

# CPSC 453: L-Systems

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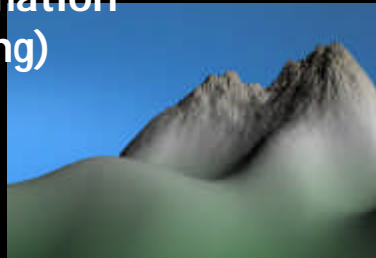
## Procedural Modeling



- Is it efficient to model these objects manually?

## Procedural Modeling

- The automatic generation of objects/animation using algorithmic techniques
- Significant time savings in modelling
- More abstract control of object shape
- Can also be used for animation (Physically Based Modeling)



(P. MacMurchy)

## L-systems

- A procedural modeling method often used for plants
- Invented by A. Lindenmayer  
Mathematical Models for Cellular Interaction in Development, Part I and II, Journal of Theoretical Biology, 18, 280-315, 1968

- Capture the development of components over time
- Division of mother cell A into two daughter cells B and C

$A \rightarrow B C$

- Parallel rewriting

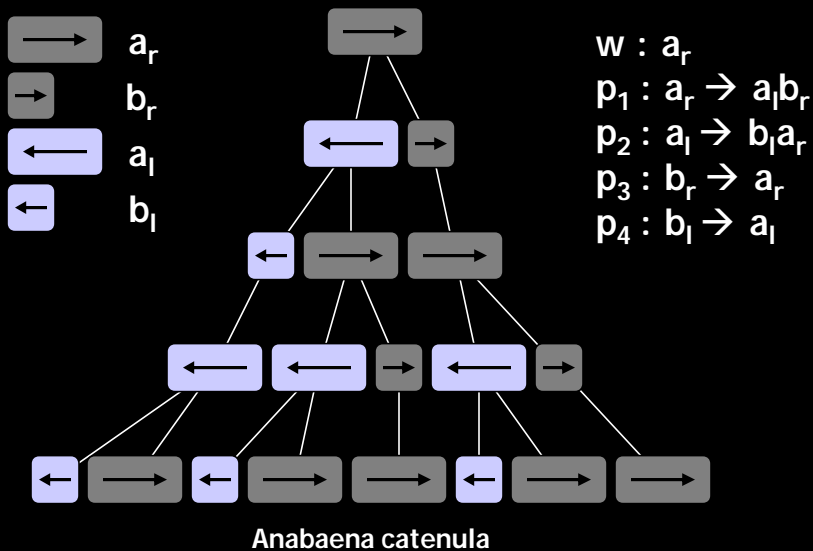
## Definition

- An L system consists of 3 things:

$$G = \langle V; \omega; P \rangle$$

- an *alphabet*:  $V$   
eg. A B C D I X
- an *axiom*:  $\omega$   
eg. C
- a set of *productions*:  $P \subseteq V \times V^*$   
eg. A  $\rightarrow$  BC  
C  $\rightarrow$  DX  
X  $\rightarrow$  A

## L-systems (cont.)





# Snowflake

$n=2, a=60^\circ$

$w : F$

$p_1 : F \rightarrow F+F--F+F$



F



F+F--F+F

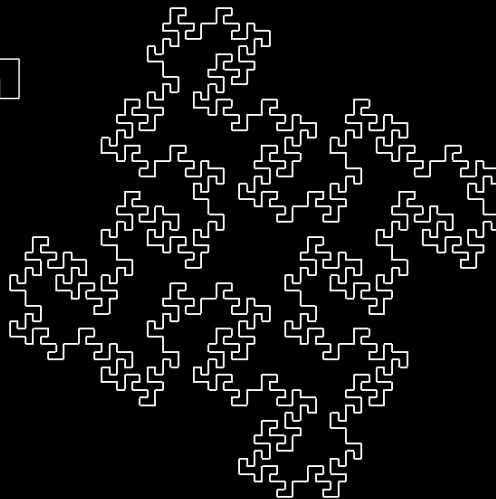
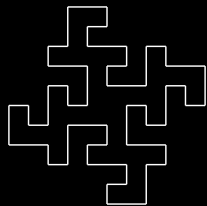


F+F--F+F+F+F  
--F+F--F+F--F+  
F+F+F--F+F



...

# Fractals

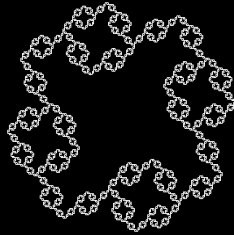


$n=2, a=90^\circ$

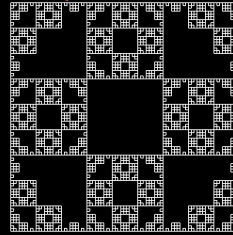
$w : F-F-F-F$

$p_1 : F \rightarrow F+FF-FF-F-F+F+FF-F-F+F+FF+FF-F$

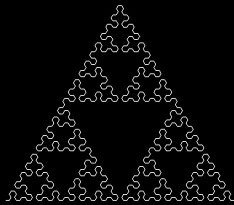
## Fractals (cont.)



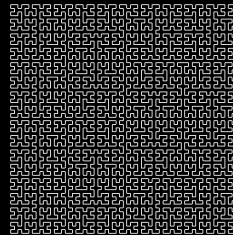
$n=4, a=90^\circ, F+F+F+F$   
 $p_1: F \rightarrow FF-F-F-F-F+F$



$n=4, a=90^\circ, F-F-F-F$   
 $p_1: F \rightarrow FF-F-F-F-F$

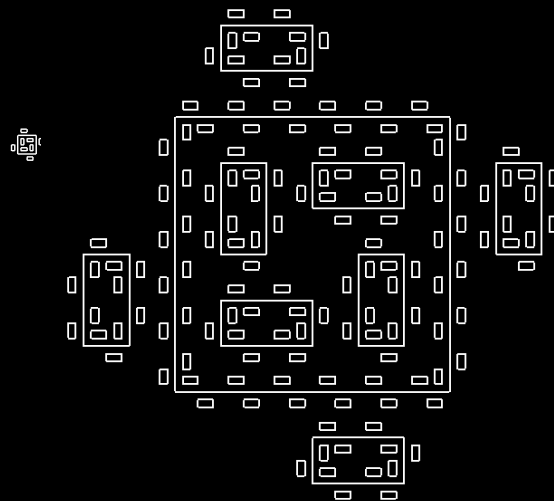


$n=6, a=60^\circ, F_r$   
 $p_1: F_l \rightarrow F_r+F_l+F_r$      $p_2: F_r \rightarrow F_l-F_r-F_l$



$n=6, a=90^\circ, L$   
 $p_1: L \rightarrow +RF-LFL-FR+$      $p_2: R \rightarrow -LF+RFR+FL-$

## Fractals (cont.)



$n=2, a=90^\circ$   
 $w: F+F+F+F$   
 $p_1: f \rightarrow ffffff$   
 $p_2: F \rightarrow F+f-FF+F+FF+Ff+FF-f+FF-F-FF-Ff-FFF$

## Graphical interpretation in 3D

*F* move forward and  
draw line

*f* move forward

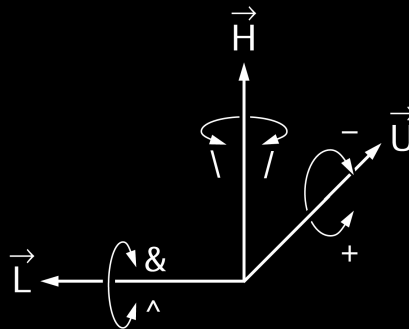
*/ \* rotate around H

*& ^* rotate around L

*+ -* rotate around U

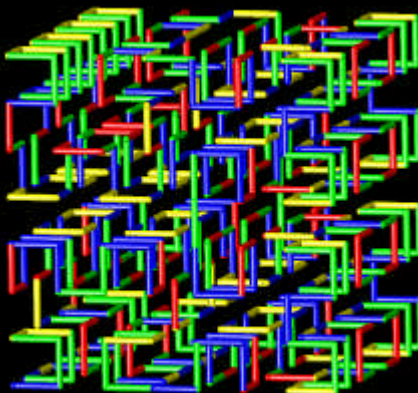
*! #* change width

*, ;* change color



Turtle

## Fractals (cont.)



$n=3, a=90^\circ, A$

$p_1: A \rightarrow B-F+CFC+F-D&F^{\wedge}D-F+&&CFC+F+B//$

$p_2: B \rightarrow A&F^{\wedge}CFB^{\wedge}F^{\wedge}D^{\wedge}F-D^{\wedge}|F^{\wedge}B|FC^{\wedge}F^{\wedge}A//$

$p_3: C \rightarrow |D^{\wedge}|F^{\wedge}B-F+C^{\wedge}F^{\wedge}A&&FA&F^{\wedge}C+F+B^{\wedge}F^{\wedge}D//$

$p_4: D \rightarrow |CFB-F+B|FA&F^{\wedge}A&&FB-F+B|FC//$

## Branching structures

- Turtle interprets a character string as a sequence of lines
- lines can intersect
- single line

## Bracketed L-systems

*F* move forward and draw line

*f* move forward

*/ \* rotate around H

*& ^* rotate around L

*+ -* rotate around U

*! #* change width

*, ;* change color

*[* start lateral branch

*]* end lateral branch

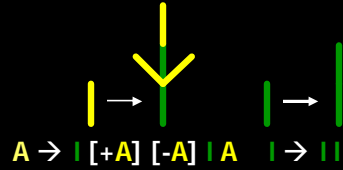


F [+ F] [- F [- F] F] F F

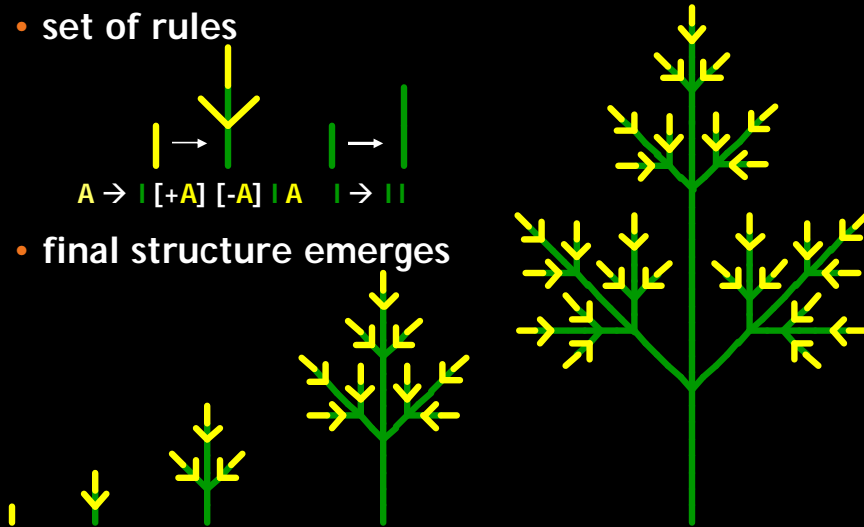


## Plant modeling

- set of rules



- final structure emerges



• P. Prusinkiewicz and A. Lindenmayer, *The Algorithmic Beauty of Plants*, Springer Verlag, 1990

## Plant-like structures

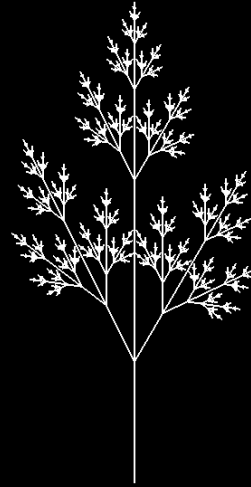


$n=3, a=25.7^\circ$

$w : F$

$p_1 : F \rightarrow F[+F]F[-F]$

## Plant-like structures (cont.)



$n=7, a=30^\circ$

$w : X$

$p_1 : X \rightarrow F[+X][-X]FX$

$p_2 : F \rightarrow FF$

## Plant-like structures (cont.)



$n=5, a=20^\circ, F$

$p_1 : F \rightarrow F[+F]F[-F][F]$



$n=4, a=22.5^\circ, F-F-F-F$

$p_1 : F \rightarrow FF[-F+F+F][+F-F-F]$



$n=7, a=20^\circ, X$

$p_1 : X \rightarrow F[+X]F[-X]+X$

$p_2 : F \rightarrow FF$



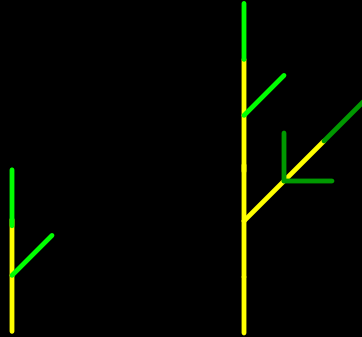
$n=5, a=22.5^\circ, X$

$p_1 : X \rightarrow F-[[X]+X]+F[+FX]-X$

$p_2 : F \rightarrow FF$

## Parametric L-systems

- extend the basic concept of parallel rewriting from strings of symbols to parametric words



$$p_1 : A \rightarrow F[-A]FA$$

$$p_2 : F \rightarrow FF$$

$$p_1 : A \rightarrow F(1)[- (30)A]F(1)A$$

$$p_2 : F(l) \rightarrow F(2^*l)$$

## Parametric L-systems (cont.)

$$r_1 \quad 0.9$$

$$r_2 \quad 0.6$$

$$a_0 \quad 45$$

$$a_2 \quad 45$$

$$d \quad 137.5$$

$$w_r \quad 0.7$$



$$r_1 \quad 0.9$$

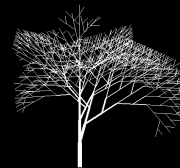
$$r_2 \quad 0.9$$

$$a_0 \quad 45$$

$$a_2 \quad 45$$

$$d \quad 137.5$$

$$w_r \quad 0.7$$



$$r_1 \quad 0.9$$

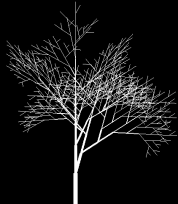
$$r_2 \quad 0.8$$

$$a_0 \quad 45$$

$$a_2 \quad 45$$

$$d \quad 137.5$$

$$w_r \quad 0.7$$



$$r_1 \quad 0.9$$

$$r_2 \quad 0.7$$

$$a_0 \quad 30$$

$$a_2 \quad -30$$

$$d \quad 137.5$$

$$w_r \quad 0.7$$



$n=10, A(1,10)$

$p_1 : A(l, w) \rightarrow !(w) F(l) [ \&(a_0) B(l^*r_2, w^*w_r) ] / (d) A(l^*r_1, w^*w_r)$

$p_2 : B(l, w) \rightarrow !(w) F(l) [ -(a_2) C(l^*r_2, w^*w_r) ] C(l^*r_1, w^*w_r)$

$p_3 : C(l, w) \rightarrow !(w) F(l) [ +(a_2) B(l^*r_2, w^*w_r) ] B(l^*r_1, w^*w_r)$

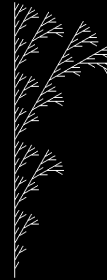
## Stochastic L-systems



$n=7, a=30^\circ, F$   
 $p_1 : F \rightarrow F[+F]F[-F]F$



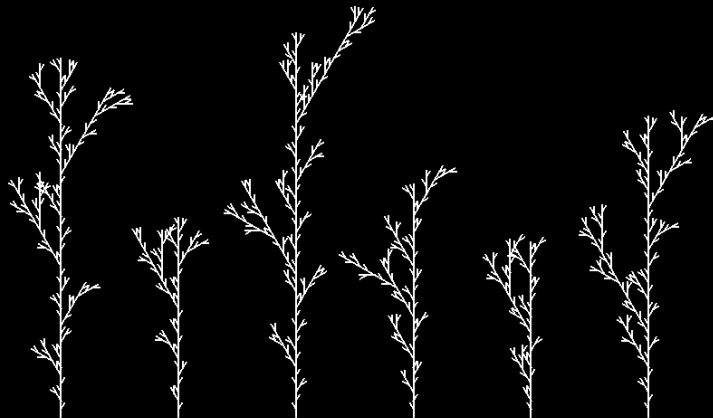
$n=7, a=30^\circ, F$   
 $p_1 : F \rightarrow F[+F]F$



$n=7, a=30^\circ, F$   
 $p_1 : F \rightarrow F[-F]F$

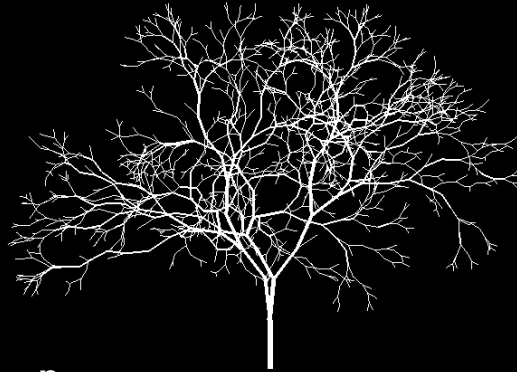
$p_1 : F \rightarrow F[+F]F[-F]F : 1/3$   
 $p_2 : F \rightarrow F[+F]F : 1/3$   
 $p_3 : F \rightarrow F[-F]F : 1/3$

## Stochastic L-systems (cont.)



$n=7, a=30^\circ, F$   
 $p_1 : F \rightarrow F[+F]F[-F]F : 1/3$   
 $p_2 : F \rightarrow F[+F]F : 1/3$   
 $p_3 : F \rightarrow F[-F]F : 1/3$

## Stochastic L-systems (cont.)



$p_1 : A \rightarrow I / [+ A] - A : p$   
 $p_2 : A \rightarrow I / [+ I] - A : q$

## Context-sensitive L-systems

- productions are context-free, i.e. applicable regardless of the context in which the predecessor appears
- interaction between plant parts
- $a_l < a > a_r : c \rightarrow x$

## Context-sensitive L-systems (cont.)

$w : B A A A A A A A A$

$p_1 : B < A \rightarrow B$

$p_2 : \quad B \rightarrow A$

$B A A A A A A A A$

$A B A A A A A A A$

$A A B A A A A A A$

$A A A B A A A A A$

$A A A A B A A A A$

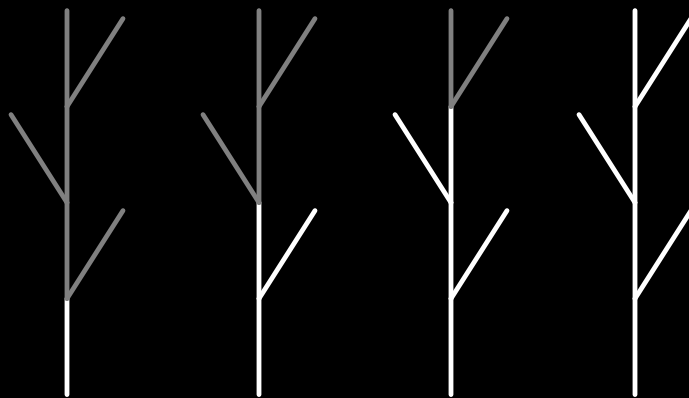
$A A A A A B A A A$

$A A A A A A B A A$

$A A A A A A A B A$

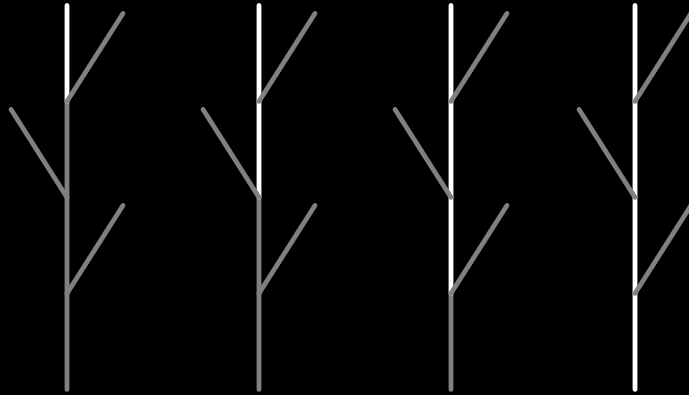
$A A A A A A A A B$

## Acropetal



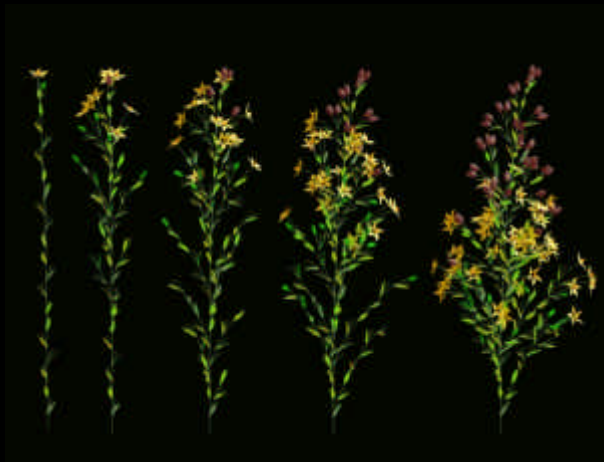
acropetal signal propagation

## Basipetal



basipetal signal propagation

## *Mycelis muralis*



P. Prusinkiewicz and J. Hanan, 1988

## Plant models



*Blechnum gibbum*  
distichious



*Antirrhinum majus*  
decussate

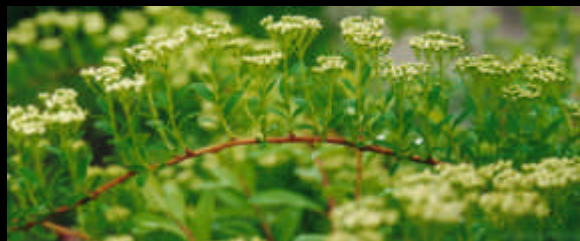
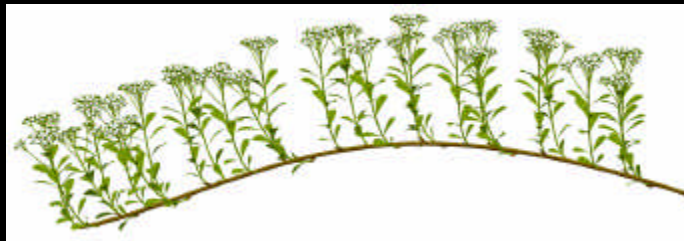


*Cassileja coccinea*  
spiral



*Pinus strobus*  
spiral

## Plant models (cont...)



*Spiraea* sp. (Spirea) twig

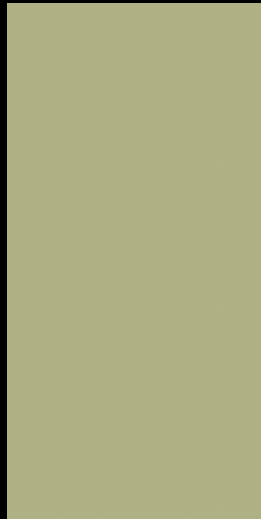


## Plant models (cont...)



*Xerophyllum tenax* (beargrass)

## Plant development



*Antirrhinum majus*

• British TV, Norwich, UK

# Plant development

plant



inflorescence



side view

top view

*Arabidopsis thaliana*

# Plant development

plant

inflorescence

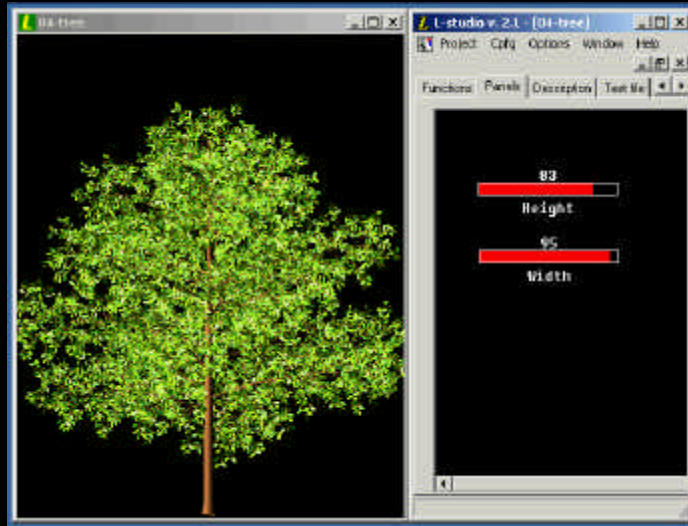
side view

top view

*Arabidopsis thaliana*

# Multiple-compound structures

DEMO



## L-systems

- **Mathematical Models for Cellular Interaction in Development**  
I. Filaments with One-sided Input  
A. Lindenmayer, *J. Theoret. Biol.*, 18, 280-299, 1968
- **Mathematical Models for Cellular Interaction in Development**  
II. Simple and Branching Filaments with Two-sided Inputs  
A. Lindenmayer, *J. Theoret. Biol.*, 18, 300-315, 1968
- **The Algorithmic Beauty of Plants**  
P. Prusinkiewicz and A. Lindenmayer, Springer, 1990
- **Models developed by P. Prusinkiewicz, L. Muendermann, B. Lane, R. Karwowski**
- **Slides and Animations Courtesy of L. Muendermann**