Affine Transformations

Graphics I
Faramarz Samavati

Today!

- Affine space
- Motivation for transformations
- 2D rotation
- Affine transformation

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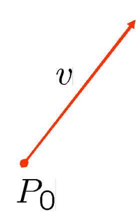
Comparison

vectors	points
Add and scale	Do not have
Direction and length	Do not have
Do not have a fixed position	Do have
Unaffected by translation	Moved by translation
u,v,w,…	P,Q,R,…

Affine Combination

- Definition
- Why does it make sense?

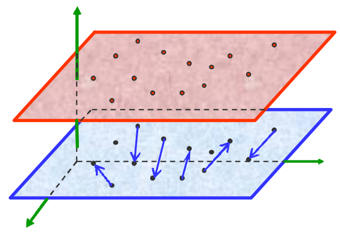
$$\sum_i \alpha_i P_i = P_0 + \underbrace{\sum_i \alpha_i (P_i - P_0)}_{v}$$



- A simple criteria to check validity of linear combinations

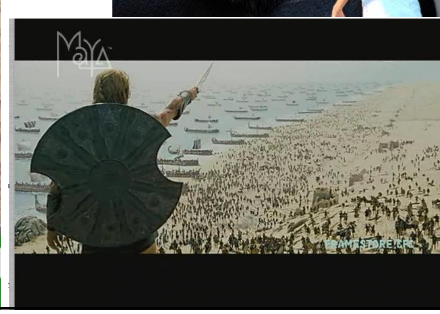
Affine Space

- A collection of elements(points)
- $\sum \alpha_i = 1$
- Associated vector space



Transformations

- Changing the size and orientation
- Placing a number of instances
- Expressing the symmetry
- Viewing
- Animation



Geri's Game Animation

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Translation

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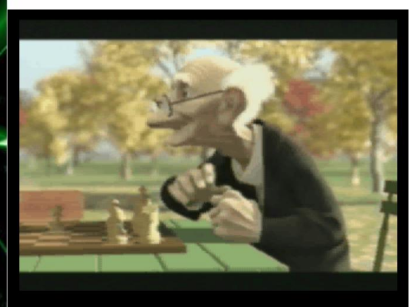
Scaling



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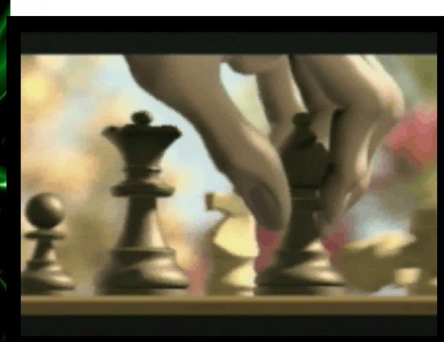
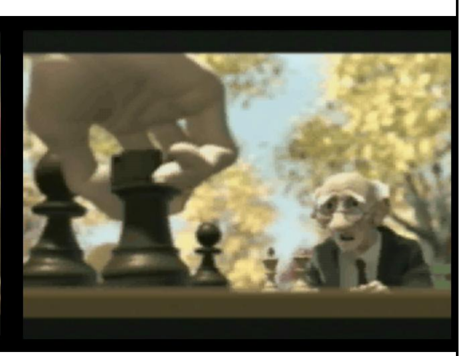
Viewing





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Rotation




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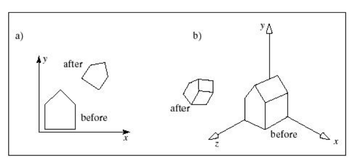
Introduction to Transformation



- Staple in computer graphics
- Example: rotation, scale, translate
- Viewing is another important transformation
- Adjust our objects in a proper size, situation and direction

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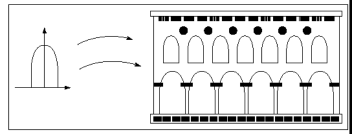
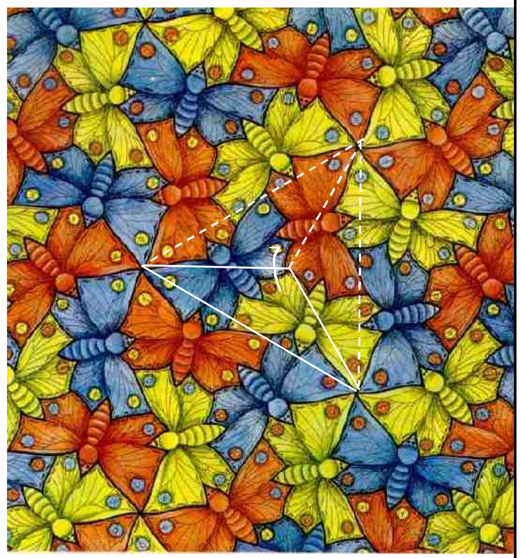
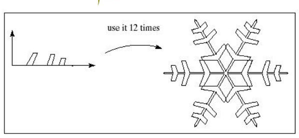
Other Benefits





- Compose a scene out of a number of similar objects
- Produce symmetric objects by designing a simple motif
- Produce complicated objects by iteration of some transformations



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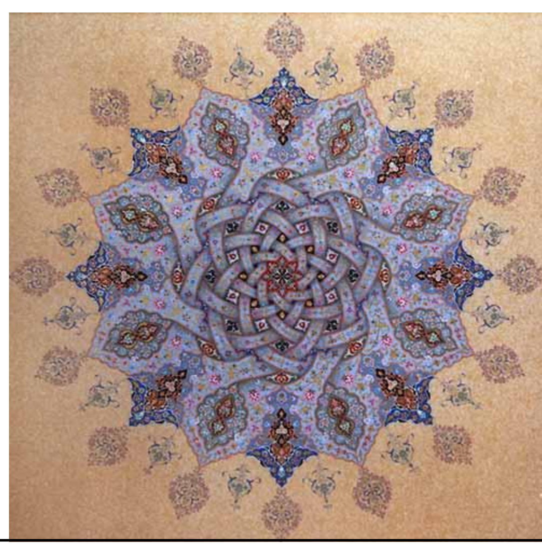
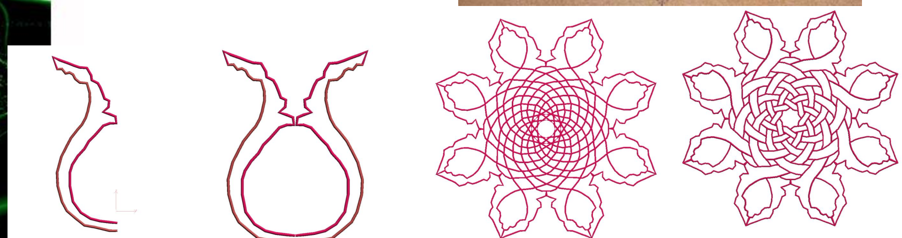
2D patterns and decorations

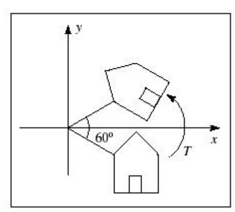


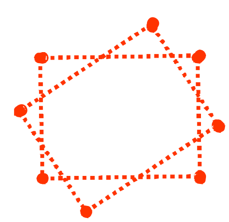
Motif



2D Rotation

- $$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$
- $P' = MP$
- It can be used for points or vectors
- Transforming endpoints is enough





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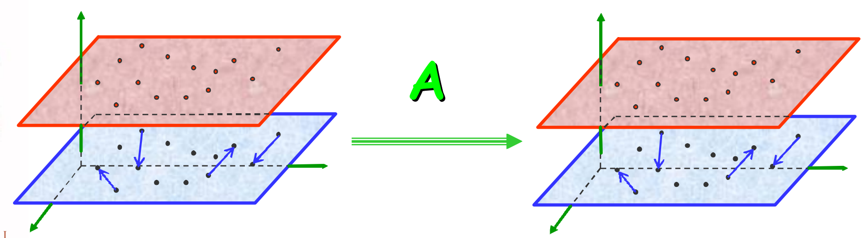
Which kind of transformations can preserve lines and poly-lines?

- Transforming end-points is enough
- Affine transformations
- Preserve Affine combination
- example



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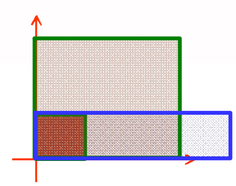
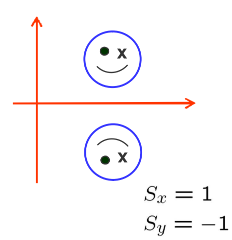
Scaling

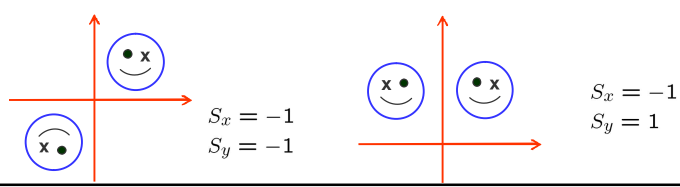


- Scaling about the origin
- Change the size of an object
- 2D matrix

$$\begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix}$$

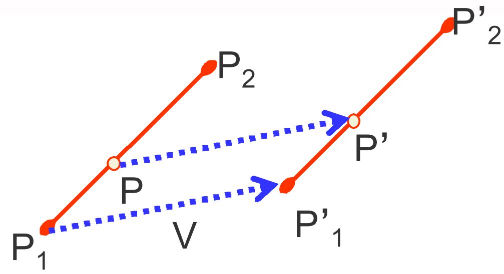
- Uniform and non-uniform scaling
- Negative factors produce reflection





Translation

- Point translation
- P'=P+v
- Matrix form?
- Is it Affine? Preserves lines?
- Does it work for points and vectors?



Answers

- No 2x2 matrix notation
- It is Affine
- Different interpretations for vectors and points
- We need a new representation

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Homogeneous Representation

shape	point •	vector →
Previous notation 2D and 3D	$\begin{bmatrix} x \\ y \end{bmatrix}$ $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$	$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$ $\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$
homogeneous 2D, 3D	$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$ $\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$	$\begin{bmatrix} v_1 \\ v_2 \\ 0 \end{bmatrix}$ $\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ 0 \end{bmatrix}$

- The reason of "Why do we need 4D coordinates?"

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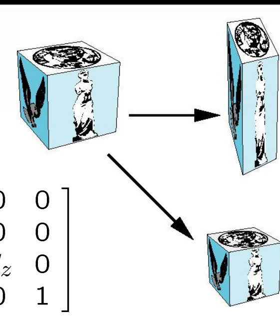
Useful Representation

- Good separation between vectors and points
- It shows directly why we have just Affine combinations
- Matrix notation for translation

Matrix Notation for Translation

- $$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & v_1 \\ 0 & 1 & v_2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$
- Points: moved by translation
- Vectors: unaffected by translation
- 2D translation in homogeneous coordinates
- 3D translation in homogeneous coordinates

3D transformations



- 3D scaling

$$\begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- 3D translation

$$\begin{bmatrix} 1 & 0 & 0 & v_1 \\ 0 & 1 & 0 & v_2 \\ 0 & 0 & 1 & v_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$