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# B-Spline Basis functions



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## Functions to Models

| Name: | Bezier | B-spline |
|---|---|---|
| Space: | Polynomials | Splines |
| Basis: | $B_{i,d}(u) = \binom{d}{i} u^i (1-u)^{d-i}$ | ??? |
| Model: Curves, surfaces, volumes | $Q(u) = \sum_{i=0}^{d} P_i B_{i,d}(u)$ | $Q(u) = \sum_{i=0}^{m} P_i N_{i,k}(u)$ |

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# B-Spline of order 1

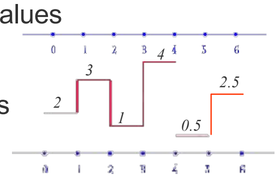
Consider a simple knot sequence : integer values

What is the Spline Space of order 1?

Piecewise constant functions, step functions

Order = 1 , degree=0

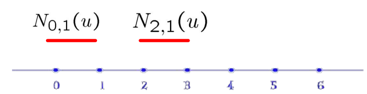
Continuity = -1 ⟶ Discontinuous




A practical example :

Any row or column of an image

B-Spline of order 1




Basis ⟶ $S(u) = 2N_{0,1}(u)+3N_{1,1}(u)+1N_{2,1}(u)+4N_{3,1}(u)+0.5N_{4,1}(u)+2.5N_{5,1}(u)$

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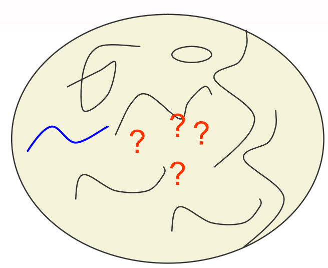
# B-Spline Basis

- ❖Basis set
- ❖many basis sets
- ❖B-Spline is the most famous basis set for Splines
- ❖The quickest way of definition




$$N_{i,1}(u) = 1 \quad u_i \le u < u_{i+1},$$

$$N_{i,1}(u) = 0 \quad otherwise.$$

$$N_{i,r-1} \xrightarrow{\omega_{i,r}} N_{i,r}$$

$$N_{i+1,r-1} \xrightarrow{1-\omega_{i+1,r}} N_{i,r} \qquad \omega_{i,r} = \frac{u-u_i}{u_{i+r-1}-u_i}$$

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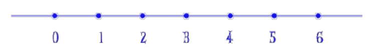
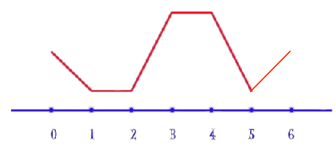
# B-Spline of order 2(Linear B-Splines)

- the same knot sequence:



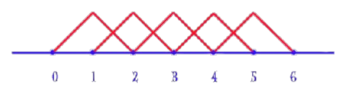

- What is the Spline Space of order 2?
- Piecewise linear functions or polyline




Order = 2 , degree = 1

Continuity = 0 , continuous

- Basis function = B-Spline of order 2
- A special situation at the start and at the end knots



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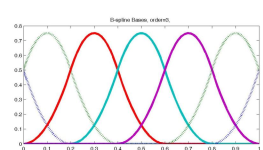
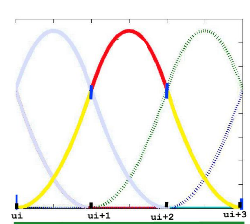
# B-Spline of order 3

- The same knot sequence
- quadratic spline
- Order = 3 , degree = 2 , continuity = ?
- 3 nonzero segments, why?



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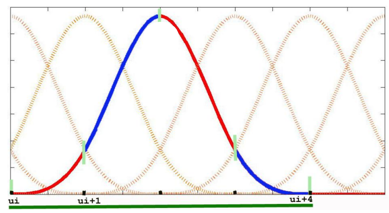
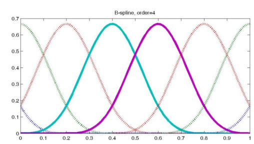
# B-Spline of Order 4(cubic B-spline)

- Piecewise cubic functions
- Order = 4, Degree = 3
- Continuity?
- 4 segments

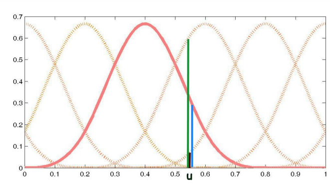


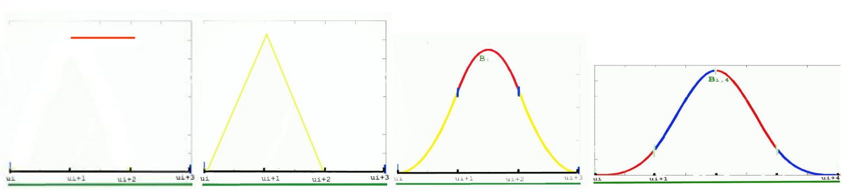





# B-Spline Properties

- Basis set for Spline space
- Positivity $N_{i,k}(u) \geq 0$
- Compact support $(u_i, u_{i+k})$
- Unit summation $\sum_i N_{i,k}(u) = 1$



# B-Spline Curves

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## Outline

- B-Spline functions (basis functions) and their properties
- B-Spline curves
- Properties of B-Spline curves
- Multiple knots
- Efficient Algorithm for B-Spline curves

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# B-Spline Curve

- Similar to Bezier curve a Linear combination of control points

$$Q(u) = \sum_{i=0}^{m} P_i N_{i,k}(u) \quad u_{k-1} \leq u \leq u_{m+1}$$

- m = number of control points (plus one!)
- Parameter range
- Knot sequence has been fixed( a set of non-decreasing real numbers)



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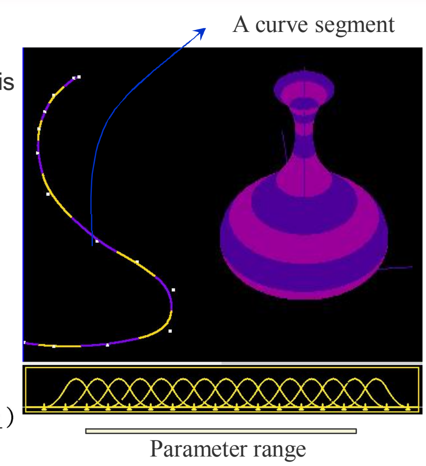
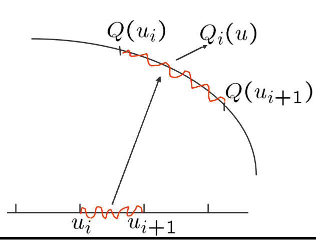
# B-Spline Curve

- m is not coupled with k
- For example : m=13 and k=3 in this curve
- Pi may has two or three components
- Parameter space, Curve space
- segments of the curve



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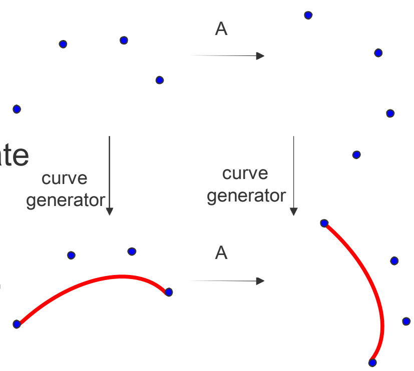
# Properties

- It is a $k^{th}$ order Spline
- Local Modification
- Convex-Hull and strong convex-Hull properties
- Affine Invariance: coordinate free definition

Affine transformation:
translation, rotation, shear, scaling.



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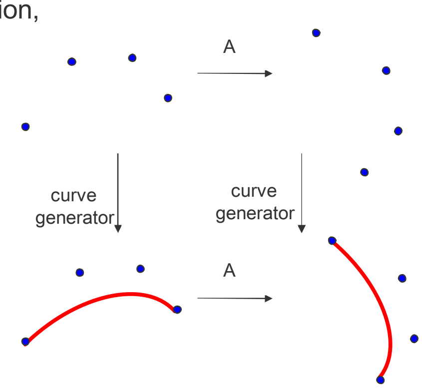
# Properties : Affine Invariance

Affine transformation (translation, rotation, shear, scaling)

A(P) = M.P + V

M: a 2x2 matrix for 2D points
or 3x3 matrix for 3D points

V: a translation vector



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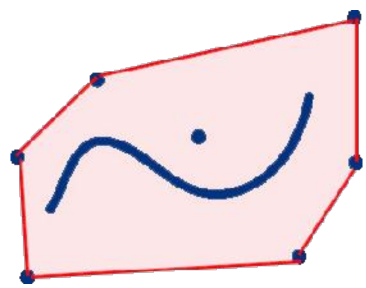
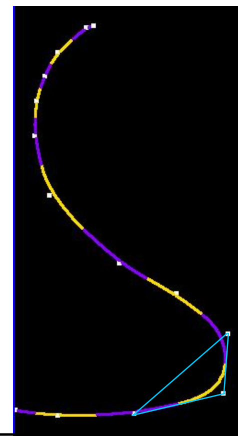
# Properties: convex hull

- Convex hull property
- Strong convex hull property





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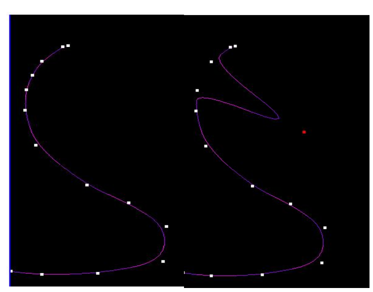
# Properties: Local control

- Moving a control points changes only k segments of the curve
- Because basis functions have local support



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# Multiple knots

- Vary the spacing between knots can control the shape
- multiple knot in the limit