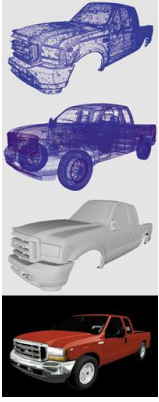
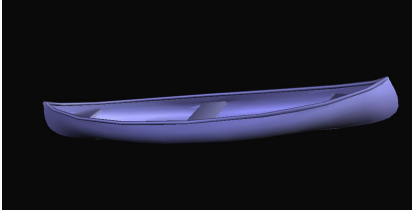



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## Parametric Curves Splines



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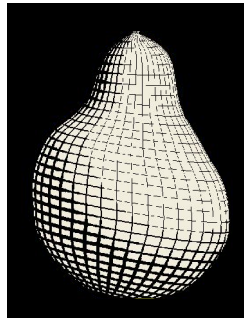
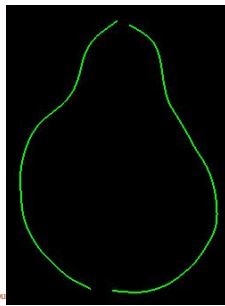
## Goal/outline

- ❖ Goals of this lecture
  - To introduce better basis functions
  - To extend polynomial curves to Splines that form most important modeling technique in graphics
- ❖ Outline
  - **Flaws of Bezier curve**
  - Wish list of basis functions
  - Spline functions
  - B-Splines

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## Be Patient: surfaces can be generated from the curves!

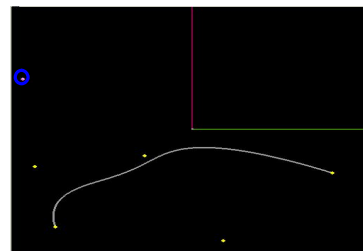
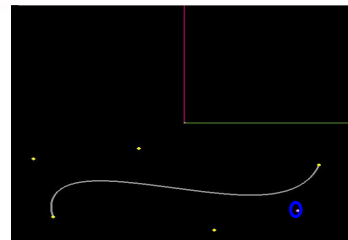
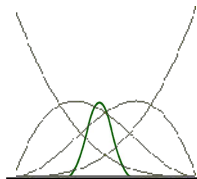


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## The lack of local control


- ❖ A change to any control point alters the entire curves
- ❖ Nature of the basis functions




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## The problem of the degree

- ❖ High degree polynomial → 
- ❖ How can we increase the control over the curve without increasing the degree?
- ❖ More control = higher degree = more computation and more round-off error accumulation

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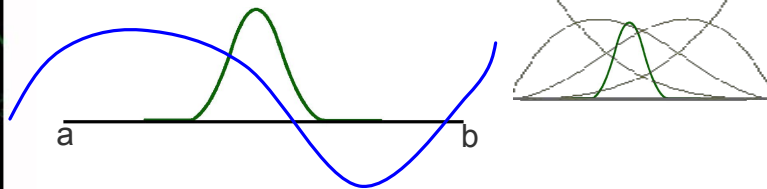


## B-Spline Basis functions

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## Wish list for basis functions

- ❖ Curve construction:  $Q(u) = \sum_{i=0}^d P_i B_{i,d}(u)$  ,  $a \leq u \leq b$
- ❖ Change of the Basis functions
- ❖ The basis functions should:
  - Be easy to compute
  - Sum to unity at every  $u$
  - Have support only a small portion of  $[a,b]$
  - Be smooth enough to produce smooth curves



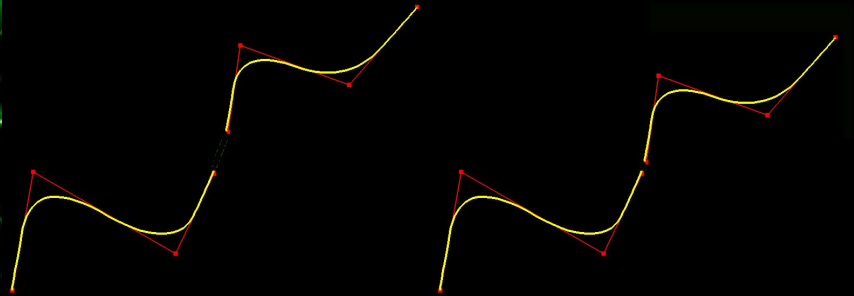
## No chance for polynomials

- ❖ Low degree polynomial can be easily compute
- ❖ Can we have all properties for them?
- ❖ A polynomial can not have a local support (why?)



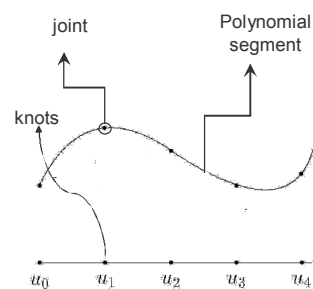
## Motivation

- ❖ Composite Bezier curve: join low degree Bezier curves in a smooth manner.



## Piecewise polynomial functions


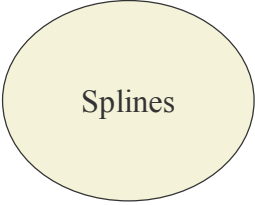
- ❖ Consists of several polynomials
- ❖ Segment polynomials
- ❖ Joint
- ❖ Knot
- ❖ Smoothness (or continuity) at knots



$$f(u) = \begin{cases} u^2 & u \geq 0 \\ -u^2 & u < 0 \end{cases}$$

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## Functions to Models

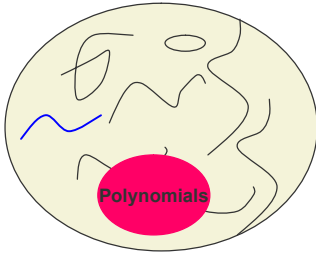
Name:	Bezier	B-spline
Space:		
Basis:	$B_{i,d}(u) = \binom{d}{i} u^i (1-u)^{d-i}$	???
Model: Curves, surfaces, volumes	$Q(u) = \sum_{i=0}^d P_i B_{i,d}(u)$	$Q(u) = \sum_{i=0}^m P_i N_{i,k}(u)$

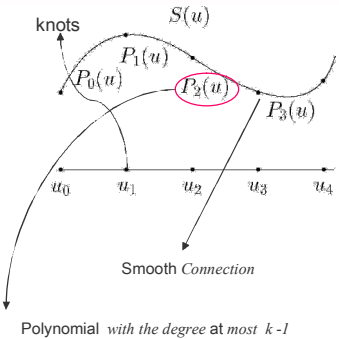
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## Splines Space

- ❖ Relationship of the spaces
- ❖ Basis set?





Smooth Connection

Polynomial with the degree at most  $k-1$

How many functions do live in that world!?

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