# CPSC 453:
# L-Systems

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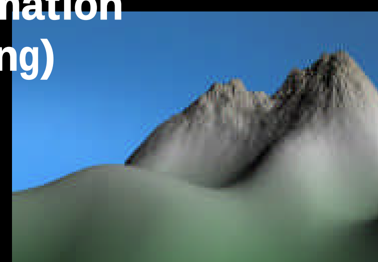
## Procedural Modeling

- Is it efficient to model these objects manually?

## Procedural Modeling

- The automatic generation of objects/animation using algorithmic techniques
- Significant time savings in modelling
- More abstract control of object shape
- Can also be used for animation (Physically Based Modeling)



(P. MacMurchy)

## L-systems

- A procedural modeling method often used for plants
- Invented by A. Lindenmayer
  Mathematical Models for Cellular Interaction in Development, Part I and II, Journal of Theoretical Biology, 18, 280-315, 1968
- Capture the development of components over time
- Division of mother cell A into two daughter cells B and C

$$A \rightarrow B\ C$$

- Parallel rewriting

# Definition

- **An L system consists of 3 things:**

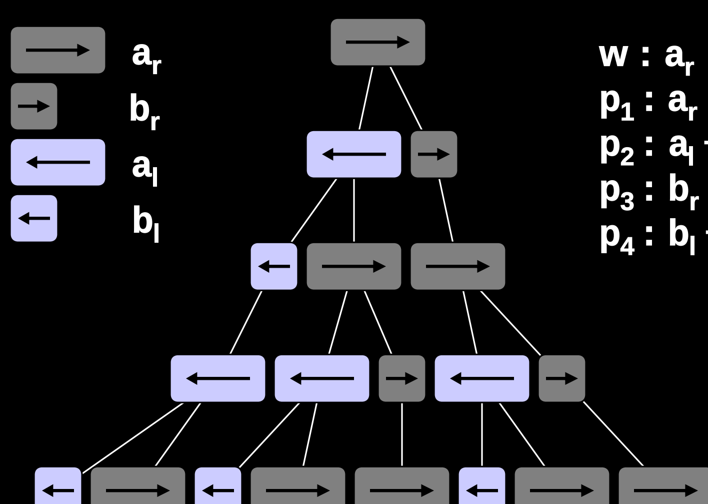
  G = hV; ! ; P i

  - **an *alphabet:*** V
    **eg. A B C D I X**

  - **an *axiom:*** !
    ***eg. C***

  - **a set of *productions:*** P ½ V £ V ¤
    **eg. A? BC**
    **C? DX**
    **X? A**

# L-systems (cont.)




**w : $a_r$**
**$p_1$ : $a_r \rightarrow a_l b_r$**
**$p_2$ : $a_l \rightarrow b_l a_r$**
**$p_3$ : $b_r \rightarrow a_r$**
**$p_4$ : $b_l \rightarrow a_l$**

**Anabaena catenula**

# Basic notation

string$_i$ $b_l\ a_r\ a_r$

productions

$p_1 : a_r \rightarrow a_l\ b_r$
$p_2 : a_l \rightarrow b_l\ a_r$
$p_3 : b_r \rightarrow a_r$
$p_4 : b_l \rightarrow a_l$

predecessor successor

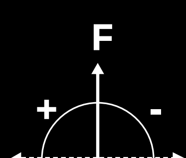
string$_{i+1}$ $a_l\ a_l\ b_r\ a_l\ b_r$

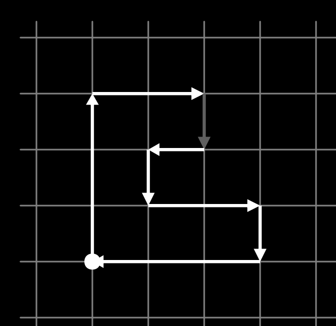
derivation step

# Graphical interpretation

- Turtle geometry

| | |
|---|---|
| *F* | move forward and draw line |
| *f* | move forward |
| + | rotate left |
| - | rotate right |



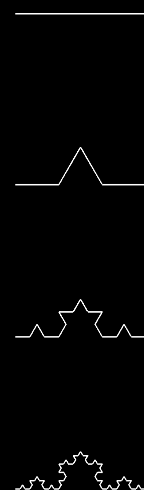



FFF-FF-f-F+F+FF-F-FFF


- H. Abelson and A. diSessa, Turtle geometry, MIT Press, Cambridge, 1982
- P. Prusinkiewicz, Graphical Application of L-systems, Graphics Interface, 1986

## Snowflake

$n=2$, $\alpha=60°$
w : F
$p_1$: F → F+F--F+F



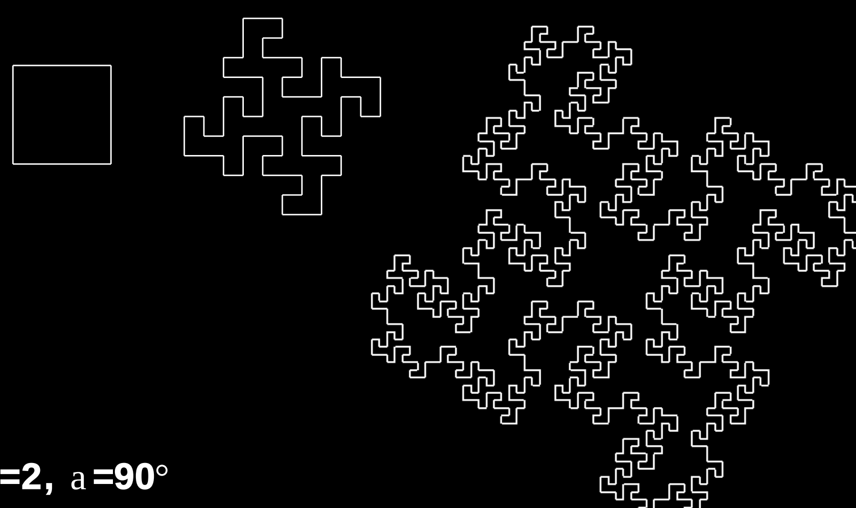
F

F+F--F+F

F+F--F+F+F+F
--F+F--F+F--F+
F+F+F--F+F

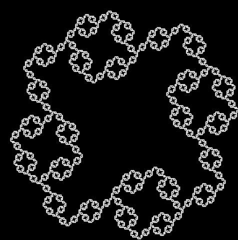
...

## Fractals

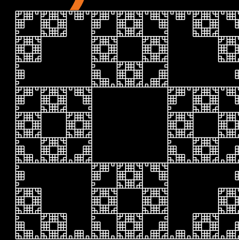


$n=2$, $\alpha=90°$
w : F-F-F-F
$p_1$ : F → F+FF-FF-F-F+F+FF-F-F+F+FF+FF-F

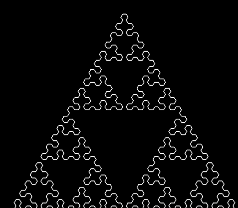
# Fractals (cont.)



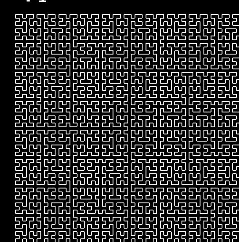
n=4, a=90°, F+F+F+F
$p_1$ : F → FF-F-F-F-F-F+F



n=4, a=90°, F-F-F-F
$p_1$ : F → FF-F-F-F-FF

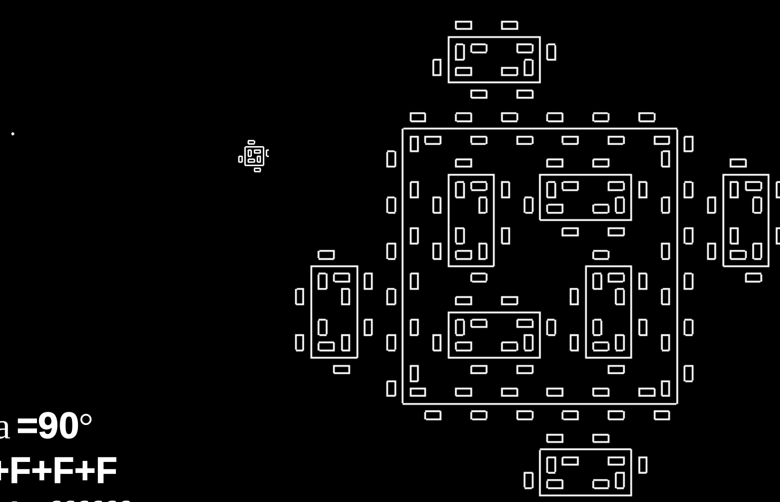


n=6, a=60°, $F_r$
$p_1$ : $F_l$ → $F_r$+$F_l$+$F_r$   $p_2$: $F_r$ → $F_l$-$F_r$-$F_l$



n=6, a=90°, L
$p_1$ : L → +RF-LFL-FR+   $p_2$ : R → -LF+RFR+FL-

# Fractals (cont.)



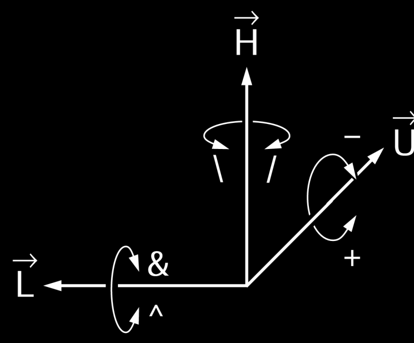
n=2, a=90°
w : F+F+F+F
$p_1$ : f → ffffff
$p_2$ : F → F+f-FF+F+FF+Ff+FF-f+FF-F-FF-Ff-FFF

# Graphical interpretation in 3D

*F* move forward and draw line
*f* move forward

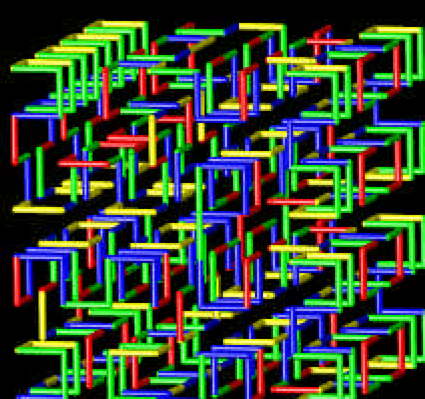
/ \ rotate around H
& ^ rotate around L
\+ - rotate around U

! # change width
, ; change color




**Turtle**

# Fractals (cont.)



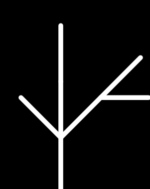
n=3, a =90°, A
$p_1$ : A → B-F+CFC+F-D&F^D-F+&&CFC+F+B//
$p_2$ : B → A&F^CFB^F^D^^-F-D^|F^B|FC^F^A//
$p_3$ : C → |D^|F^B-F+C^F^A&&FA&F^C+F+B^F^D//
$p_4$ : D → |CFB-F+B|FA&F^A&&FB-F+B|FC//

# Branching structures

- Turtle interprets a character string as a sequence of lines
- lines can intersect
- single line

# Bracketed L-systems

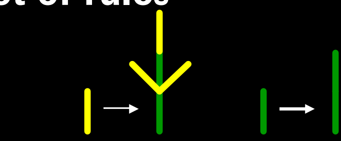
| | |
|---|---|
| *F* | move forward and draw line |
| *f* | move forward |
| */  * | rotate around H |
| *& ^* | rotate around L |
| *+  -* | rotate around U |
| *!  #* | change width |
| *,  ;* | change color |
| *[* | **start lateral branch** |
| *]* | **end lateral branch** |



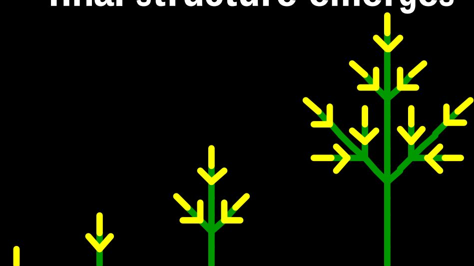
**F [+ F] [- F [- F] F] F F**

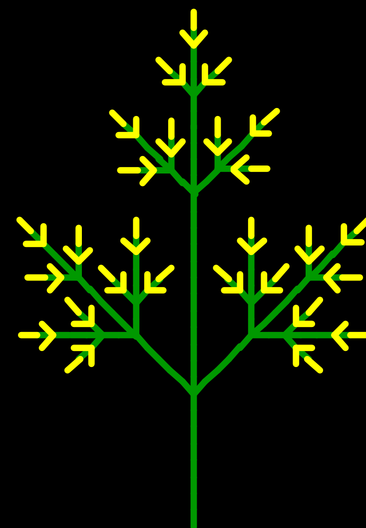
# Plant modeling

- set of rules




- final structure emerges





- P. Prusinkiewicz and A. Lindenmayer, The Algorithmic Beauty of Plants, Springer Verlag, 1990

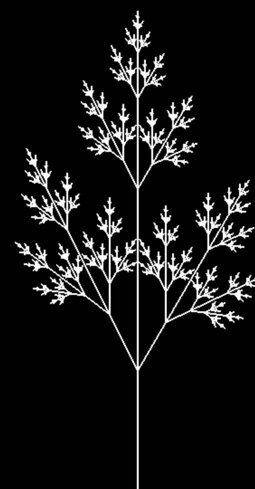
# Plant-like structures

n=3, a =25.7°

w : F

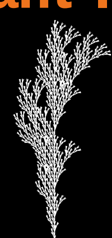
$p_1$ : F → F[+F]F[-F]

# Plant-like structures (cont.)

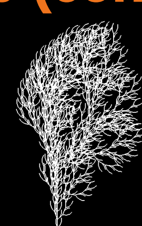


n=7, a =30°
w : X
$p_1$ : X → F[+X][-X]FX
$p_2$ : F → FF

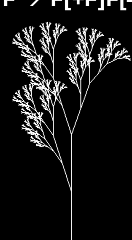
# Plant-like structures (cont.)



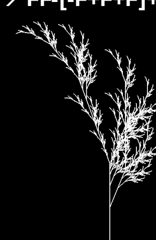
n=5, a =20°, F
$p_1$ : F → F[+F]F[-F][F]



n=4, a =22.5°, F-F-F-F
$p_1$ : F → FF-[-F+F+F]+[+F-F-F]



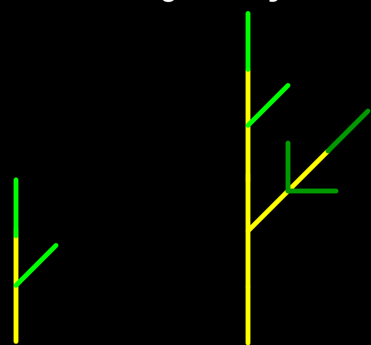
n=7, a =20°, X
$p_1$ : X → F[+X]F[-X]+X   $p_2$: F → FF



n=5, a =22.5°, X
$p_1$ : X → F-[[X]+X]+F[+FX]-X   $p_2$ : F → FF

# Parametric L-systems

- extend the basic concept of parallel rewriting from strings of symbols to parametric words

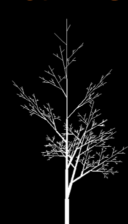


$p_1$ : A → F[-A]FA
$p_2$ : F → FF

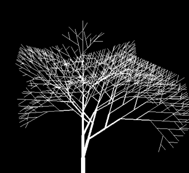
$p_1$ : A → F(1)[-(30)A]F(1)A
$p_2$ : F(l) → F(2*l)

# Parametric L-systems (cont.)

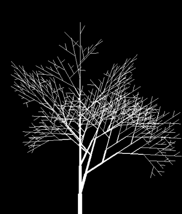
$r_1$ 0.9
$r_2$ 0.6
$a_0$ 45
$a_2$ 45
d 137.5
$w_r$ 0.7



$r_1$ 0.9
$r_2$ 0.9
$a_0$ 45
$a_2$ 45
d 137.5
$w_r$ 0.7



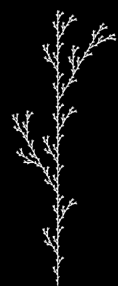
$r_1$ 0.9
$r_2$ 0.8
$a_0$ 45
$a_2$ 45
d 137.5
$w_r$ 0.7



$r_1$ 0.9
$r_2$ 0.7
$a_0$ 30
$a_2$ -30
d 137.5
$w_r$ 0.7

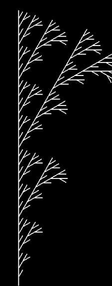
n=10, A(1,10)
$p_1$ : A(l,w) → !(w) F(l) [&($a_0$) B(l*$r_2$,w*$w_r$)] /(d) A(l*$r_1$,w*$w_r$)
$p_2$ : B(l,w) → !(w) F(l) [-($a_2$) C(l*$r_2$,w*$w_r$)] C(l*$r_1$,w*$w_r$)
$p_3$ : C(l,w) → !(w) F(l) [+($a_2$) B(l*$r_2$,w*$w_r$)] B(l*$r_1$,w*$w_r$)

# Stochastic L-systems



n=7, α=30°, F
$p_1$ : F → F[+F]F[-F]F

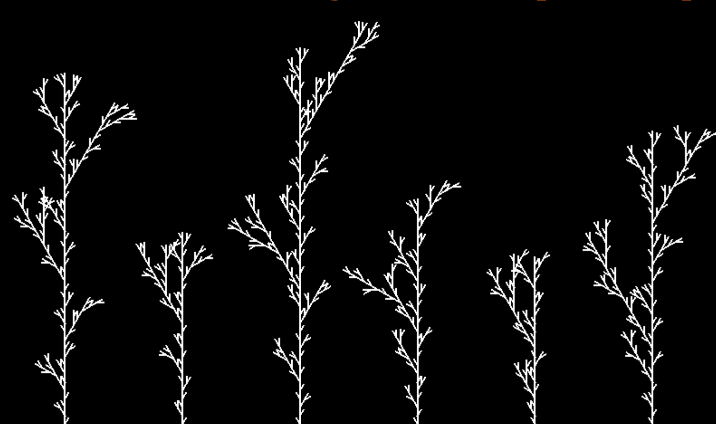
n=7, α=30°, F
$p_1$ : F → F[+F]F



n=7, α=30°, F
$p_1$ : F → F[-F]F

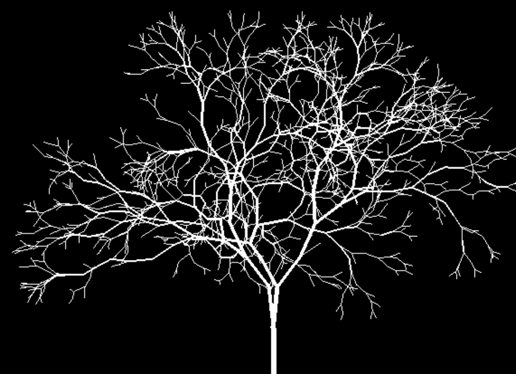
$p_1$ : F → F[+F]F[-F]F : 1/3
$p_2$ : F → F[+F]F : 1/3
$p_3$ : F → F[-F]F : 1/3

# Stochastic L-systems (cont.)



n=7, α=30°, F
$p_1$ : F → F[+F]F[-F]F : 1/3
$p_2$ : F → F[+F]F : 1/3
$p_3$ : F → F[-F]F : 1/3

## Stochastic L-systems (cont.)



$p_1$ : A → I / [+ A] - A : p
$p_2$ : A → I / [+ I ] - A : q

## Context-sensitive L-systems

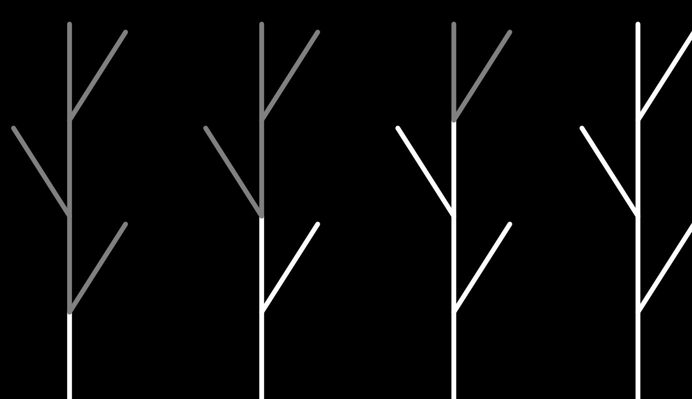
- productions are context-free, i.e. applicable regardless of the context in which the predecessor appears
- interaction between plant parts
- $a_l < a > a_r : c \rightarrow x$

# Context-sensitive L-systems (cont.)

w : B A A A A A A A A
$p_1$ : B < A → B
$p_2$ : B → A

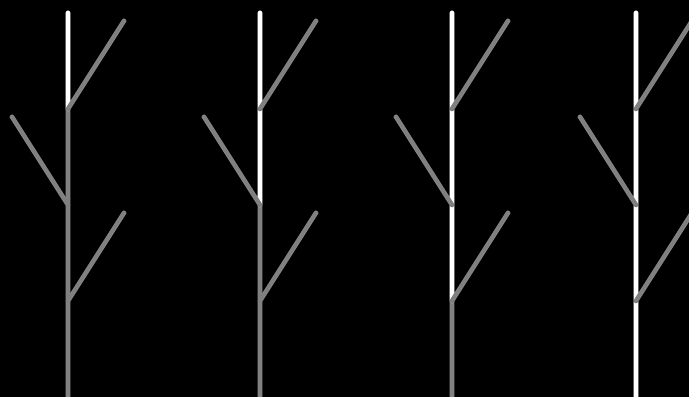
B A A A A A A A A
A B A A A A A A A
A A B A A A A A A
A A A B A A A A A
A A A A B A A A A
A A A A A B A A A
A A A A A A B A A
A A A A A A A B A
A A A A A A A A B

# Acropetal



acropetal signal propagation

## Basipetal

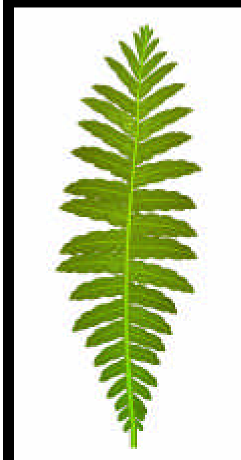


basipetal signal propagation

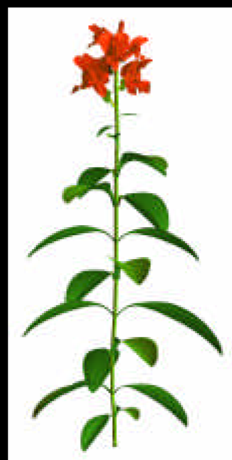
## Mycelis muralis

P. Prusinkiewicz and J. Hanan, 1988

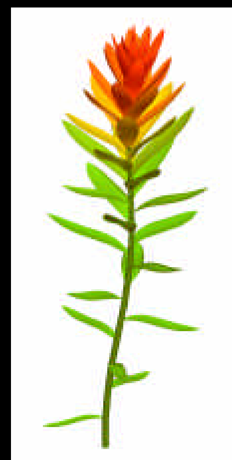
# Plant models



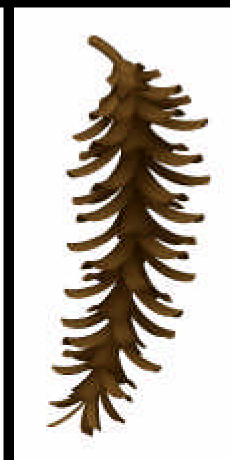
Blechnum gibbum
distichious



Antirrhinum majus
decussate

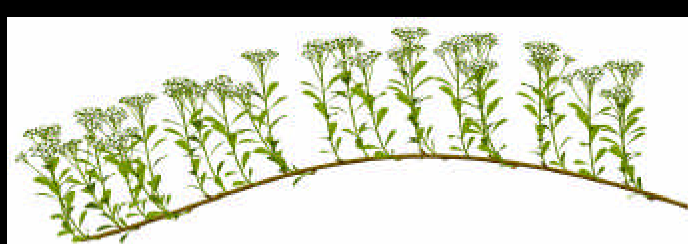


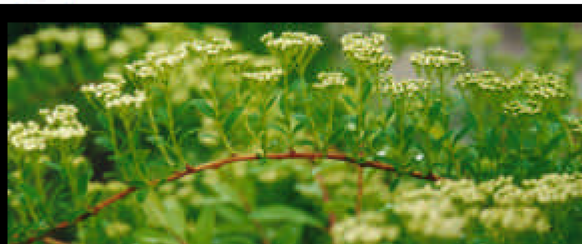
Cassilleja coccinea
spiral



Pinus strobus
spiral

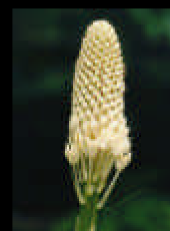
# Plant models (cont...)

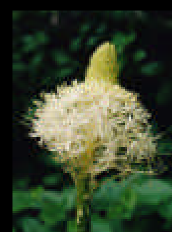




Spiraea sp. (Spirea) twig

# Plant models (cont...)





Xerophyllum tenax (beargrass)

# Plant development

Antirrhinum majus

- British TV, Norwich, UK

# Plant development

plant

inflorescence

side view

top view

Arabidopsis thaliana

# Plant development

plant

inflorescence

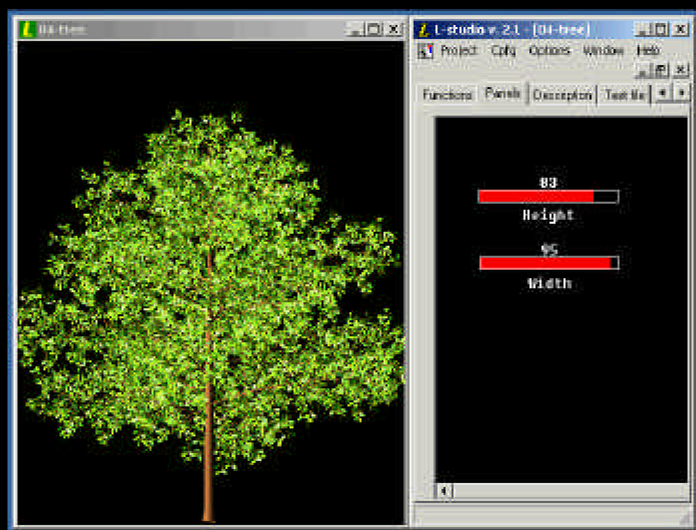
side view

top view

Arabidopsis thaliana

# Multiple-compound structures

DEMO




# L-systems

- Mathematical Models for Cellular Interaction in Development
  I. Filaments with One-sided Input
  A. Lindenmayer, J. Theoret. Biol., 18, 280-299, 1968
- Mathematical Models for Cellular Interaction in Development
  II. Simple and Branching Filaments with Two-sided Inputs
  A. Lindenmayer, J. Theoret. Biol., 18, 300-315, 1968
- The Algorithmic Beauty of Plants
  P. Prusinkiewicz and A. Lindenmayer, Springer, 1990
- Models developed by P. Prusinkiewicz, L. Muendermann, B. Lane, R. Karwowski
- Slides and Animations Courtesy of L. Muendermann