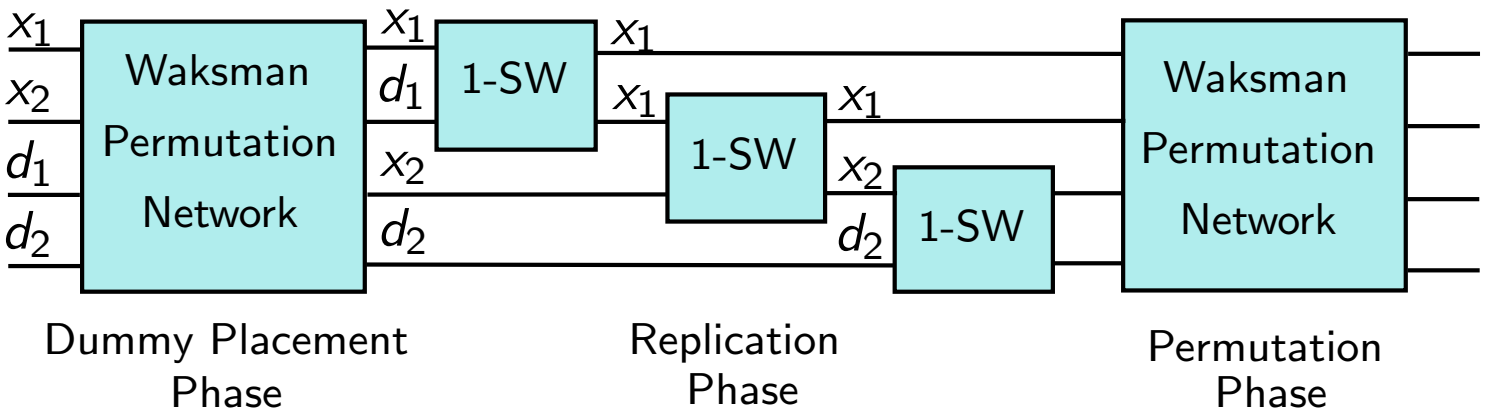

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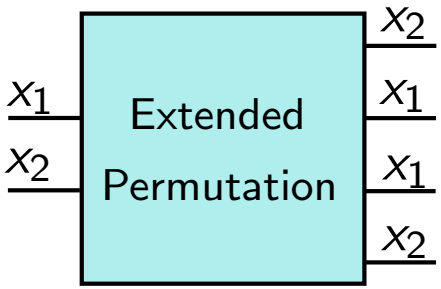

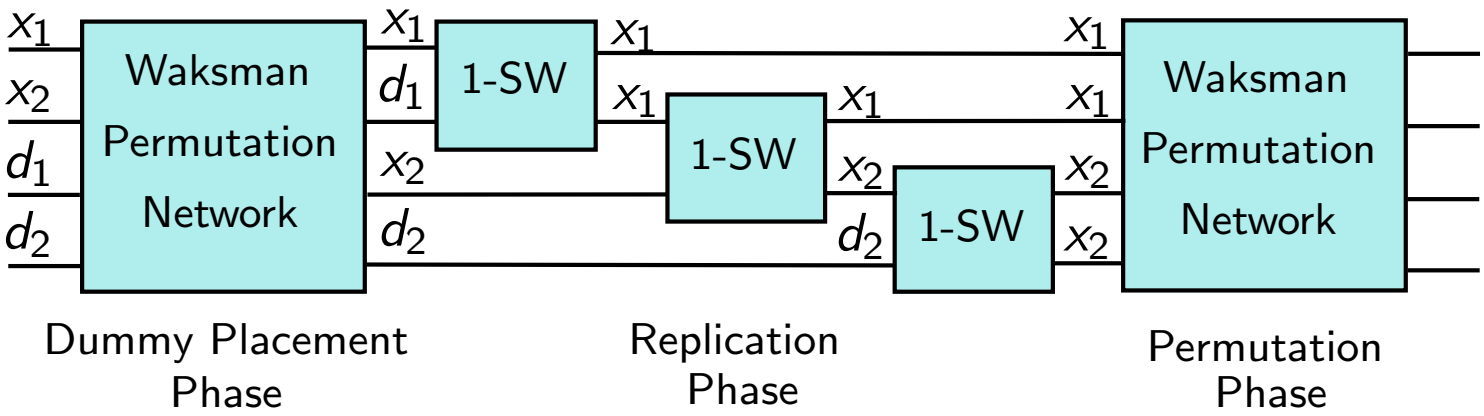

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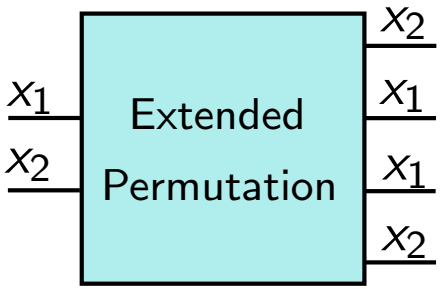

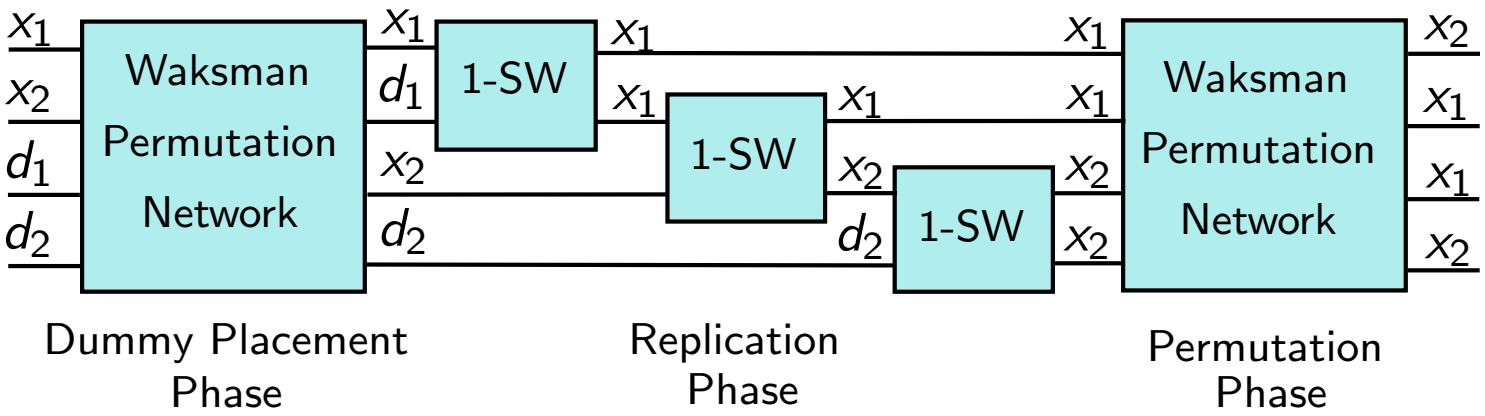

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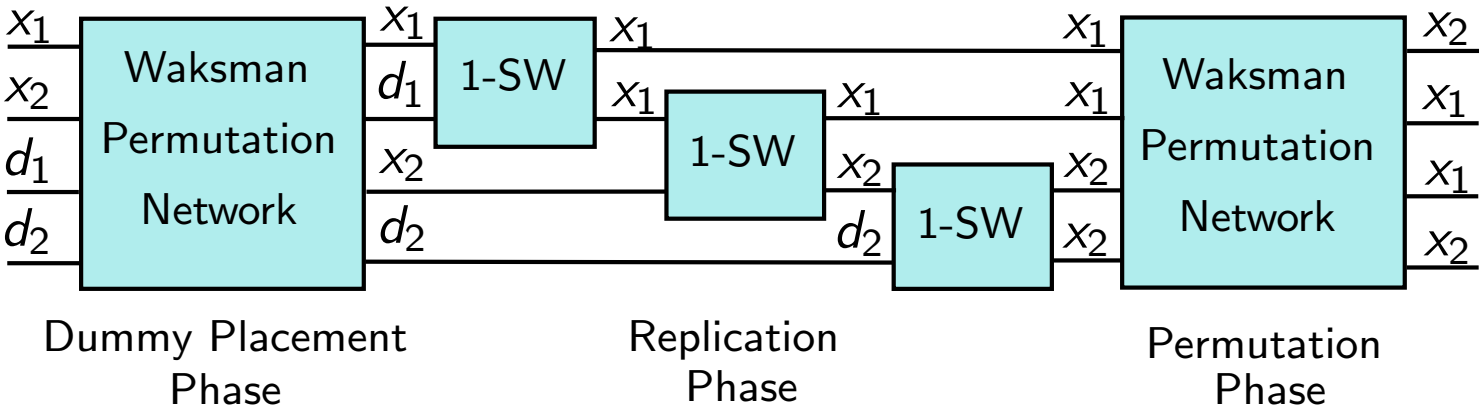
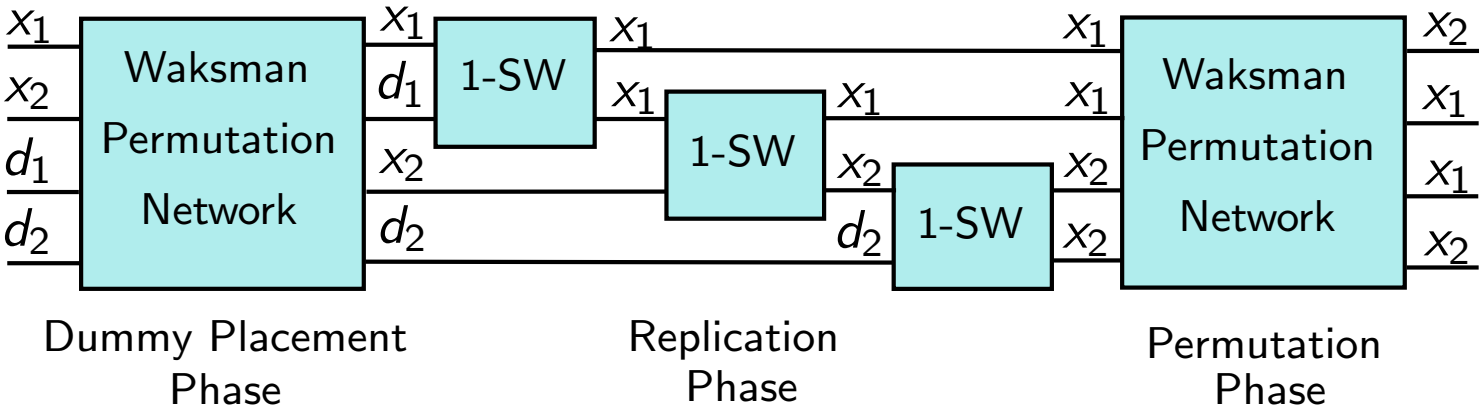


- Size of Waksman network: $N \log N - N + 1$.

Extended Permutation Network

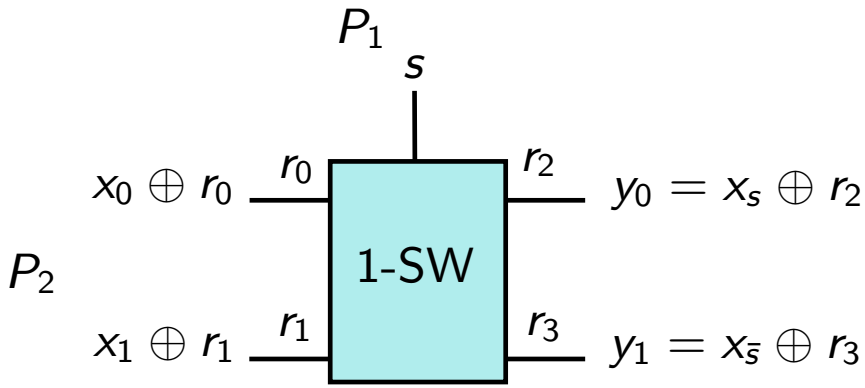


- Size of Waksman network: $N \log N - N + 1$.
- Size of EP network: $2(N \log N - N + 1) + N - 1 = 2N \log N - N + 1$

Oblivious Evaluation of Switching Network

Oblivious Switching Network Evaluation Protocol

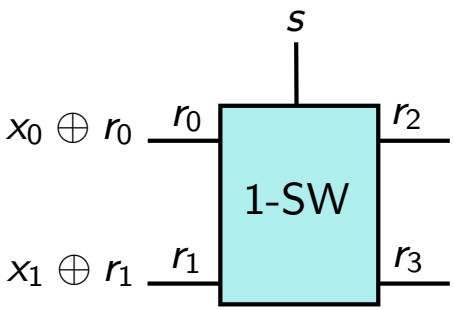
- Evaluate each switch



- Extend to the whole network

Oblivious Evaluation of Switch

- P_2 generates random values for each wire and prepares the following table and inputs. He then sends the inputs to P_1.



s		
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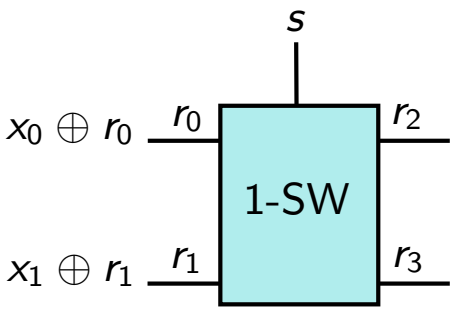
Oblivious Evaluation of Switch

- P_2 generates random values for each wire and prepares the following table and inputs. He then sends the inputs to P_1.



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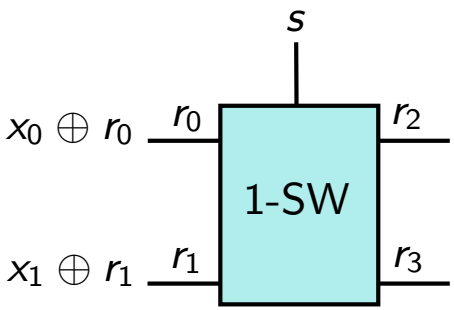

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- P_1 applies the switch and sends back the received inputs XORed with result of OT and his blinding. For $s = 0$ and y_0:

$$y_0 = (x_0 \oplus r_0) \oplus (r_0 \oplus r_2) = x_0 \oplus r_2$$

Improved Oblivious Shuffling

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- A subprotocol used for private set intersection

Oblivious Shuffling Prot.	Asymptotic Complexity	Eff. Gain
HE-Based	$O(N)$ Asym.	175
GC-Based [HEK12]	$(\frac{4\ell(N \log N - N + 1)}{3} + 2N\ell)$ Sym. $+ O(k)$ Asym.	25
OSN-Based (This work)	$(2N \log N - 2N + 2)$ Sym. $+ O(k)$ Asym.	1

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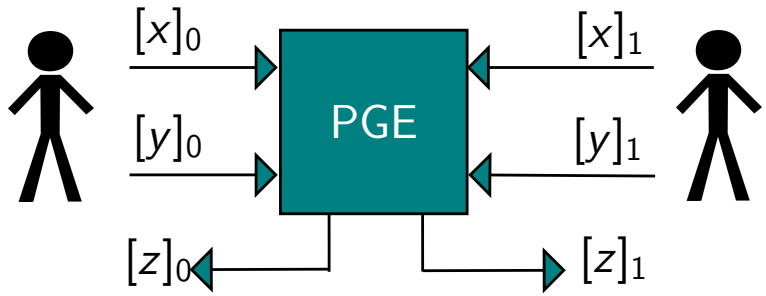

  - Create this table and use OT with the shares:

$[x]_1$	$[y]_1$	Output
0	0	$[z]_0 \oplus f([x]_0 \oplus 0, [y]_0 \oplus 0)$
0	1	$[z]_0 \oplus f([x]_0 \oplus 0, [y]_0 \oplus 1)$
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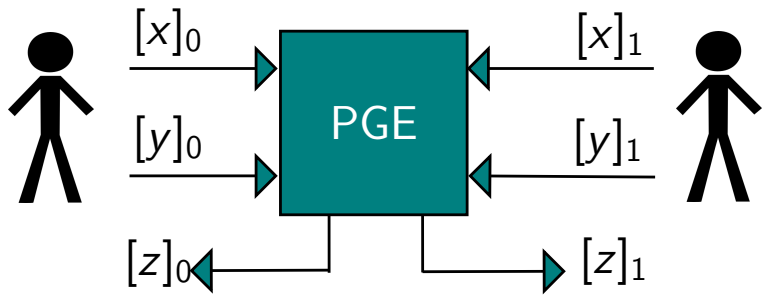

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- Plug into the framework

Multi-Party PFE

Multi-Party PFE	Complexity
[KS08] Universal Circuits	$O(m^2 g \log^2 g)$ Sym. + $O(k)$ Asym.
[Val76] Universal Circuits	$O(m^2 g \log g)$ Sym. + $O(k)$ Asym.
GMW-PFE (SN-OEP)	$O(m^2 g + mg \log g)$ Sym. + $O(k)$ Asym.
GMW-PFE (HE-OEP)	$O(m^2 g)$ Sym. + $O(mg)$ HE. + $O(k)$ Asym.

Constant round Two-Party PFE

2-Party PFE	Complexity
[KS08]	$(1.5g \log^2 g)$ sym. + $O(k)$ Asym.
[Val76]	$(19g \log g)$ sym. + $O(k)$ Asym.
[KM11]	$O(g)$ Sym. + $O(g)$ (HE+HM+HA) + $O(k)$ Asym.
Yao-PFE (HE-OEP)	$O(g)$ Sym. + $O(g)$ (HE+HA) + $O(k)$ Asym.
Yao-PFE (SN-OEP)	$(8g \log 2g)$ Sym. + $O(k)$ Asym.

PFE for Arithmetic Circuits

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Universal Arithmetic Circuits

While universal arithmetic circuits exist, their sizes are too large for any practical purpose (e.g. as high as $O(g^5)$).

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- Thank you !


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