Interference-Free Energy Efficient Scheduling in Wireless Ad Hoc Networks

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Abstract—This paper studies the problem of interference-free broadcast in wireless ad hoc networks. In particular, we are interested in asymmetric power assignments so that the induced broadcast communication graph is both, energy efficient and has a short collision-free broadcast schedule. We consider both random and deterministic node layouts and develop four different broadcast schemes with provable performance guarantees on three optimization objectives simultaneously: total energy consumption, network lifetime and collision-free schedule length. We also show some numerical results which support our findings.

I. INTRODUCTION

The technological and theoretical advances in the study of wireless communications lead to a rapid introduction of wireless ad hoc networks to a wide spectrum of applications, from scientific monitoring to military and rescue operations. As wireless nodes are often deployed in areas where battery replacement is infeasible, energy efficiency remains one of the most critical issues in any wireless network design [7]. The high levels of energy consumption are mainly due to the power used for wireless communication. It is customary to assume that the power required to transmit to distance $r$ is proportional to $r^\alpha$, where $\alpha$ is the distance-power gradient. In perfect conditions $\alpha = 2$, however in more realistic settings (in presence of obstructions) it can have a value between 2 and 4 (see [18]). Thus, the assignment of transmission powers and the relative node disposition constitute the communication backbone of the network. In this paper we assume $\alpha = 2$ for simplicity, although our results could be easily generalized for any $\alpha > 2$. Energy efficiency is usually measured through two parameters: total energy consumption [8] (also referred as total cost) and network lifetime [21]. The former refers to the total energy used for a specific network task (e.g. broadcast), while the latter estimates the expected endurance of network nodes until they run out of battery charge (typically, the network lifetime is defined as the time it takes the first node to run out of its battery charge).

Energy efficiency is not the only challenge faced by the network designer. As nodes communicate through radio signals, wireless interference becomes inevitable. Simultaneous transmissions are sensed at every node, which may lead to incorrect signal receptions. We consider omnidirectional antennas, where the transmission of a single node is propagated in all directions. The level of interference depends on the transmitting nodes proximity and the transmission ranges. High levels of interference decrease the number of transmissions that can happen simultaneously, which has a direct affect on the schedule length of the network, which is the required number of time slots for the message to propagate from the source to all the other nodes in the network. It should be noted that traditional works which aim to minimize the hop-diameter in order to minimize the schedule length fail to do so as they neglect the presence of interference.

In this paper we consider a fundamental topology control problem which is to induce an energy efficient broadcast communication backbone with a low schedule length. That is, given a special source node $s$ (also referred to as the root node), we wish to induce a communication graph by adjusting transmission powers, so that there is a directed path from $s$ to every other node in the network; the total energy consumption, network lifetime and feasible schedule length are used to measure the efficiency of the scheme.

The broadcast problem, under various optimization criteria has been extensively studied in the research community. It was first introduced by Wieselthier et al. ([26], [27]), where the authors proposed several heuristics (MST, SPT and BIP), without provable bounds, for the minimum total energy broadcast problem. Wan et al. [25] presented the first analytical results for this problem; in particular, they showed that the approximation ratio of an MST is between 6 and 12, for BIP it is between $\frac{12}{7}$ and 12 and for SPT it is at least $\frac{n}{2}$, where $n$ is the number of receiving nodes. Cagalj et al. [6] gave a proof of NP-hardness of the minimum-energy broadcast problem in a Euclidean space and Ambühl et al. [1] showed that the MST heuristics achieves an approximation ratio of 6, which matched the
lower bound by [25]. All of the results above address only the issue of energy efficiency without considering the schedule length in the presence of interference and the network lifetime. One possible approach to minimize the schedule length is to bound the hop-diameter of the induced communication graph [3]. This alone, however, is not sufficient as possible collisions need to be taken into account. There has been some extensive work on collision-free scheduling as well. Parthasarathy et al. [19] show that minimum schedule length broadcasting is NP-hard for ad hoc wireless networks. They present a distributed collision-free broadcasting algorithm with an approximation ratio of $O(1)$ on the schedule length and the number of message propagations in the case that the transmission ranges are bounded. Onus et al. [17] develop efficient self-stabilizing collision-free broadcasting and data gathering algorithms. See some additional results in [11], [12]. These works however do not address the energy efficiency of the proposed schemes. The works [4] and [28] address a minimum energy delay constrained broadcast scheduling problem without taking interference or network lifetime into consideration. See [15] and [30] for some results on interference and energy aware scheduling in networks with directional antennas.

Our main contribution in this paper is the development of interference and energy aware broadcast schedules with provable analytical bounds for three optimization criteria simultaneously: total energy consumption, network lifetime and schedule length. To the best of our knowledge, this is the first paper to consider all three objectives simultaneously. A summary of our results appears in Table I. Note that the total cost and network lifetime are compared with the optimal cost and network lifetime of any broadcast, without schedule length restrictions, in all cases except for the FORK scheme, where the network lifetime is compared with the optimal network lifetime of a broadcast schedule of length $k$. As it can be seen the first two schemes, GBT and MST are developed for the case of random node distribution in a unit square, while the latter two, HCT and FORK, consider deterministic node layouts.

The rest of this paper is organized as follows. Some definitions, the wireless model, and problem formulation are given in Section II. Then, in Section III we present some preliminary results, followed by Sections IV and V, which present our results for the random and deterministic cases, respectively. In Section VI we verify our results through simulations. Finally, in Section VII we conclude our work and discuss some possible future research directions.

## II. System settings

Let $G_V = (V, E_V)$ be a complete directed graph of the wireless nodes $V$, $|V| = n$, positioned in the plane. We define the weight function, $w : E_V \rightarrow \mathbb{R}^+$, on the edge set $E_V$ as $w(u, v) = d(u, v)^\alpha$, where $d(u, v)$ is the Euclidean distance between $u$ and $v$. Note that the weight of an edge $(u, v)$ matches the amount of energy which is required to transmit from $u$ to $v$.

We proceed by first presenting some general graph theory related definitions. This is followed by the wireless ad-hoc network model used in the paper. In the end, we formally define the Minimum Energy and Schedule Broadcast problem.

### A. General definitions

The following definitions are used for both, directed and undirected graphs. Let $G = (V, E)$ be a subgraph of $G_V$. Denote the total weight of $G$ by $w(G) = \sum_{e \in E} w(e)$.

For an edge $e = (u, v) \in E$, let $|e| = d(u, v)$. Also, let $e^*(G)$ be the longest outgoing edge from $u$ in $G$, and by $e^*(G)$, $|e^*(G)| = \max_{u \in V} |e^*_u(G)|$, the longest edge in $G$. Let $MST_V$ be a minimum weight spanning tree of the undirected version of $G_V$ (which is obtained easily by omitting the edge directions). The hop-distance from $u$ to $v$ in $G$, $h_{u, v}(G)$, is defined as the minimum number of edges in any path from $u$ to $v$. The height of $u$ in $G$, $h_u(G)$, is the maximum hop-distance from $u$ to any node, i.e. $h_u(G) = \max_{v \in V} h_{u, v}(G)$.

The hop-diameter of $G$ is defined as $h(G) = \max_{u \in V} h_u(G)$. A directed graph $G = (V, E)$ is a broadcast arborescence rooted at $s \in V$ if for any node $u \in V$ there is a path from $s$ to $u$ in $G$.

We assume the following scenario of a broadcast task. A message from a source $s \in V$ to all the other nodes in the network over a broadcast arborescence $G$ is propagated over a directed subtree of $G$ (in the tree there is exactly one path from $s$ to every other node). Once a node receives a message from its parent, it forwards it to all its children.

### B. Wireless ad-hoc network model

A power assignment is a function $p : V \rightarrow \mathbb{R}^+$, which assigns each node $v \in V$ a transmission range $r_v = \sqrt[p]{p(v)}$. The transmission possibilities resulting from a power assignment induce a directed communication graph $H_p = (V, E_p)$, where $E_p = \{(u, v) : r_u \geq d_{u, v}\}$ is a set of directed edges. Recall that we assume that $\alpha = 2$.

The total energy consumption, also referred to as the cost, of the power assignment is given by $c(p) = \sum_{v \in V} p(v)$. Each node $v$ has some initial battery charge $b(v)$, which is sufficient for a limited amount of time, proportional to the power assignment $p(v)$. It is common to take the lifetime of a wireless node $v$ to be $l(v) = b(v)/p(v)$. The network lifetime is defined as the time it
takes the first node to run out of its battery charge. For a power assignment \( p \) and initial battery charges \( b \), the network lifetime is defined as \( l(p) = \min_{v \in V} l(v) \). Again, for simplicity we assume \( b(v) = 1 \) for every \( v \in V \).

Interference is a direct consequence of any power assignment \( p \). A signal transmitted over one communication link may interfere with the correct reception of a transmission over some other link. We adopt the protocol interference model which defines for each node \( u \) a set of nodes, \( I_p(u,T) \), referred to as the conflict set of \( u \), which consists of nodes which cannot be scheduled to transmit simultaneously with \( u \) because of interference to either the recipients of \( u \) or \( v \), in a broadcast subtree \( T \) of \( H_p \) which is used for the broadcast task. That is, node \( u \) cannot be scheduled simultaneously with \( v \) iff there exists a child of \( u \) in \( T \) which is interfered by \( v \) or vice versa (a child of \( v \) interfered by \( u \)). To simplify the notations we use \( I_p(u) \) instead of \( I_p(u,T) \) throughout the paper as the broadcast subtree is clear from the context. There are several variations for the definition of \( I_p(\cdot) \) ([5], [13], [16]); for simplicity we use the following definition, however it can be easily generalized.

**Definition II.1.** For a power assignment \( p \) and a broadcast subtree \( T = (V,E_T) \subseteq H_p \), \( v \in I_p(u) \) iff there exists \((u,x) \in E_T \) such that \((v,x) \in E_p \) or there exists \((v,y) \in E_T \) such that \((u,y) \in E_p \).

Note that \( v \in I_p(u) \) iff \( u \in I_p(v) \). Let \( I^*_p \) be the maximum size conflict set for a power assignment \( p \), i.e., \( |I^*_p| = \max_{u \in V} |I_p(u)| \).

For the broadcast operation we assume a time slot based scheduling, where links are assigned to time slots. Let \( S = \{U_1, U_2, \ldots, U_k\} \) be a schedule based on some communication graph \( H_p \), where \( U_i \subseteq V \), \( 1 \leq i \leq k \), is a non-empty set of nodes scheduled to transmit at time slot \( i \). The length of \( S \) is defined as \( |S| = k \). A schedule \( S \) is feasible if all the transmitting nodes in every time slot are non-interfering, i.e., for any two nodes, \( u,v \in U_i \), scheduled at the same time slot \( i \), it holds \( u \notin I_p(v) \) and \( v \notin I_p(u) \). When schedule \( S \) is executed, all the nodes in \( U_i \), \( 1 \leq i \leq k \), are activated (simultaneously) before any of the nodes in \( U_j \), \( i < j \leq k \), are allowed to transmit. Below we give a definition of a feasible broadcast schedule.

**Definition II.2.** \( S \) is a feasible broadcast schedule for a broadcast arborescence \( G = (V,E) \) rooted at \( s \in V \) if \( S \) is a feasible schedule and it is possible to broadcast a message from \( s \) to all the nodes in \( V \setminus \{s\} \) when \( S \) is executed.

Let \( FBS(G,s) \) be a set of feasible broadcast schedules for a broadcast arborescence \( G \) rooted at \( s \). The scheduling efficiency of a broadcast arborescence \( G \) rooted at \( s \) is the length of the shortest feasible broadcast schedule, \( \text{Len}(G, s) = \min_{S \in FBS(G,s)} |S| \).

### C. Problem definition

The problem we address in this paper is defined as follows.

**Problem.** Given a set \( V \) of wireless nodes in the Euclidean plane, and a source node \( s \in V \), find a power assignment \( p \), such that \( H_p \) is a broadcast arborescence rooted at \( s \). The objectives are: minimize \( \text{Len}(H_p, s) \), minimize \( c(p) \), and maximize \( l(p) \).

Note that the optimization objectives can be contradicting. In our solutions we provide analytical guarantees for all three measures. The cost and network lifetime of our power assignments is compared to the best possible for any broadcast arborescence, i.e., for any \( s \in V \), let \( p^*_C \) and \( p^*_L \) be the minimum cost power and maximum network lifetime power assignments, respectively so that \( H_{p^*_C} \) and \( H_{p^*_L} \) are broadcast arborescences rooted at \( s \). We define \( c^*_s = c(p^*_C) \) and \( l^*_s = l(p^*_L) \).

### III. Preliminaries

In what follows we present several theoretical results which are used throughout the paper. The cited theorems and lemmas are adapted to our model.

In [25] the authors derived a bound on the minimum cost of any power assignment that induces a broadcast arborescence.

**Theorem III.1** ([25]). \( c^*_s = \Omega(w(MST_V)) \), for any \( s \in V \).

Interestingly, using the edges of the minimum spanning tree produces an optimal network lifetime for broadcast if all the initial battery charges are equal \((\forall u \in V, b(u) = 1)\), as shown in the next theorem.

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Schedule length</th>
<th>Total cost (approx. ratio)</th>
<th>Lifetime (approx. ratio)</th>
<th>Node layout</th>
<th>Section</th>
</tr>
</thead>
<tbody>
<tr>
<td>GBT</td>
<td>( O\left(\sqrt{n/\log n}\right))</td>
<td>( O(1))</td>
<td>( \Omega(1))</td>
<td>Random</td>
<td>IV-A</td>
</tr>
<tr>
<td>MST</td>
<td>( O(\log n \cdot h(MST_V)))</td>
<td>( O(1))</td>
<td>1</td>
<td>Random</td>
<td>IV-B</td>
</tr>
<tr>
<td>HCT</td>
<td>( [(n-1)/k] )</td>
<td>( O(k))</td>
<td>( \Omega(1/k^2))</td>
<td>Arbitrary</td>
<td>V-A</td>
</tr>
<tr>
<td>FORK</td>
<td>( k )</td>
<td>( O(1))</td>
<td>1 {for a schedule with length ( k }</td>
<td>Grid</td>
<td>V-B</td>
</tr>
</tbody>
</table>

**TABLE I**

Summary of our contribution.
Theorem III.2. \( l_s^* \leq 1/w(e^*(MST_V)) \), for any \( s \in V \).

Proof: Let \( p \) be a maximum network lifetime power assignment such that \( H_p \) is a broadcast arborescence rooted at \( s \) and \( l_s^* = l(p) \). Let \( T \) be a minimum bottleneck directed spanning tree of \( H_p \), so that the weight of the maximum weight edge in \( T \) is minimized. Clearly \( l(p) \leq 1/w(e^*(T)) \). A well known property of \( MST_V \) is that its bottleneck edge is the minimum possible one of all spanning trees. Therefore, \( 1/w(e^*(T)) \leq 1/w(e^*(MST_V)) \), which rests our proof. 

Two major factors have influence on the schedule efficiency of a broadcast arborescence; these are the hop-diameter of the arborescence and the conflict sets of communication links. In the next theorem we derive a simple upper bound on the schedule length as a function of the hop-diameter and the maximum size of a conflict set.

Theorem III.3. If a power assignment \( p \) induces a broadcast arborescence \( H_p \) rooted at node \( s \in V \) then there exists a feasible broadcast schedule \( S \) so that \( Len(S) \leq h_s(H_p) \cdot (|I_p^*| + 1) \).

Proof: Let \( T \) be a directed spanning tree of \( H_p \) rooted at \( s \) which results from running the BFS algorithm (9) from \( s \) (all the edges are directed from the parent to the child nodes). Clearly, \( h_s(T) = h(H_p) \). Let \( L_i \), \( 0 \leq i \leq h_s(T) - 1 \), be a set of nodes which are at distance \( i \) from \( s \), i.e. for \( u \in L_i \), \( h_s(u)(T) = i \). We show that it is possible to schedule every set of nodes \( L_i \) in \(|I_p^*| + 1\) time slots.

For every \( L_i \), \( 0 \leq i \leq h_s(T) - 1 \), we divide the nodes into \( l \) time slots \( U_1, U_2, \ldots, U_l \), which form a feasible schedule. The sets \( U_j \), \( 1 \leq j \leq l \), are constructed in incremental order; first \( U_1 \), then \( U_2 \), etc. Each set \( U_j \) is constructed in a greedy fashion as follows. Step 1: Initialize \( U_j \leftarrow \emptyset \) and \( L \leftarrow L \setminus (U_1 \cup U_2 \ldots \cup U_{j-1}) \). Step 2: Pick an arbitrary node \( u \in L \) and add it to \( U_j \). Step 3: Remove \( u \) and \( I(u) \) from \( L \). Step 4: If \( L \neq \emptyset \) goto step 2.

The above routine is executed until all the nodes are assigned to one of the time slots \( U_j \), \( 1 \leq j \leq l \). It is important to note that this assignment of nodes to time slots forms a feasible schedule since all the sets are disjoint and in each set \( U_j \) there are no interfering nodes due to step 3. To upper bound \( l \) we first show that for any \( j \), \( 1 \leq j \leq l \),

\[
|U_1 \cup U_2 \ldots \cup U_j| \geq \frac{|L_i| \cdot j}{|I_p^*| + 1}.
\]

We show this fact by induction. In order to simplify the notation we denote \( m = |L_i| \) and \( k = |I_p^*| \). It is easy to verify that \( |U_1| \geq m/(k+1) \) as for each node which is picked in step 2, at most \( k+1 \) nodes are removed from \( L \), which is initialized to be \( L_i \) for the construction of \( U_j \). Therefore, step 2 will be repeated at least \( m/(k+1) \) times.

Suppose that the inequality holds for some \( j \); we will prove it for \( j + 1 \). Let \( L \) be a set of nodes which are yet to be assigned a time slot after the construction of \( U_1, U_2, \ldots, U_j \) (as initialized in step 1). According to the induction assumption

\[
|L| \leq m - m \cdot j \frac{k + 1}{k + 1} = \frac{m \cdot (k - j + 1)}{k + 1}.
\]

We observe that every node \( u \in L \) was not assigned to any of the time slots \( 1, \ldots, j \). This could happen only if it was removed in step 3 of the construction of each of the node sets \( U_1, U_2, \ldots, U_j \). From the definition of \( I_p(v) \) it follows that \( u \in I_p(v) \) iff \( v \in I_p(u) \), and therefore for any \( U_q, 1 \leq q \leq j \), there exists a node \( v \in U_q \) so that \( v \in I_p(u) \). We conclude that for any node \( u \in L \) holds \(|I_p(u) \cap L| \leq k - j \). Following the same reasoning as for \( U_1, |U_{j+1}| \geq \frac{|L|}{k-2} \). As a result,

\[
|L| - |U_{j+1}| \leq |L| - \frac{|L|}{k-2} = \frac{|L|(k-j)}{k-2} + 1 \leq \frac{m \cdot (k-j)}{k+1}.
\]

Note that \( L \setminus U_{j+1} \) are exactly the nodes which are left unassigned after \( U_{j+1} \) is constructed. Therefore,

\[
|U_1 \cup U_2 \cup \ldots \cup U_{j+1}| \geq m - (|L| - |U_{j+1}|) \geq \frac{m \cdot (j + 1)}{k+1}.
\]

Clearly, after at most \( k + 1 \) rounds all nodes will be assigned and therefore \( l \leq k + 1 = |I_p^*| + 1 \).

To create a feasible broadcast schedule for \( H_p \) we simply schedule the node sets \( U_i \), \( 0 \leq i \leq h_s(T) - 1 \) consecutively so that each node set is scheduled as described above. It is easy to verify that the schedule is a feasible broadcast schedule and its length is at most \( h_s(H_p) \cdot (|I_p^*| + 1) \).

IV. RANDOM NODE LAYOUT

In this section we consider a random network, where \( n \) wireless nodes are uniformly and independently distributed in a unit square. Due to the probabilistic nature of the network, the results are with high probability, or in short w.h.p., which means that the probability of the result converges to one as the number of network nodes, \( n \), increases. First, we introduce some bounds which are used throughout the section. Then, we present two power assignment schemes (GBT and MST), with different performance guarantees. The theorem below derives an additional lower bound on the minimum cost of a power assignment which induces a broadcast arborescence.

Theorem IV.1. \( c^*_s = \Omega(1) \), for any \( s \in V \).
Let us denote by $N(i,j)$ all the nodes which are in a grid cell $(i,j)$, $1 \leq i,j \leq 1/\delta$. Note that due to Lemma IV.3, each grid cell contains at least one node.

**Step 2** We arbitrarily pick one representative node $u_{i,j}$ from each grid cell $N(i,j)$, $1 \leq i,j \leq 1/\delta$, with one exception. If the cell contains $s$ we pick $s$.

**Step 3** Let $H = (V,E)$ be an undirected graph with two types of edges, $E = E_1 \cup E_2$. The first type of edges is between the representative nodes to other nodes in their cell. $E_1 = \{(u_{i,j},v) : v \in N(i,v), 1 \leq i,j \leq 1/\delta \}$. The second type of edges connect between representatives of adjacent cells. Two cells $N(i,j)$ and $N(k,l)$ are adjacent if they have a common wall; this is denoted by $(i,j) \perp (k,l)$. $E_2 = \{(u_{i,j},u_{k,l}) : (i,j) \perp (k,l), 1 \leq i,j,k,l \leq 1/\delta \}$. An example of $H$ is given in Fig. 1 for a grid with 9 cells.

**Step 4** Let $T$ be a directed spanning tree of $H$ rooted at $s$ which results from running the BFS algorithm from $s$.

**Step 5** Finally, we can define the power assignment $p_1$. For every $u \in V$, $p_1(u) = w(e^*_s(T))$.

The performance analysis of $p_1$ is based on the following two lemmas.

**Lemma IV.4.** $c(p_1) = O(1) \cdot c^*_s$ and $l(p_1) = \Omega(1) \cdot l^*_s$.

**Proof:** From the construction of $T$ it follows that $|e^*(T)| \leq \sqrt{5\delta}$. Therefore, for any $u \in V$, $p(u) \leq 5\delta^2$, which immediately results in $l(p_1) = \Omega(1) \cdot l^*_s$ due to Corollary IV.2.

To bound the cost of $p_1$ we look closely at the tree $T$. Note that nodes which are not cell representatives have no outgoing edges in $T$. This is due to the fact that the degree of any not representative node in $H$ is 1. Since $s$ is a representative node, every non representative must be a leaf in $T$. Thus, only cell representatives will have some power assigned. There are $1/\delta^2$ cells, each assigned a power of at most $5\delta^2$ and hence $c(p_1) = O(1)$. In conjunction with Theorem IV.1 we obtain $c(p_1) = O(1) \cdot c^*_s$.

**Lemma IV.5.** $\text{Len}(H_{p_1},s) = O\left(\sqrt{\frac{n}{\log n}}\right)$.

**Proof:** Clearly $h(T) \leq 2/\delta = \sqrt{\frac{8 \log n}{n}}$ as every pair of adjacent cells is connected. To bound the length of a feasible schedule we argue that $I^p_{p_1} = O(1)$. As it was stated in the proof of Lemma IV.4, representative nodes are the only ones to transmit and their transmission range is at most $\sqrt{5\delta}$. We can derive that each representative nodes interferes with and is interfered by at most a constant number of nodes and hence $|I^p_{p_1}| = O(1)$. By using Theorem III.3 we obtain $\text{Len}(H_{p_1},s) = O\left(\sqrt{\frac{n}{\log n}}\right)$.

The properties of $p_1$ are summarized in the following theorem.

![Grid based construction](image)
Theorem IV.6. For n wireless nodes, randomly, uniformly, and independently distributed in a unit square and a source node s, the power assignment \( p_2 \) induces a broadcast arborescence \( H_{p_2} \) rooted at s such that \( \text{Len}(H_{p_2}, s) = O\left(\frac{n}{\log n}\right) \), \( c(p_1) = O(1) \cdot c^*_s \), and \( l(p_2) = \Omega(1) \cdot l^*_s \).

B. MST based broadcast tree (MST)

The second scheme (MST) is based on a minimum spanning tree \( \text{MST}_V \). We describe a power assignment \( p_2 \) which was first introduced by Kirousis [14]. For every \( u \in V \) we define \( p(u) = w(e^*_u(\text{MST}_V)) \). In [14] the authors showed that \( c(p_2) \leq 2w(\text{MST}_V) \). The energy efficiency of \( p_2 \) is easily derived in the following lemma based on Theorems III.1 and III.2.

Lemma IV.7. \( c(p_2) = O(1) \cdot c^*_s \) and \( l(p_2) = l^*_s \).

Next we analyze the length of the shortest schedule from any source node \( s \in V \) we first derive the following technical lemma by applying the “balls and bins” analysis from [20].

Lemma IV.8. \( |I_{p_2}^*| = O(\log n) \).

Proof: Note that from the definition of \( I_{p_2}(\cdot) \) it follows that two nodes cannot be scheduled to transmit simultaneously if there exists a designated recipient that is within the transmission range of both nodes. Which means that two conflicting nodes can be at a distance of at most twice the maximum transmission range. Therefore, according to the construction of \( p_2 \), for any node \( u \), the nodes in \( I_{p_2}(u) \) are within distance \( 2|e^*(\text{MST}_V)| \) from \( u \). Since \( w(e^*(\text{MST}_V)) = \Theta\left(\frac{\log n}{n}\right) \), for any \( u \in V \) and \( v \in I_{p_2}(u) \), \( d(u, v) \leq a \cdot \log n/n \), where \( a > 1 \) is some constant.

Next we apply the “balls and bins” analysis. Divide the unit square into \( \frac{n}{\log n} \) grid cells, each of size \( \sqrt{\frac{\log n}{n}} \times \sqrt{\frac{\log n}{n}} \). We can look at the distribution of nodes in the grid as a random process where we independently and uniformly throw \( n \) balls into \( \frac{n}{\log n} \) bins. The authors in [20] analyzed the maximum number of balls in each bin (in our case, nodes in a grid cell). They showed that w.h.p. each grid cell contains at most \( O(\log n) \) nodes. We can conclude that for each node \( u \) there are at most \( O(\log n) \) nodes at distance \( a \cdot \log n/n \) from it. Thus, \( |I_{p_2}^*| = O(\log n) \).

By using Theorem III.3 and Lemmas IV.7, IV.8 we can easily obtain the properties of \( p_2 \).

Theorem IV.9. For any source node \( s \in V \), \( \text{Len}(H_{p_2}, s) = O(\log n \cdot h(\text{MST}_V)) \), \( c(p_2) = O(1) \cdot c^*_s \), and \( l(p_2) = l^*_s \).

V. Deterministic node layout

In this section we present two broadcast schemes for deterministic node layout, i.e. the nodes are not placed randomly. First we develop a Hamiltonian circuit based scheme (HCT) for an arbitrary node layout, and then present the FORK scheme for a grid layout of nodes.

A. Hamilton circuit based broadcast tree (HCT)

Our third construction (HCT) is based on a Hamiltonian circuit. Sekanina [22] showed that the cube of any tree in \( \tau = (V, E) \), with \( |V| \geq 3 \), is Hamiltonian. Andrea and Bandelt [2] give a linear time algorithm for the construction of the Hamiltonian circuit \( P_\text{h} \) in \( T^3 \), given \( T \). They also show that \( w(h) \leq w(T) \cdot \left(\frac{3}{2}\tau^2 + \frac{1}{2}\tau\right) \), where \( \tau \) is the weak triangle inequality parameter. Note that \( \tau = 2^{n-1} = 2 \) under our assumption that \( \alpha = 2 \) (recall that \( \alpha \) is the distance-power gradient). Moreover, it can be shown that the weight of the longest edge in \( P_\text{h} \) is at most \( O(1) \) times the weight of the longest edge in \( T \). The following theorem applies the above on \( T = \text{MST}_V \).

Theorem V.1 ([2]). Let \( P_\text{h} = \langle u_0 = s, u_1, \ldots, u_n = u_0 \rangle \), where \( u_i \in V \) for \( 0 \leq i \leq n \), be a Hamiltonian circuit as a result of applying the construction in [2] on \( \text{MST}_V \). Let \( H \) be a path graph which is obtained by taking \( P_\text{h} \) and removing the last edge. Then \( w(H) = O(w(\text{MST}_V)) \) and \( w(e^*(H)) = O(w(e^*(\text{MST}_V))) \).

We define the power assignment \( p_3 \) as follows. For any \( u_{ik}, 0 \leq l \leq \lfloor(n-1)/k\rfloor, \)

\[
p_3(u_{ik}) = \max_{1 \leq i \leq \min\{k, n-1-lk\}} d(u_{ik}, u_{ik+i})^2.
\]

That is, every node \( u_{ik}, 0 \leq l \leq \lfloor(n-1)/k\rfloor, \) is assigned to reach the next \( k \) nodes on the Hamiltonian path constructed in the theorem above. All the other nodes have a power assignment of 0. We can easily obtain the following theorem.

Theorem V.2. \( c(p_3) = O(k) \cdot c^*_s \), \( l(p_3) = \Omega(1/k^2) \cdot l^*_s \), \( \text{Len}(H_{p_3}, s) = \Theta((n-1)/k) \).

Proof: Note that for any node which is assigned a power, \( u_{ik} \) it holds

\[
p(u_{ik}) = \left(\sum_{i=0}^{\min\{k, n-1-lk\}} d(u_{ik+i}, u_{ik+i+1})\right)^2 \leq k \sum_{i=0}^{\min\{k, n-1-lk\}} w(u_{ik+i}, u_{ik+i+1}).
\]

Therefore, \( w(e^*(H_{p_3})) \leq k^2 w(e^*(\text{MST}_V)) \) and \( c(p_3) = O(k)w(\text{MST}_V) \). This proves the bounds of total cost and

\(^2\)In the cube of a tree \( T \) there are edges between the endpoints of any three- and two-hop paths, in addition to the original one.
Therefore, when a cell leader with energy \((k)\) then analyze its efficiency. We divide the grid to a smaller scheme without using the power assignment notations and we assume the scheduling length is fixed). Execution is distributed, and partially synchronous (since and almost optimal power assignment. The algorithm execution is distributed, and partially synchronous (since we assume the scheduling length is fixed).

\[ \text{The FORK scheme} \]

Our fourth construction is developed for a grid node layout. It is dubbed FORK as the tree resembles a fork (see Fig. 2(b)). We assume that the underlying grid has a fixed scheduling requirement \(k\) and show that there exists a broadcast tree with scheduling length \(k\), optimal lifetime, and almost optimal power assignment. The algorithm execution is distributed, and partially synchronous (since we assume the scheduling length is fixed).

To simplify the notation we describe the transmission scheme without using the power assignment notations and then analyze its efficiency. We divide the grid to a smaller \(k \times k\) grid with equal size cells, for each cell \(i, j\) in the new \(k \times k\) grid we select a cell leader, \(\text{leader}_{i,j}\), the cell leader is the node located at the bottom left corner of any cell (For example, in Fig. 2(a) we have \(\sqrt{n} = 14\), and \(k = 7\); the selected cell leaders are colored in Grey). Without loss of generality, we set the root node \(r\) (\(\text{leader}_{0,0}\)) to be the node at the bottom left corner of \(G\) (i.e., \(r\) has coordinates \((0, 0)\)). The progress of the broadcast algorithm is as follows, \(r\) transmits the first message with power \((\frac{\sqrt{n}}{k})^2\), when a cell leader \(\text{leader}_{i,j}\) receives the message from cell leader \(\text{leader}_{k,l}\) at the first time, it transmits the message with energy \((\frac{\sqrt{n}}{k})^2\) if either a.) \(i = k + 1\) and \(j = l + 1\) or b.) \(i = 0\) and \(j \equiv 2\) mod 3 or \(j = 0\) and \(i \equiv 2\) mod 3. If c.) \(i = 0\) and \(j \equiv 0\) mod 3 or \(j = 0\) and \(i \equiv 0\) mod 3, the node transmits the message with energy \((\frac{\sqrt{n}}{k})^2\) note that the fringe cell leaders (cell leaders with either \(i = 0\) or \(j = 0\)) behaves differently and some nodes receives the transmission twice, but without interference in the second time.

A sample scheduling schema is depict in Fig. 2(b), the number inside each cell leader illustrates the message transmission round, the directed arrows illustrate the radius of transmission (i.e., the transmitted message arrives to all the nodes whose distance is less then or equal to the radius). Note that the scheduling length is \(k\), the maximum transmission power of any node is at most \((\frac{\sqrt{n}}{k})^2\), and the total cost of is therefore \(k^2(\frac{\sqrt{n}}{k})^2 = 2n\). The next theorem summarizes the properties of FORK.

**Theorem V.3.** The schedule length of FORK is \(k\), the network lifetime is optimal for a schedule of length \(k\) and the total power is at most \(O(1)\) times the optimal.

**Proof:** By definition, the schedule length is \(k\). According to III.1 we can derive that the cost of the optimal solution is at least \(\Omega(n)\) and therefore the scheme achieves a constant factor approximation for the total cost. Regarding the network lifetime, any optimal broadcast tree which has a schedule length of \(k\) has at most \(k\) hops and therefore there exists a node which must transmit to distance \(\frac{\sqrt{n}}{k}\) and therefore the network lifetime of FORK is optimal. \(\blacksquare\)

**VI. Simulation results**

The performance of several of our algorithms is evaluated through simulations. We compare the grid based (GBT), MST based (MST), and Hamiltonian circuit based (HCT) with \(k = \sqrt{n}\) power assignments \(p_1, p_2, p_3\), respectively, and measure their total energy consumption and schedule length. The simulations have been carried out for values of \(n\) ranging from 200 to 2000 with steps of 200. Each point in the plot is an average of 5 tries. The
source node \( s \) is randomly picked out of the \( n \) nodes. The evaluation results appear in Fig. 3(a) and Fig. 3(b).

In terms of total energy (Fig. 3(a)), all three power assignments show a constant approximation ratio over the optimal lower bound (OPT). We can see that the MST is the most energy constrained, followed by GBT and HCT. In can be seen that the ratio between the power consumption of different power assignments to the optimum is at most \( \approx 8 \). Surprisingly, even though we picked a very high value of \( k = \sqrt{n} \) for HCT, the energy consumption is far below the theoretical bound of \( O(k)w(MST) \).

We estimated the schedule length of the power assignment through measuring the diameter of the network and the anticipated conflict sets (Fig. 3(b)). It can be seen that even though MST outperforms the other power assignments in terms of energy consumption, it trails behind both, GBT and HCT in terms of schedule length, with HCT naturally being the best.

VII. Conclusions and Future Work

In this paper, we studied the interference-free broadcast problem in wireless ad hoc networks. We considered both random networks and the non-random case where the nodes could have arbitrary positions or a grid layout. We developed four broadcast schemes and analyzed their energy efficiency (through total energy consumption and network lifetime) and schedule length. To the best of our knowledge, this is the first paper to consider all three objectives simultaneously.

A possible future research direction would be to improve the asymptotic bounds which we derived throughout the paper. It would also be of interest to consider additional interference models, such as the SINR model.

References


