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Strategyproof Wireless Spectrum Auctions with Interference

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***Abstract*—Wireless spectrum is a regulated resource, whose control and usage is regulated by government agencies. The allocation of spectrum to interested parties is usually conducted through auctions, and are an important source of income for these regulatory agencies. However, previous spectrum auction design fail to take into consideration the effect of interference, which can adversely affect the truthfulness of an auction. In this paper, we explicitly consider interference effects, and design truthful auctions for maximizing social welfare. Since the spectrum allocation problem is NP-Hard, we first show how to compute an approximate spectrum allocation scheme that is within a constant factor of the optimal solution, under certain simplifying assumptions on the interference graph. We then proceed to make this scheme strategyproof by tailoring a payment scheme based on the idea of minimum bids. A naive method to compute such a payment scheme requires $O(n)$ iterations of the spectrum allocation algorithm, where n is the number of bidders. We show how to reduce the complexity to $O(1)$ iterations instead. We conclude by discussing possible directions for future research in this area.**

I. INTRODUCTION

Since 1994, the Federal Communications Commission (FCC) has conducted a series of auctions to allocate wireless spectrum to telecommunication companies. Such auctions have generated billions of dollars in revenue [1], and many governmental institutions around the world have since followed suit. Such auctions are usually characterized by long bidding periods and financial investments on a large scale. Hence, in these auctions, the focus has usually been on attracting companies to bid, and preventing collusion among these companies during the bidding process [1]. However, spectrum auction design should also take into consideration the unique nature of the spectrum allocation problem. Spectrum auctions are unique in the sense that (1) transmissions within the same spectrum range suffer from *interference*, and (2) a given spectrum range can be sold to multiple bidders, subject to interference constraints. Auctions that fail to take these properties into account may not be robust against *strategic behaviour* by bidders.

Mathematically, including interference constraints when allocating discrete spectrum ranges can be modeled as the weighted independent set problem, which is known to be NP-Hard [2]. This latter fact complicates matters somewhat, since most auctions are designed to use the well known Vickrey-Clarke-Groves (VCG) mechanism [3]–[5] in order to ensure bidders refrain from misreporting their true valuation for winning the auction. An auction that guarantees truthful behaviour is said to be *strategyproof*. However, VCG schemes lose this important strategyproof property when applied to sub-optimal solutions [6,7]. Our goal in this paper is to design auctions that are truthful, while explicitly taking interference effects into consideration.

In this work, we will assume there is a spectrum owner, who wishes to auction discrete ranges of spectrum, *i.e.* discrete channels. Each ISP has a privately known *valuation*, which indicates how much the ISP is willing to pay for the channel. The goal of the auctioneer is to allocate channels to ISPs in an interference-free fashion, while maximizing *social welfare*. The social welfare of an auction is the total utility gained by all participants given the auction outcome. Since the spectrum allocation problem is NP-Hard, we first show how to compute an *approximately optimal* allocation using a simple greedy technique. We prove the approximation factor of our technique using a well known property of unit disk graphs [8]. We then proceed to show how to elicit the valuations of ISPs truthfully when computing this approximately optimal allocation, via the use of a payment scheme. Our payment scheme relies on the notion of charging each ISP that is allocated a channel a *minimum bid*, which is *independent* of the ISP's own bid. A naive approach to compute this payment scheme requires running our approximation algorithm as many times as there are ISPs. However, we present an algorithm that reduces this computational overhead significantly by requiring only a single iteration of the channel allocation algorithm to compute payments for all ISPs. Our focus in this work is on auctions that maximize social welfare. Such auctions have poor revenue generating properties, and hence we conclude the paper with a discussion on auction design when the goal is *revenue maximization*.

The rest of the paper is organized as follows; in Sec. II, we discuss some background and related research. We then introduce our model and notation in Sec. III. We present a simple greedy channel allocation algorithm in Sec. IV. We then describe a payment scheme to ensure truthful behaviour by ISPs when this greedy channel allocation algorithm is used in Sec. V, before concluding in Sec. VI.

II. PREVIOUS RESEARCH

A *mechanism* is a protocol for implementing a desired *social choice* function. Social choice theory has its origins

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in the field of economics, where the goal is to design a set of rules for achieving a desired social choice outcome in the presence of strategic behaviour by agents. The classic negative result of Gibbard [9] and Satterthwaite [10], which derives from Arrow's impossibility theorem [11], essentially states that under very general conditions, every social choice function that can be implemented in dominant strategies is *dictatorial*. A function is said to be dictatorial if there exists an agent whose preference is always chosen as the outcome of the mechanism. Much of modern mechanism design circumvents this impossibility result through the use of an additional degree of freedom in the outcome of a mechanism. Frequently, this degree of freedom is in the form of *payments* (both positive and negative) to the agent by the mechanism designer. Mechanisms employing payments to agents (such as auctions) are said to be quasi-linear.

Within the quasi-linear domain, the most important results in the literature come from the sequence of work by Vickrey [3], Clarke [4] and Groves [5]. Vickrey studied auction mechanisms, and showed how to design truthful auctions by charging the winning bidder the second highest bid. Vickrey's pricing rule for strategyproof auctions was generalized by Clarke within the context of taxing public goods [4]. This line of research finally culminated in the general VCG mechanism, first presented by Groves [5]. The VCG mechanism is the best known strategyproof mechanism, and in some cases, is known to be the only method for ensuring truthful behaviour by agents [7]. However, VCG mechanisms are known to have a number of drawbacks [12]. Crucially, VCG mechanisms lose the strategyproofness property when applied to sub-optimal solutions, rendering it unsuitable for designing truthful mechanisms when dealing with NP-Hard problems [6,7].

In this paper, we are primarily interested in mechanisms that are auctions. Auctions are also sometimes referred to as *direct revelation* mechanisms, to reflect the fact that in such mechanisms the only strategy available to an agent is to reveal her valuation, or *bid*, for the one or more items being auctioned. The monograph of Krishna [13] provides a classic treatment of auction theory, while Klemperer [1] surveys practical and theoretical techniques used in modern auctions, which includes a comprehensive description of 3G wireless spectrum auctions for mobile networks. Cramton *et al.* [14] on the other hand focus on combinatorial and computational aspects of auctions.

Wireless spectrum auctions have previously been studied by Jia *et al.* [15] and Zhou *et al.* [16]. Jia *et al.* [15] consider a dynamic spectrum allocation market, and show how to incorporate Myerson's virtual valuation [17] into their model for use in a VCG auction with the goal of maximizing revenue. In contrast, Zhou *et al.* [16] focus on designing strategyproof mechanisms for maximizing social welfare, and show how their scheme can be adapted for use with any channel allocation algorithm. They further derive a closed form expression for maximizing the revenue among all social welfare maximizing auctions, through an appropriate choice of the number of channels available in the auction.

III. Preliminaries

We denote the set of ISPs interested in obtaining spectrum as $\mathcal{S}$, and hereafter refer to each ISP as an *agent*. We will further assume that each ISP or agent owns a single base station, which are distributed geographically, and we will denote by $d(i,j)$ the physical distance between the base stations of i and j. The wireless service coverage of each agent is a circle with fixed radius r. Each agent $i \in \mathcal{S}$ is interested in purchasing spectrum from the spectrum owner, hereafter referred to as the *auctioneer*. We will assume that the available spectrum has been divided into K discrete channels, with the set of channels denoted as $\mathcal{K}$. Agents are interested in *at most one* channel. We will discuss extensions to this model in Sec. VI. We say agents i and j *interfere*, if both agents are allocated a given channel $k \in \mathcal{K}$, and $d(i,j) \leq 2r$, i.e., the coverage area of agents i and j overlap. We can model this interference pattern using the conflict graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where for $i,j \in \mathcal{V}$, $ij \in \mathcal{E} \iff d(i,j) \leq 2r$. The assumption that the coverage radius r is uniform for all i implies that $\mathcal{G}$ is a *unit disk graph* [8]. We will denote by $\mathcal{N}(i)$ the set of i's neighbours in the conflict graph $\mathcal{G}$.

Each agent i has a *valuation* v_i for receiving a channel from the auctioneer. Without loss of generality, v_i is assumed to be expressed in monetary units. Each agent is charged a payment p_i by the auctioneer, for receiving a channel. In this case, we can state the *utility* of an agent i as:

$$u_i = \begin{cases} v_i - p_i & \text{if } i \text{ receives a channel} \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

Adopting standard assumptions from the field of mechanism design to model *strategic behaviour*, agents are assumed to be *selfish* and *rational*. Hence, each agent i behaves strategically with the aim of maximizing Eq. (1).

We will be focusing on sealed-bid auctions[1]. In this setting, each agent is required to disclose her valuation v_i to the auctioneer at the beginning of the auction. In terms of strategy, the sole degree of freedom for any agent is the value of her bid. Consequently, agents may choose to misreport her valuation to the auctioneer. To distinguish between bids and valuations, we will use b_i to denote agent i's bid. The vector of bids by all agents will be denoted by $\mathbf{b}$, while the vector $\mathbf{b}_{-i}$ will be used to denote the bids of all agents excluding i. Since the outcome of an auction is a function of the bids of all agents, we can denote agent i's utility by $u_i(b_i, \mathbf{b}_{-i})$ to more precisely reflect this. An auction is said to be *dominant-strategy truthful* if reporting her true valuation is the dominant strategy for an agent i, regardless of the bids of other agents, *i.e.* for all $b_i \neq v_i$ and for any $\mathbf{b}_{-i}$, the following always holds:

$$u_i(v_i, \mathbf{b}_{-i}) \geq u_i(b_i, \mathbf{b}_{-i}) \tag{2}$$

The goal of the auctioneer is to design an auction mechanism that simultaneously computes a channel allocation that is *interference-free*, and a payment scheme that *maximizes*

[1] By the revelation principle [17], this assumption is without loss of generality

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social welfare. The space of all possible mechanisms may be quite large, but we will restrict ourselves to mechanisms for which there is a *dominant-strategy implementation*. This is a reasonable assumption, as it ensures that in any outcome of the auction, each agent is playing her dominant strategy. This property in turn provides us with a platform from which we may make concrete guarantees about the performance of our mechanism. We will further focus on designing auctions that are dominant-strategy truthful. The *revelation principle* [6,17], which states that any dominant-strategy mechanism can be reduced to a dominant-strategy *truthful* one, reassures us that our focus on the latter is without loss of coverage. Aside from truthfulness, we will also require that our auction satisfies the properties of *individual rationality*:

$$u_i \geq 0 \quad \forall i \tag{3}$$

as well as *no positive transfers* [12]:

$$p_i \geq 0 \quad \forall i \tag{4}$$

The first property ensures that agents never lose from participating in the auction, while the second forbids the auctioneer from making payments to agents.

We will make heavy use of the following characterization of truthful or strategyproof mechanisms, essentially due to Myerson [17] but stated here in the simpler form as expressed by Archer and Tardos [18]:

Lemma 1 ***[Myerson, 1981]*** *Let $x_i(b_i)$ be the allocation function used in an auction for bidder i with bid b_i. A mechanism is strategyproof if and only if the following hold for a fixed $\mathbf{b}_{-i}$:*

- *$x_i(b_i)$ is monotonically non-decreasing in b_i*
- *Bidder i bidding b_i is charged $b_i x_i(b_i) - \int_0^{b_i} x_i(z)dz$*

Note that by Lemma 1, once an allocation rule for agent i $x_i(.)$ is fixed, this rule completely determines her payment. Conversely, given a fixed payment rule, the allocation rule is fixed. Lemma 1 thus provides us with two equivalent ways to view a truthful mechanism; (i) there exists a *minimum bid* b'_i such that if i bids at least b_i, then she will be allocated a channel, while bidding less than b'_i results in i not being allocated, or (ii) the payment charged to i for a fixed $\mathbf{b}_{-i}$ is independent of b_i. We will employ this view of truthful mechanisms to prove that the payment scheme we design in Sec. V is strategyproof.

IV. Approximately optimal channel allocation

In this section, we will first show a simple approximation algorithm for computing a feasible channel allocation scheme that maximizes *social welfare*. A feasible channel allocation scheme is one that is interference-free. In the next section, we will design a payment scheme tailored to this channel allocation scheme that guarantees agents have no incentive to lie when submitting their bids for channels.

The social welfare of a system is the sum of the utilities of all agents as well as the auctioneer. Since the utility of an auctioneer is the sum of all payments collected from agents, one can express the social welfare of a system W, as

$$W = \sum_{i \in \mathcal{S}'} v_i \tag{5}$$

where $\mathcal{S}' \subseteq \mathcal{S}$ is the set of agents allocated a channel. Let the binary variable x_i^k indicate if agent i is allocated channel k. We can express the social welfare maximization problem as the following integer linear program (ILP):

$$\text{Maximize} \sum_k \sum_i v_i x_i^k \tag{6}$$

Subject To:

$$\begin{aligned} \sum_{k \in \mathcal{K}} x_i^k &\leq 1 \quad \forall i \\ x_i^k + \sum_{j \in \mathcal{N}(i)} x_j^k &\leq 1 \quad \forall i, \forall k \\ x_i^k \in \{0,1\} & \quad \forall k, \forall i \end{aligned}$$

The first constraint reflects the demand of each agent, which is at most one channel. The second requirement captures the interference constraint – an agent i can only be allocated channel k if no other agent in i's neighbourhood has been assigned this channel. It is not hard to see that the social welfare maximization problem is NP-Hard, since the special case when $K = 1$ and all agents have the same valuation reduces to the well known Maximum Independent Set problem, which is also NP-Hard [2].

Nevertheless, our assumption that the conflict graph $\mathcal{G}$ can be modeled as a unit disk graph allows us to design a simple, greedy algorithm with a constant approximation ratio. This scheme is shown in Algorithm 1. We essentially sort all agent valuations, and greedily assign channels to agents in this order, subject to interference constraints. The approximation factor of this algorithm relies on the following simple property of unit disk graphs, due to Marathe *et al.* [8]:

Algorithm 1: Simple 5-approximation algorithm for social welfare maximization

Input: Conflict graph $\mathcal{G}$, set of channels $\mathcal{K}$, set of agents $\mathcal{S}$ with valuations $\mathbf{v}$
Output: Allocation matrix $\mathbf{x}$

Initialize $x_i^k := 0 \quad \forall i, \forall k$;
while $S \neq \emptyset$ **do**
  $i := \arg\max_j v_j$ such that $v_j \in S$;
  $S := S \setminus \{i\}$;
  foreach $k \in \mathcal{K}$ **do**
    if $\sum_{j \in \mathcal{N}(i)} x_j^k = 0$ **then**
      $x_i^k := 1$;
      break ;

Lemma 2 *In a unit disk graph, there is no induced subgraph isomorphic to $K_{1,6}$*

Proof: The $K_{1,6}$ graph is bipartite graph in which each partition contains 1 and 6 nodes respectively. Lemma 2 is

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a consequence of simple geometry, and we will prove it by contradiction. Assume the lemma is not true, and that there is a subgraph in a unit disk graph isomorphic to $K_{1,6}$. Let u_1 be the node in the first partition, and let $u_2 \ldots u_7$ be nodes in the second partition. Clearly, $u_2 \ldots u_7$ is at distance at most $2r$ from u_1. Since there are 6 nodes, there must be at least two nodes u_i and u_j such that the angle between the lines $u_1 u_i$ and $u_1 u_j$ is less than $60°$, and hence $d(u_i, u_j) \leq 2r$, which is a contradiction. □

Lemma 2 implies that the maximum independent set in the neighbourhood of any node is at most 5, from which we can trivially deduce the following:

Theorem 1 *Algorithm 1 is a 5-approximation algorithm for computing a social welfare maximizing channel allocation.*

Algorithm 1 finds a channel allocation that approximately maximizes social welfare, under the assumption that agent valuations are known. In reality, agent valuations are private information known only to the agent. Hence, in the next section, we will design a payment scheme that elicits bids truthfully when Algorithm 1 is used to allocate channels.

V. Strategyproof Approximately Optimal Channel Allocation

Algorithm 1 works if agents declare their valuations truthfully. In a strategic setting, we need to design a *payment scheme* to be used in conjunction with Algorithm 1 such that the entire mechanism is strategyproof. The first candidate that comes to mind is the well known Vickrey-Clarke-Groves (VCG) payment scheme [3]–[5]. Unfortunately, VCG schemes applied directly to sub-optimal solutions are in general not strategyproof [6,7]. Hence, we will design a simple payment scheme built on the key ideas of Lemma 1. Recall that in a truthful mechanism, there exists a minimum bid b'_i, that is independent of b_i, and is such that i is only allocated the item if i bids at least b'_i. This perspective points to a simple recipe for creating strategyproof mechanisms – charge each agent this minimum bid.

The minimum bid of an agent is closely tied to the allocation rule used. In the context of Algorithm 1, the greedy nature of the algorithm allows us to pinpoint the minimum bid for agent i. It is the valuation of some agent j that causes i's available, interference-free channels to be exhausted in a run of Algorithm 1 without the participation of i. Clearly, if such a j exists, then bidding below v_j causes i to not be allocated by Algorithm 1, while bidding above this value ensures that i is allocated a channel. If no such j exists, then i should be charged 0. It is not hard to see that under this protocol, bidding truthfully is a dominant strategy, and we will prove this in a rigorous fashion later.

The obvious way of computing this minimum bid is to run Algorithm 1 once for all agents in $\mathcal{S}$, and then once without agent i for each $i \in \mathcal{S}$, resulting in $|\mathcal{S}|+1$ iterations. However, we can reduce this computational overhead by updating payments for agent i when we remove the agent j from $\mathcal{S}$ whose bid represents the minimum bid required for i

Algorithm 2: Strategyproof 5-approximation algorithm for social welfare maximization

Input: Conflict graph $\mathcal{G}$, set of channels $\mathcal{K}$, set of agents $\mathcal{S}$ with valuations $\mathbf{v}$

Output: Allocation matrix $\mathbf{x}$, payment vector $\mathbf{p}$

Initialize $x_i^k := 0 \quad \forall i, \forall k$;
Initialize $p_i := 0 \quad \forall i$;
Initialize $paid[i] := 0 \quad \forall i$;
Initialize $sat[i] := 0 \quad \forall i$;
while $S \neq \emptyset$ **do**
  $i := \arg\max_j v_j$ such that $v_j \in S$;
  $S := S \setminus \{i\}$;
  if $sat[i] = 1$ **then**
    $\mathcal{M} := \{j | j \in N(i) \wedge sat[j] := 1 \wedge paid[j] = 0\}$;
    $l := \arg\min_j \{v_j | j \in \mathcal{M}\}$;
    $p_l := v_i$;
    $paid[l] := 1$;
  foreach $k \in \mathcal{K}$ **do**
    if $\sum_{j \in \mathcal{N}(i)} x_j^k = 0$ **then**
      $x_i^k := 1$;
      foreach $j \in \{\mathcal{N}(i) \cup i\}$ **do**
        if $\sum_{k \in \mathcal{K}} \left(\sum_{l \in \mathcal{N}(j)} x_l^k + x_j^k \right) = K$ **then**
          $sat[j] := 1$;
      break ;

to be allocated a channel. In order to determine the minimum bid of some agent i, the following lemma will prove useful:

Lemma 3 *If v_j is the minimum bid for some agent i, then when i gets allocated by bidding v_i, j will not be allocated a channel.*

Proof: By definition, if v_j is i's minimum bid, it must mean that when i bids less than v_j, i does not get allocated a channel. By way of contradiction, assume the lemma is not true. That is, j gets allocated when i bids v_i, as well as when i bids less than v_j. Let i's bid in the latter case be $v_j - \epsilon$ for small enough ϵ so that i is considered for allocation right after j. Let k be the channel assigned to i when i bids v_i. Then since all other bids are fixed, if i bids $v_j - \epsilon$, it must be that j gets allocated k, by definition of the minimum bid. But if j is allocated even when i bids v_i, then she must be allocated some channel $k' > k$, which means that i would have been allocated k' for bidding $v_j - \epsilon$, which is a contradiction. □

Employing Lemma 3, we show a modified version of Algorithm 1 in Algorithm 2, which also simultaneously computes payments for agents. We will say an agent i is *saturated* if i together with i's neighbours have been allocated all K channels. In Algorithm 2, if an agent i is saturated when being considered for allocation, then she cannot be allocated a channel. For each such agent i, we build a set consisting of i's neighbours j that have been allocated a channel and are saturated, but whose payment has not been computed. We then find the agent l in this set with the minimum valuation, and charge l the valuation of i, v_i. Clearly, if l bids lower than v_i, then i would be allocated ahead of l. Before we prove that this payment scheme always ensures truthful bidding, we first require the following observation on the property of Algorithm 2:

Lemma 4 *The channel allocation rule used in Algorithm 2 for some agent i is monotonically non-decreasing in b_i for fixed $\mathbf{b}_{-i}$. That is, if i is allocated a channel when bidding b_i, then she is guaranteed to be allocated a channel when bidding $b'_i \geq b_i$.*

Lemma 4 is a direct consequence of the greedy allocation rule used in Algorithm 2. We are now ready to prove the next theorem:

Theorem 2 *Algorithm 2 is a strategyproof mechanism for approximately maximizing social welfare.*

Proof: Fix all other bids at $\mathbf{b}_{-i}$, and let $y = \sum_{k \in \mathcal{K}} x_i^k$ be the allocation of agent i. Let v_i and b_i be i's true valuation and declared bid respectively. Similarly, let y_v and y_b be i's allocation when bidding v_i and b_i respectively. First, assume that $b_i < v_i$. If $y_v = 1$ and $y_b = 0$, then bidder i does not gain from lying. If $y_v = 0$ and $y_b = 0$, then bidding truthfully is still a dominant strategy. If $y_v = 0$ and $y_b = 1$, we get a contradiction due to Lemma 4. If $y_b = y_v = 1$, observe that the payment is independent of i's bid, and hence i's utility stays the same. Next, consider the case when $v_i < b_i$. If $y_v = y_b = 0$, then i does not gain from lying. If $y_v = 1$ and $y_b = 0$, we get a contradiction due to Lemma 4. If $y_v = y_b = 1$, then again, since the payment is bid independent, bidding truthfully is still a dominant strategy. If $y_v = 0$ and $y_b = 1$, then it means that i does not get allocated when bidding truthfully, for a net utility of 0. When she bids b_i and gets allocated, then it must be that this causes some other bidder j to not be allocated this channel, and it must further be that $v_j > v_i$. In this case. Algorithm 2 will charge i the minimum bid v_j, resulting in negative utility for i. $\square$

The payment scheme described ensures agents have no incentive to lie when a greedy allocation algorithm is used for computing a channel allocation that maximizes social welfare. However, this payment scheme does not make any guarantees on the *revenue* obtained from this auction. Indeed if the goal is revenue maximization, the auctioneer must resort to other means. In particular, under the (realistic) assumption that the auctioneer has no prior information about the distribution of agent valuations, maximizing revenue while simultaneously ensuring strategyproofness requires the use of *randomized auctions* [19]. Such auctions advocate randomly sampling agents to determine a good revenue maximizing price. We are currently studying how similar techniques can be used for the purpose of wireless spectrum auctions with interference.

VI. Conclusion

To summarize, we have studied truthful spectrum auctions, while explicitly taking into consideration interference effects. We first showed how to allocate channels to approximately maximize social welfare by exploiting a simple property of unit disk graphs. Since the VCG mechanism is not truthful when applied to sub-optimal solutions, we designed a payment scheme to ensure that our greedy allocation scheme can be made strategyproof. The naive method of computing the payments required as many iterations of the allocation scheme as there are ISPs. Hence, we proposed an algorithm to reduce this complexity to just a single iteration of the allocation algorithm.

In the future, we intend to generalize the model considered in this paper. In particular, we intend to design approximately optimal and truthful wireless spectrum auctions for the general case when ISPs have multiple channels demands, with multiple base stations. In this work, we have assumed that channels are indistinguishable in terms of quality and bandwidth. One can relax this assumption, and consider auctions for channels with different quality of service guarantees. In this model, an ISP's valuation for a channel will depend on the channel quality. Consequently, new schemes need to be designed to ensure spectrum auctions under interference can be made truthful. Finally, the goal of maximizing revenue for wireless auctions under interference is yet another avenue for future research, one which we are currently pursuing actively.

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