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Parametric Curves

Modeling for computer graphics
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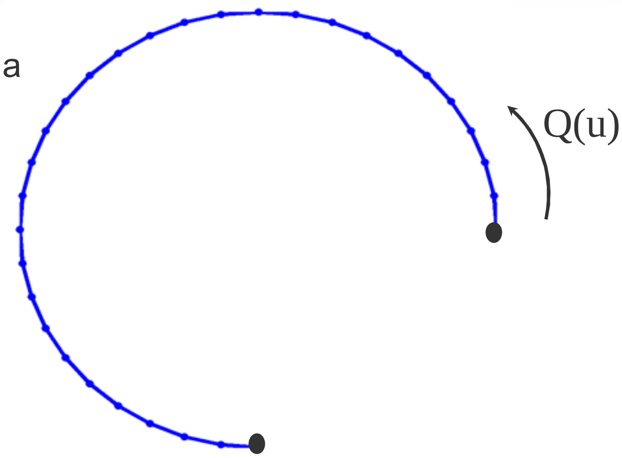
Goal/importance/outline

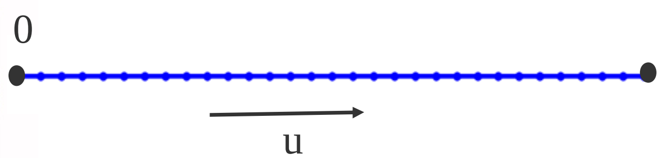
- Goals
  - To provide necessary and general knowledge about parametric curves
  - To determine the key property of curves for computer Graphics
  - To introduce interactive parametric curves starting with Bezier curve
- Outline
  - A review on parametric curves
  - Geometric continuity and parametric continuity
  - Polynomial curves

Parametric Curves

- Continuous deformation of a line segment
- u is parameter
- Vary u, find (x,y) from the formula, plot(x,y)



$$Q(u) = (X(u), Y(u))$$
$$= (R\cos u, R\sin u)$$
$$0 \leq u \leq \tfrac{3}{2}\Pi$$



Motion representation

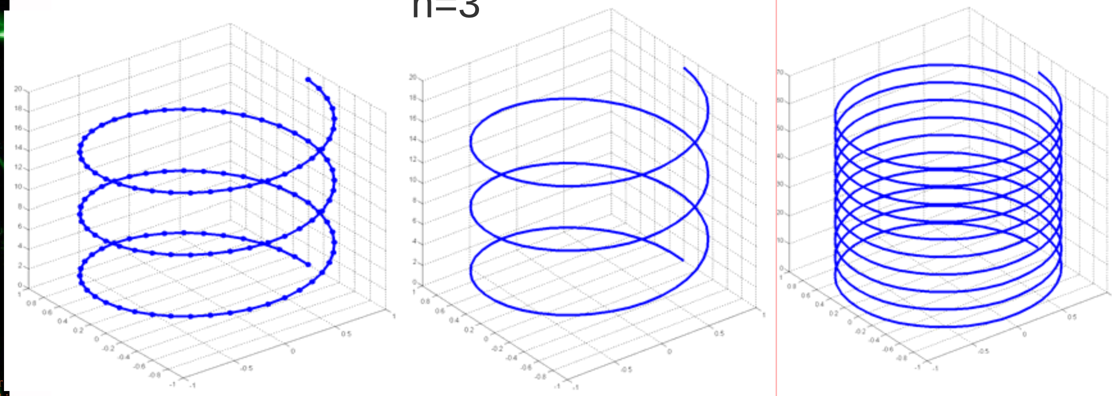
- Change the notation, u by t, Q(t)=(x(t), y(t))
- t: time and Q: path or trajectory of a moving particle
- Parametric representation can be also used for moving object and camera in animation

3D Parametric Curves

- Q(u)=(x(u), y(u), z(u))
- Example: helix (visualization of DNA)
- Q(u)=(cos(u), sin(u), u)
- u in [0, n*2Pi]

n=3

n=10



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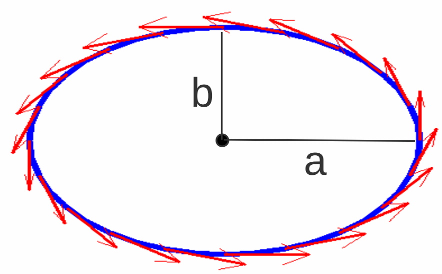
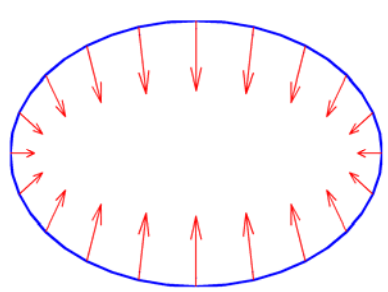
Features: Tangent vectors

- Many features can be found from parameterization
- As an example: Tangent vector
- T(u)=(x'(u), y'(u))
- For any u, T(u) is a vector
- In motion, T(u) represents velocity vector at each u
- Example: tangent vector for ellipse
- T(u)=(-a sin u, b cos u)



Normal vectors (for 2D curves)

- Rotate T(u) with 90 degrees
- N(u)=(-y'(u), x'(u))
- Example: ellipse
- N(u)=(-b cos u, -a sin u)



Goal/importance/outline

- Goals of this lecture
  - To provide necessary and general knowledge about parametric curves
  - To determine the key property of curves for computer Graphics
  - To introduce interactive parametric curves starting with Bezier curves
- Importance of this lecture
  - high: to have a general knowledge about parametric curves
- Outline
  - Review on parametric curves
  - Geometric continuity and parametric continuity
  - Polynomial curves

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Uniqueness of parametric representations!?

- Same path but with different ways of moving
- Example !?

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Uniqueness of parametric representation!?

- Many ways of motion along a given path (starting, stopping, accelerating at different points!)
- Those cars which travel the same highway at different speeds and stopping and starting at different places
- Same trajectory but different parameterizations
- A smooth trajectory can have a non-smooth parameterization

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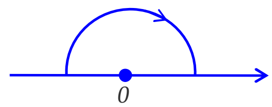
Example

- The unit circle with two different parameterization

$$Q_1(t) = \begin{bmatrix} \sin(t) \\ \cos(t) \end{bmatrix}, \quad -\frac{\pi}{2} \le t \le \frac{\pi}{2}$$

$$Q_2(t) = \begin{bmatrix} t-1 \\ 2t-t^2 \end{bmatrix}, \quad 0 \le t \le 2$$



Parametric versus geometric continuity

- Parametric continuity : smoothness of motion
- Geometric continuity : smoothness of the trajectory(curve)

Polynomial curves

- Polynomials are fundamental mathematical objects
- Easy and efficient to compute
- standard representation $P(u) = a_0 + a_1 u + \dots a_n u^n$
- Polynomial curves of degree one , two and three

$$Q(u) = P_0 + P_1 u + \dots P_n u^n$$

where

$$P_i = \begin{bmatrix} a_i \\ b_i \end{bmatrix}$$

User input for a polynomial curve

- Formula!!! No way
- Giving 2D(or 3D) points as Pi "a good idea"
- Pi : point => arbitrary combination of them is not possible. Why?
- Affine combination
- It is not possible to satisfy this condition by this set of polynomials

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Bernstein basis functions

- Terms in the expansion of: $1 \equiv ((1-u)+u)^n$
- Second degree polynomials
- Third degree polynomials
- Resulting curves