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# Modeling Curves and Surfaces

Graphics I  
Faramarz Samavati



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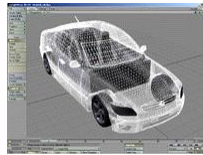
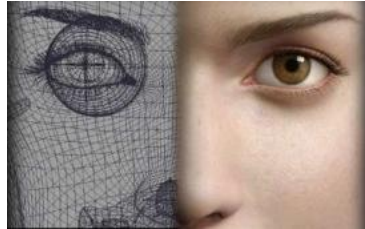
# Modeling for Computer Graphics!?



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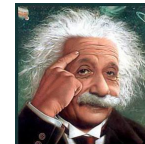
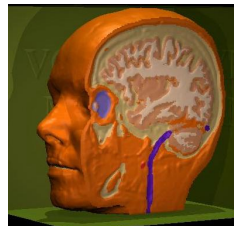
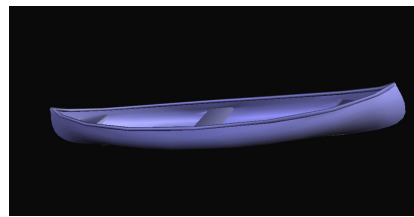
# How can we generate this kind of objects?



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# Umm!?! Mathematical Modeling!



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Do not worry too much about your difficulties in mathematics,  
I can assure you that mine are still greater.

## Our modeling plan

- ❖ Parametric curves
  - Bezier
  - B-Splines (subdivision methods)
- ❖ How to extend curves to the surfaces, images, volumetric data sets



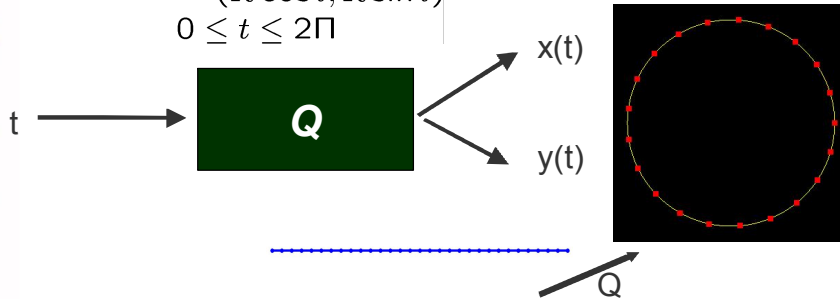
## Parametric versus implicit modeling

- ❖ Simple example: Circle
  - parametric

$$Q(t) = (X(t), Y(t))$$

$$= (R \cos t, R \sin t)$$

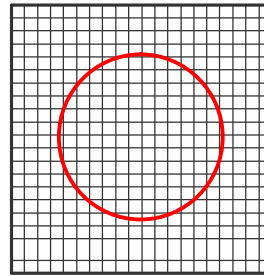
$$0 \leq t \leq 2\pi$$



## Parametric versus implicit modeling

$$f(x, y) = x^2 + y^2 - 1$$

all  $(x, y)$  such that  $f(x, y) = 0$



## Polynomial curves

- ❖ Polynomials are fundamental mathematical objects
- ❖ Easy and efficient to compute  $P(u) = a_0 + a_1u + \dots + a_nu^n$
- ❖ standard representation
- ❖ Polynomial curves of degree one,  
 $Q(u) = P_0 + P_1u + \dots + P_nu^n$

$$P_i = \begin{bmatrix} a_i \\ b_i \end{bmatrix}$$



## User input for a polynomial curve

- ❖ Formula!!! No way
- ❖ Giving 2D(or 3D) points as  $P_i$  “a good idea”
- ❖  $P_i$  : point  $\Rightarrow$  arbitrary combination of them is not possible. Why?
- ❖ Affine combination
- ❖ It is not possible to satisfy this condition by this set of polynomials



## Bernstein basis functions

- ❖ Terms in the expansion of:  $1 \equiv ((1 - u) + u)^n$
- ❖ Second degree polynomials
- ❖ Third degree polynomials
- ❖ Resulting curves

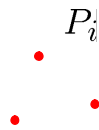


# Bezier Model

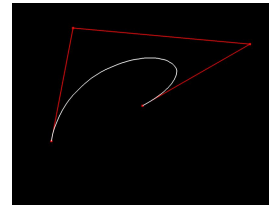


- ❖ P. Bezier,
- ❖ 1960, Unisurf in Renault automobile designing.
- ❖ Mathematical definition

$Q(u)$



Modeling



Point in  $\mathbb{R}^2$  or  $\mathbb{R}^3$

curve

- ❖ Old CAD and graphics software
- ❖ Postscript
- ❖ METAFONT