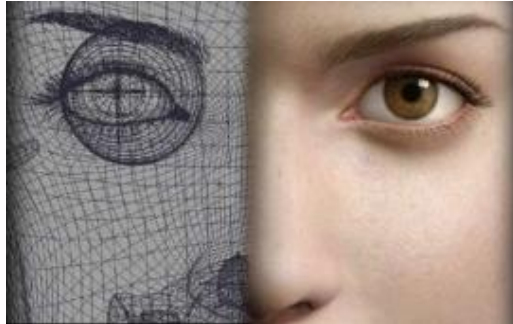


## Geometric Objects



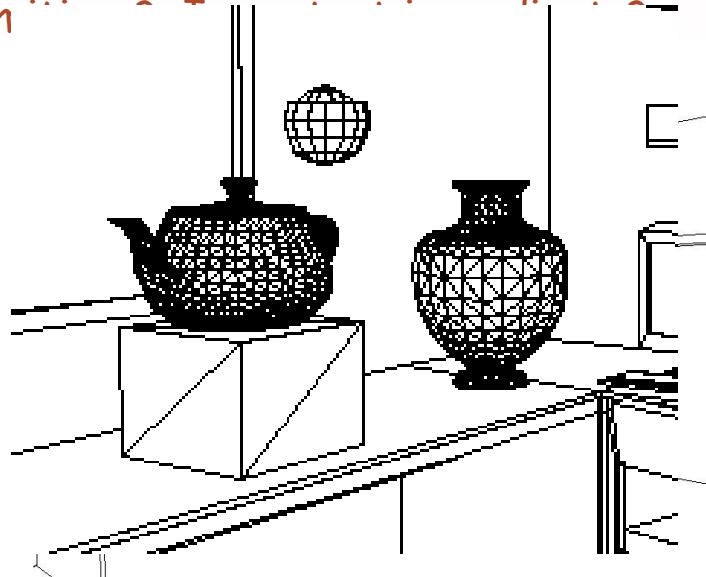
Geometry is the right foundation of all painting.

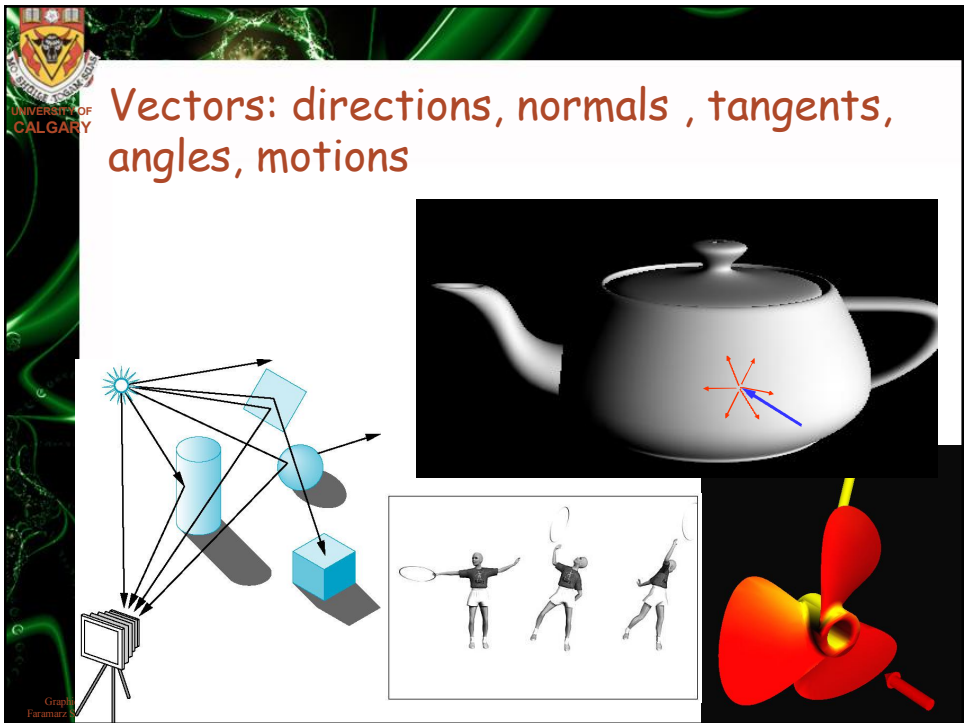
**Albrecht Dürer**

Profession: artist. Born 1471. Died 1528.

## What is the ambient space in graphics?

Prin





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# Transformations

- ❖ Changing the size and orientation
- ❖ Placing a number of instances
- ❖ Expressing the symmetry
- ❖ Viewing
- ❖ Animation



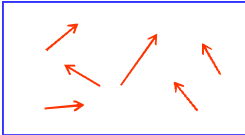


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# Geometric Objects

- ❖ Points, vectors, scalars
- ❖ Vectors in 2D
- ❖ Vectors in 3D
- ❖ Vector space

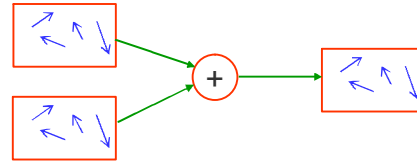
Vector space

<p style="text-align: center;">Entities (vectors)</p> 	<p style="text-align: center;">Scalars</p> <p style="text-align: center;"><math>\alpha, \beta, \gamma, \dots</math></p>
<p style="text-align: center;">Operations</p> <p style="text-align: center;">+ •</p>	

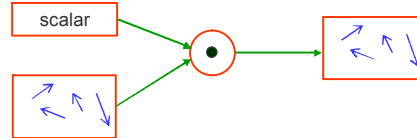
Graphics 1  
Farmanur Shamvati

## Properties

❖ Vector-vector addition



❖ Scalar multiplication



❖ Commutative

❖ Zero

❖ Associative

❖ Inverse

## Magnitude

❖ Length or magnitude of a vector

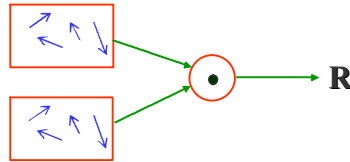
❖ Unit vector

❖ Normalized vector

❖ Direction

## Dot Product

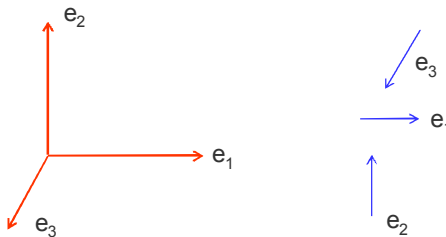
- ❖ Dot product or inner product



- ❖ Definition in 2D and 3D
- ❖ Properties

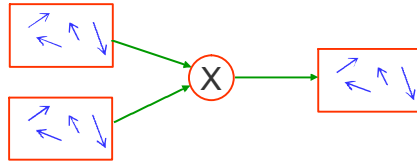
## Angle Between Two Vectors

- ❖ Angle between two vectors
- ❖ Orthogonality condition
- ❖ Orthonormal



## Cross Product

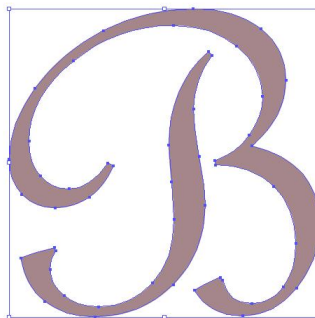
- ❖ Just for 3D



- ❖ Formula
- ❖ Geometric interpretation

## Points

- ❖ Geometric objects are based on points
- ❖ Location in plane or space
- ❖ Point-vector addition
- ❖ Point-point subtraction
- ❖ Point-point addition?!



## Comparison

vectors



Add and scale

Direction and length

Do not have a fixed position

Unaffected by translation

$u, v, w, \dots$

points



Do not have

Direction and length

Do have

Moved by translation

$P, Q, R, \dots$

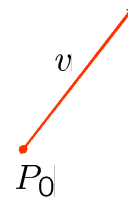
## Could we add points?

- ❖ Arbitrary linear combinations of points doesn't have a clear meaning
- ❖  $\frac{1}{2}P + \frac{1}{2}Q$  does have!
- ❖ Line as linear combinations of two points

## Affine Combination

- ❖ Definition
- ❖ Why does it make sense?

$$\sum_i \alpha_i P_i = P_0 + \underbrace{\sum_i \alpha_i (P_i - P_0)}_v$$



- ❖ A simple criteria to check validity of linear combinations

## Affine Space

- ❖ A collection of elements(points)

$$\sum \alpha_i = 1$$

- ❖ Associated vector space

