This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE Secon 2010 proceedings.

Throughput and Energy Efficiency in Wireless Ad Hoc Networks with Gaussian Channels

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***Abstract*—This paper studies the problem of topology control in random wireless ad hoc networks through power assignment for n nodes uniformly distributed in a unit square. We require that the network is strongly connected and look to maximize the minimum throughput (or capacity) link in the case that all the nodes *transmit simultaneously*. According to the Gaussian channel model, the throughput of a wireless link (u, v) is $B \log(1 + S/N)$ bps, where B is the channel bandwidth and S/N is the signal to noise ratio.**

We distinguish between two types of power assignments: homogeneous (all nodes have the same power level) and heterogeneous (nodes may have different power levels) cases. For the homogeneous case we give lower and upper bounds on the minimum capacity link. In the heterogeneous case we develop an energy efficient power assignment algorithm which achieves a minimum throughput of $\Omega(B \log(1 + 1/\sqrt{n} \log^2 n))$ and also discuss how to implement this algorithm in a distributed fashion. Finally, we present some simulation results.

To the best of our knowledge, these are the first provable bounds for capacity in wireless networks, when nodes are allowed to transmit simultaneously.

I. Introduction

A wireless ad hoc network consists of several transceivers (nodes) located in the plane, communicating by radio. Unlike wired networks, in which the link topology is fixed at the time the network is deployed, wireless ad hoc networks have no fixed underlying topology. In addition, the relational disposition of wireless nodes is constantly changing. The temporary physical topology of the network is determined by the distribution of the wireless nodes, as well as the transmission range of each node. The ranges determine a directed communication graph, in which nodes correspond to the transceivers and edges correspond to the communication links.

The key difference between wireless ad hoc networks and "conventional" communication structures, from the designer's point of view, is in the *power assignment model*. Each node decides on a transmission power level, and a transmission from node u can be received at node v if the transmission power of u is at least $d(u, v)^\kappa$, where $d(u, v)$ is the Euclidean distance between u and v, where κ is a constant representing the *distance-power gradient*, usually taken to be in the interval $[2, 4]$ (see [1]).

The increase in popularity of wireless ad hoc networks lead to an extensive research of many important functional properties of these networks, with one of the central issues being the *network capacity* (or throughput). Roughly speaking, network capacity defines the amount of data which can be transported during a certain time frame. As one of the main factors effecting the network capacity is radio interference caused by simultaneous transmissions, it has been shown ([2]) that the capacity of the network with n nodes is $\Theta(B\sqrt{n})$ bits per second (bps), where B is the bandwidth of the communication channel, even if the nodes are optimally placed in a disk of unit area, the transmissions are optimally assigned, and traffic patterns are optimally chosen. Thus, if the capacity is equally divided between the nodes, the per-node throughput scales as $\Theta(B/\sqrt{n})$, i.e. decreases at the rate of $1/\sqrt{n}$ as the number of nodes, n, increases. This surprising result emphasizes the destructive impact radio interference has on the network capacity.

Current existing works ([2]–[12]) consider smart scheduling algorithms to improve the overall capacity by lowering the cumulative interference of concurrent transmissions. Nodes are assigned *transmission time slots*, so that at any given time slot, only a subset of nodes are active while the others are idle. This scheme requires a synchronization mechanism, such as TDMA, which is impractical in many wireless applications.

In this paper we study the capacity of the communication links, when all the nodes *transmit simultaneously*. By allowing simultaneous transmissions we drop the need for synchronization, which allows easier network configuration and essentially better fits the nature of ad-hoc networks. We adopt the *Gaussian channel* model, which determines, the link capacity as a continuous function of SINR (Signal to Interference plus Noise Ratio) on the receiver's side. The Gaussian channel better characterizes the physical layer of wireless networks then the more common, *threshold-based* models [2]. A formal definition of the Gaussian channel model appears in Section I-A.

Energy efficiency is another critical issue in wireless networks design. The energy consumption of a single node is proportional to the power it is assigned – a higher transmission range requires more energy, which grows nonlinearly with the distance. Nodes in a wireless network are typically battery-powered and have an initial battery charge which is sufficient for a limited amount of time. As it is usually impossible to re-charge or replace the battery, energy efficiency becomes crucial. We evaluate energy efficiency through two measures:

total energy consumption and network lifetime, which is the time until the first battery charge depletion. Note that there is a strong correlation between the two measures.

This paper studies the problem of topology control through power assignments in random wireless networks. We assume that n wireless nodes are randomly, uniformly and independently distributed in a unit square, and look for an *energy efficient* (in terms of total energy and network lifetime) power assignment such that the communication graph is strongly connected,[1] and the *capacity of the minimum capacity link is maximized*. We consider two types of power assignments: homogeneous (all nodes have the same power level) and heterogeneous (nodes may have different power levels). To the best of our knowledge, we present the first provable bounds for capacity in wireless networks, when *all* nodes are allowed to transmit simultaneously.

The paper is organized as follows. In the rest of this section we present the model, discuss previous work and describe our contribution. In Section II we introduce some definitions and statements which we use throughout the paper. Section III studies the homogeneous power assignment case, followed by Section IV which addresses the heterogeneous case. Finally, we present some simulation results in Section V.

A. Network model

Let V be the n wireless nodes randomly, uniformly and independently distributed in a unit square. A power assignment is a function $p : V \to \mathbb{R}^+$. The transmission possibilities resulting from a power assignment p induce a directed communication graph $H_p = (V, E_p)$, where

$$E_p = \{(u, v) : p(u) \geq d(u, v)^\kappa\}$$

is a set of directed edges. The graph H_p is strongly connected if for every pair of nodes $u, v \in V$, there exists a directed path from u to v in H_p. The total energy consumption, also referred to as the cost, of the power assignment is given by $c(p) = \sum_{u \in V} p(u)$.

Each node u has some initial battery charge $b(u)$, which is sufficient for a limited amount of time, proportional to the power assignment $p(u)$. It is common to take the lifetime of a wireless node v to be $l(u) = b(u)/p(u)$. The network lifetime is defined as the time it takes the first node to run out of its battery charge. For a power assignment p and initial battery charges b, the network lifetime is defined as $l(p) = \min_{u \in V} l(u)$. In this paper we assume unit initial battery charges $b \equiv 1$, that is $b(u) = 1$, for every $u \in V$.

According to the Gaussian channel model, node v can successfully receive a transmission from u over a wireless communication link (u, v) at a data rate

$$Cap(u, v) = B \log(1 + SINR(u, v))$$

bits per second, where B is the channel bandwidth, and

$$SINR(u, v) = \frac{p(u)/d(u, v)^\kappa}{N_0 + \sum_{w \in V \setminus \{u,v\}} p(w)/d(w, v)^\kappa},$$

[1] A graph is strongly connected if there is a path between any pair of nodes.

with N_0 being the ambient noise power. That is, the receiver achieves the Shannon's capacity for a wireless channel with additive Gaussian noise [13]. A closer look at the expression $SINR(u, v)$ reveals that it consists of two parts: the signal strength in the numerator, and the interference in the denominator. In our scenario, the ambient noise, N_0, is negligible comparing to the interference caused by other transmitting nodes, which we denote as $I(u, v) = \sum_{w \in V \setminus \{u,v\}} p(w)/d(w, v)^\kappa$. As the expression for $SINR(u, v)$ is the only variable which effects the capacity of the link (u, v), most of the paper is dedicated to its analysis, and in particular to analyzing $I(u, v)$.

The capacity of a path P in a communication graph H is defined as the capacity of the minimum capacity link in P, that is $Cap(P) = \min_{(x,y) \in P} Cap(x, y)$. For a pair of nodes $u, v \in V$, we define the feasible throughput between u and v in a H as $Cap(H, u, v) = \max\{Cap(P), P$ is a path from u to v in $H\}$. Finally, the capacity of a communication graph H is defined as the feasible throughput between a pair of nodes with the minimum feasible throughput,

$$Cap(H) = \min_{u,v \in V} Cap(H, u, v).$$

Formally, this paper addresses the following problem.

Problem 1.

Input:	*Wireless nodes, V, randomly, uniformly, and independently distributed in a unit square.*
Output:	*A power assignment p, such that H_p is strongly connected.*
Objective:	*Maximize $Cap(H_p)$, minimize $c(p)$, and maximize $l(p)$.*

We assume $\kappa = 2$ for simplicity, although our results can be easily extended to any constant $\kappa \geq 2$.

B. Previous work

This paper combines between several different areas of research in wireless networks, which can be roughly divided into *capacity in wireless networks* and *energy efficient topology control*. We briefly sketch current developments in each of the areas.

1) Capacity in wireless networks: The asymptotic capacity for wireless ad hoc networks has been intensively studied under different channel models. The existing research can be divided into two main channel models: the threshold-based channel model and the Gaussian channel model.

The threshold model: The threshold-based model is subdivided into two interference models - the protocol model (PR) and the physical model (PH). In the former, a transmission by node u is successfully received by a target node v iff node v is sufficiently apart from the source of any other simultaneous transmission. In the latter, a transmission from u is received at v if $SINR(u, v)$ is above a certain threshold. These models were introduced in the ground breaking work of Gupta and Kumar [2], where they studied the asymptotic capacity of the network for unicast multi-hop sessions, under the PR and PH models, where each node chooses a random destination and is transmitting at B bits per second. They showed that a transport capacity of $\Theta(B/\sqrt{n})$ per node is feasible for arbitrary placement of nodes. They also considered random homogeneous networks and showed that a throughput of $O(B/\sqrt{n \log n})$ is feasible. Grossglauser and Tse [3] demonstrate how mobility can be used to increase the throughput. Gastpar and Vetterli [4] addressed the problem of only one active source/destination pair, while all other nodes assist this transmission, for both PR and PH models. Moscibroda and Wattenhofer [5] studied the scheduling complexity of connectivity, i.e., the minimal amount of time required until a connected structure can be scheduled under the PH model, and presented an $O(\log^4 n)$ scheduling algorithm. See additional results in [6], [7].

Gaussian channel model: Much less research was done in the context of Gaussian channels. Franceschetti et al. [8] shows that a throughput of $\Omega(1/\sqrt{n})$ can be achieved for random networks and random unicast sessions. Zheng [9] studied the data dissemination capacity in power-constrained networks. The author showed that the total broadcast capacity is $\Theta(P/\log n)$, when each node transmits at a power P. Keshavarz-Haddad and Riedi [10] study static wireless networks with the goal of assessing the impact of topology and traffic pattern on capacity. For additional results see [11], [12].

The results above use channel access methods, such as the TDMA scheme to schedule node transmissions, which requires *synchronization* and does not allow simultaneous transmissions of *all* nodes simultaneously.

2) Energy efficient topology control: The first to initiate the study of topology control through varying the power assignment of wireless nodes were Chen and Huang [14]. They addressed the problem of **minimizing the total energy** of a strongly connected graph, and gave a 2-approximation algorithm based on finding the minimum spanning tree and showed that the problem is NP-hard. Kirousis et al. showed the problem is NP-hard for the 3-dimensional Euclidean space for any value of κ. The NP-hardness for the 2-dimensional Euclidean space for any value of κ was proved in [15]. An excellent survey may be found in [16]. **Maximizing the network lifetime** in the case of uniform battery charges is equivalent to minimizing the maximum power level assigned to any node. The first to study this problem were Ramanathan and Hain [17], who provided an optimal polynomial time algorithm for this problem under the strong connectivity property. A general approach, which leads to polynomial time algorithms was developed in [18]. In [19], a PTAS for the problem under various network tasks was developed by devising an LP formulation for the problem. For additional results, see [20], [21].

C. Our contribution

This paper studies the problem of topology control through power assignments in random wireless networks. We assume that n wireless nodes are randomly, uniformly and independently distributed in a unit square, and look for an energy efficient power assignment such that the communication graph is strongly connected, and the capacity of the minimum capacity link is maximized. We distinguish between two types of power assignments: homogeneous and heterogeneous. In particular, our contribution is as follows:

- We show that for any power assignment p', which is homogeneous, it holds:
$Cap(H_{p'}) = O\left(B \log\left(1 + \frac{1}{\log^2 n}\right)\right)$ and
$Cap(H_{p'}) = \Omega\left(B \log\left(1 + \frac{1}{n\phi(n)\log^2 n}\right)\right)$, where $\phi(n)$ is any function with $\lim_{n\to\infty} \phi(n) = \infty$.
- For the heterogeneous case we develop a power assignment algorithm, CBPA, which constructs a power assignment p'', so that $Cap(H_{p''}) = \Omega\left(B \log\left(1 + \frac{1}{\sqrt{n}\log^2 n}\right)\right)$, $c(p'') = O(\log n)OPT_c$, and $l(p'') = \Omega(1)OPT_l$, where OPT_c and OPT_l are the minimum possible cost and maximum possible lifetime, respectively, of a power assignment that induces a strongly connected communication graph.
- We argue that CBPA can be implemented distributively.
- Our simulations show that CBPA outperforms several power assignment algorithms which induce a strongly connected communication graph.

Due to the probabilistic nature of the problem, all our statements are with high probability, that is the probability of the statement converges to one as the number of network nodes, n, increases. To the best of our knowledge, these are the first provable bounds for capacity in wireless networks with unsynchronized nodes.

II. Preliminaries

In this section we present some definitions and statements that are used throughout the rest of the paper.

Let $G_V = (V, E_V)$ be a complete directed graph, where V is a set of the wireless nodes. We define the weight function, $w : E_V \to \mathbb{R}^+$, on the edge set E_V as $w(u,v) = d(u,v)^2$, where $d(u,v)$ is the Euclidean distance between u and v. Note that the weight of an edge (u,v) matches the amount of energy which is required to transmit from u to v. Let MST_V be the minimum weight spanning tree of the undirected version of G_V (which is obtained easily by omitting the edge directions). Let e^* be the maximum length edge in MST_V. For every subgraph H of G_V, let $E(H)$ be the edge set of H. The weight of H is given by $w(H) = \sum_{e\in E(H)} w(e)$. For any edge $e = (u,v)$, its length is denoted by $|e| = d(u,v)$.

Let p^* be the minimum cost power assignment so that H_{p^*} is strongly connected. Chen and Huang [14], and later Kirousis et al. [22] made the following statement, which already became a common folklore in the study of wireless networks.

Theorem II.1 ([14]). $c(p^*) \geq w(MST_V)$.

Let p^{**} be the maximum lifetime power assignment so that $H_{p^{**}}$ is strongly connected. In [23] the authors showed the following lemma for the case that all the initial battery charges are equal ($\forall u \in V, b(u) = 1$).

Theorem II.2 ([23]). $l(p^{**}) \leq \frac{1}{w(e^*)}$.

In this paper we consider a wireless ad-hoc network with nodes distributed uniformly and independently in a unit square. We make use of several relevant theoretical results, which apply to the random distribution. The following corollary was derived in [24].

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Corollary II.3 ([25])**.** $w(e^*) = \Theta\left(\frac{\log n}{n}\right)$.

Zhang and Hou in [26] derived a lower bound on the cost of a power assignment required to induce a k fault resistant strongly connected communication graph (in our case $k = 1$) under the assumption that the nodes form a homogeneous Poisson point process with density λ. They also mentioned that according to [27] it is well accepted that n nodes whose locations are independent random variables, each with a uniform distribution over the unit square, are essentially a Poisson process with $\lambda = n$, for large values of n. In the next theorem we bring the main result of [26] adapted to the case of $k = 1$.

Theorem II.4 ([26])**.** $c(p^*) = \Omega(1)$.

For any $u \in V$, let $d_1(u) = \min_{v \in V \setminus \{u\} d(u,v)}$ be the distance from u to its nearest neighbor. Berend et al. [28] bounded the distance to the k-th nearest neighbor from any node. The theorem below is adapted to the case of $k = 1$.

Theorem II.5 ([28])**.** *For every node $v \in V$,*

$$\sqrt{\frac{1}{2\pi(n-1)n\phi(n)}} \leq d_1(v) \leq 2\sqrt{\frac{2\log n}{\pi(n-1)}},$$

where $\phi(n)$ is any function with $\lim_{n\to\infty} \phi(n) = \infty$.

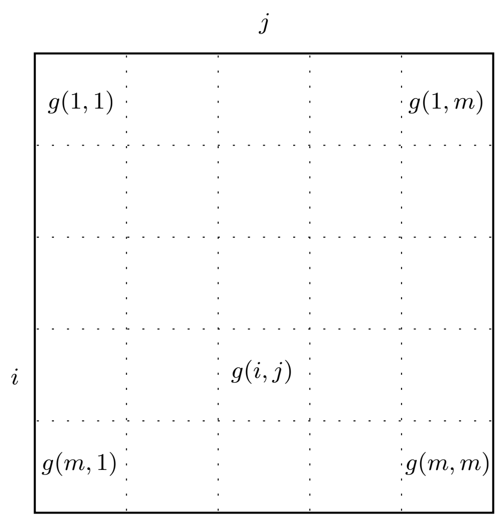


Fig. 1. The grid cells in a unit square ($m = \sqrt{\frac{n}{16\log n}}$)

We divide the unit square into $\frac{n}{16\log n}$ grid cells[2], each of size $\sqrt{\frac{16\log n}{n}} \times \sqrt{\frac{16\log n}{n}}$. Let $g(1,1)$ and $g\left(\sqrt{\frac{n}{16\log n}}, \sqrt{\frac{n}{16\log n}}\right)$ be the leftmost top and rightmost bottom cells, respectively. The rest of the cells are indexed as depicted in Fig. 1. Let $N(i,j)$ be the set of nodes in a grid cell $g(i,j)$, $1 \leq i,j \leq \sqrt{\frac{n}{16\log n}}$. The next lemma analyzes the number of nodes in each cell.

Lemma II.6. *There are $\Theta(\log n)$ nodes in each cell.*

[2]For convenience we omit the use of floors and ceilings, which does not effect our analysis.

Proof: The process of random placement of nodes uniformly and independently in a unit square can be viewed as Bernoulli trials with respect to grid cell $g(1,1)$. Let $X_1, X_2, \ldots, X_n$ be independent Bernoulli trials, where $X_i = 1$ if the i-th node is placed inside $g(1,1)$, and $X_i = 0$ otherwise. Clearly $\Pr[X_i = 1] = \frac{16\log n}{n}$ for every i, $1 \leq i \leq n$. Let $X = \sum_{i=1}^{n} X_i$. We make use of the well known Chernoff bounds to bound X. The expected value of X is given by $E[X] = 16\log n$. First we compute the lower bound of X. For any $\delta \in (0,1]$:

$$\Pr[X < (1-\delta)E[X]] < \exp(-E[X]\delta^2/2).$$

By setting $\delta = 1/2$ we obtain $\Pr[X < 8\log n] < 1/n^2$. The upper bound is computed in a similar manner. For any $\delta > 0$:

$$\Pr[X > (1+\delta)E[X]] < \left(\frac{e^\delta}{(1+\delta)^{\delta+1}}\right)^{E[X]}.$$

By setting $\delta = e - 1$ we obtain $\Pr[X > e \cdot E[X]] < \exp(-16\log n) = 1/n^{16}$. As the total number of cells is $\frac{n}{16\log n}$ we conclude that there are $\Theta(\log n)$ nodes in each cell. ∎

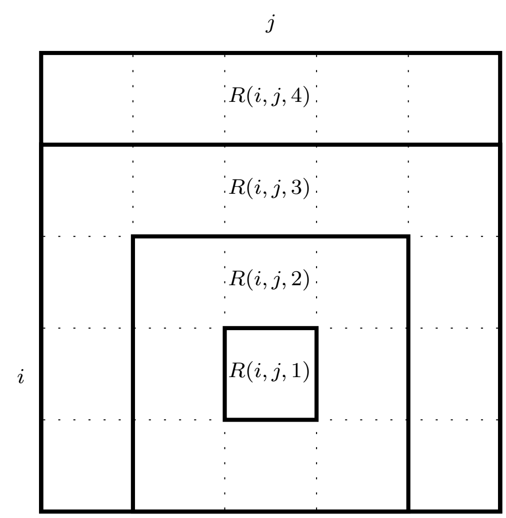


Fig. 2. The rings around grid cell $g(i,j)$

Later in this paper we use the notion of rings of cells, which are defined "around" a certain grid cell $g(i,j)$. The nodes which reside in the k-th ring, $k \geq 1$, around $g(i,j)$ are defined as

$$R(i,j,k) = \{v : v \in N(l,m), \max\{|i-l|, |j-m|\} = k-1\}.$$

It is possible that some rings have cells which "fall out" of the unit square. To simplify the notation we define $N(i,j) = \emptyset$ if $\min\{i,j\} \leq 0$ or $\max\{i,j\} > \sqrt{\frac{n}{16\log n}}$. An example of the rings is given in Fig. 2 (note that starting from the third ring some of the cells are "outside" the unit square).

III. Homogeneous power assignment

In this section we consider the case when all the nodes are assigned the same power level. First, we prove lower and upper bounds of the throughput in any strongly connected topology. Then, we shortly discuss how to compute an optimum routing which maximizes the throughput between any pair of nodes.

A. Throughput bounds

Suppose that the nodes are assignment a transmission power level of $p(u) = \gamma$, for every $u \in V$. Then,

$$SINR(u, v) = \frac{\gamma \cdot d(u, v)^{-2}}{N_0 + \gamma D(u, v)},$$

where $(u, v) \in E_p$ and $D(u, v) = \sum_{w \in V \setminus \{u,v\}} d(w, v)^{-2}$. As the noise N_0 is negligible, we have the following corollary.

Corollary III.1. *Given a homogeneous power assignment p, for every $(u, v) \in E_p$, $SINR(u, v) = \Theta(d(u, v)^{-2}/D(u, v))$.*

In our analysis we focus on the signal to interference plus noise ratio (SINR) in the worst possible case. The following two lemmas derive bounds for $D(u, v)$.

Lemma III.2. *For any $u, v \in V$, $D(u, v) = \Omega(n \log n)$.*

Proof: Denote $D(v) = \sum_{w \in V \setminus \{v\}} d(w, v)^{-2}$. We first show that $D(v) = \Omega(n \log n)$, and then conclude the same for $D(u, v)$. Recall the grid defined in Section II. Suppose that $v \in N(1, 1)$. We divide the nodes into $\sqrt{\frac{n}{16 \log n}}$, rings around $g(1, 1)$. For simplicity, let $R_k = R(1, 1, k)$, $k \in \{1, \ldots, \sqrt{\frac{n}{16 \log n}}\}$. Clearly, for any $w \in R_k$, $d(w, v) \leq k\sqrt{32 \log n/n}$. Note that in the k-th ring, R_k, there are $2k - 1$ grid cells. Combining with Lemma II.6, we can conclude that the number of nodes in R_k is at least $\Omega(k \log n)$. In fact, from the proof of Lemma II.6 it follows $|R_k| \geq 8k \log n$. Therefore,

$$\begin{aligned} D(v) &= \sum_{k=1}^{\sqrt{\frac{n}{16 \log n}}} \sum_{w \in R_k} d(w, v)^{-2} \\ &\geq \sum_{k=1}^{\sqrt{\frac{n}{16 \log n}}} \frac{|R_k| n}{32k^2 \log n} \geq n \sum_{k=1}^{\sqrt{\frac{n}{16 \log n}}} \frac{1}{4k} \\ &= \tfrac{1}{4} n H_{\sqrt{\frac{n}{16 \log n}}} = \Omega(n \log n), \end{aligned}$$

where H_m is the m-th harmonic number. It is easy to verify that the lower bound on $D(v)$ computed for $v \in N(1, 1)$ is also a lower bound for any node in any grid cell.

In our analysis we assumed that every node $w \in R_k$ is at a maximum possible distance from v. As a result, for any $u, v \in V$, $D(u, v) \geq \Omega(n \log n) - \frac{n}{32 \log n} = \Omega(n \log n)$. ∎

Lemma III.3. *For any $u, v \in V$, $D(u, v) = O(n^2 \phi(n) \log n)$, where $\phi(n)$ is any function with $\lim_{n\to\infty} \phi(n) = \infty$.*

Proof: The proof resembles the one of Lemma III.2. Again, let $D(v) = \sum_{w \in V \setminus \{v\}} d(w, v)^{-2}$. Note that for any pair of nodes, $u, v \in V$, $D(u, v) \leq D(v)$. This time we compute the upper bound for a node in the center grid cell, that is $v \in N\left(\sqrt{\frac{n}{64 \log n}}, \sqrt{\frac{n}{64 \log n}}\right)$. We divide the nodes into $\sqrt{\frac{n}{64 \log n}}$ rings around $g\left(\sqrt{\frac{n}{64 \log n}}, \sqrt{\frac{n}{64 \log n}}\right)$. For simplicity, let $R_k = R\left(\sqrt{\frac{n}{64 \log n}}, \sqrt{\frac{n}{64 \log n}}, k\right)$, $k \in \{1, \ldots, \sqrt{\frac{n}{64 \log n}}\}$.

Due to Theorem II.5, for any $w \in V$, $d(w, v) \geq \frac{1}{n\phi(n)}$, where $\phi(n)$ is any function with $\lim_{n\to\infty} \phi(n) = \infty$. In particular this holds for nodes in $R_1 \cup R_2$. For any $w \in R_k$, $k > 2$ it holds $d(w, v) \geq (k - 2)\sqrt{\frac{16 \log n}{n}}$. Note that there are 9 grid cells covered by R_1 and R_2, and at most $8(k - 1)$ grid cells in R_k, $k > 2$. Combining with Lemma II.6, there are $\Theta(k \log n)$ nodes in R_k. As a result,

$$\begin{aligned} D(v) &= \sum_{k=1}^{\sqrt{\frac{n}{64 \log n}}} \sum_{w \in R_k} d(w, v)^{-2} \\ &= \sum_{w \in R_1 \cup R_2} d(w, v)^{-2} + \sum_{k=3}^{\sqrt{\frac{n}{64 \log n}}} \sum_{w \in R_k} d(w, v)^{-2} \\ &\leq O(n^2 \phi(n) \log n) + O(n \log n). \end{aligned}$$

The bound for $\sum_{k=3}^{\sqrt{\frac{n}{64 \log n}}} \sum_{w \in R_k} d(w, v)^{-2}$ is obtained similarly to the proof of Lemma III.2. ∎

We are ready to present the main Theorem of this section.

Theorem III.4. *Let p be a power assignment so that H_p is strongly connected. Then, $Cap(H_p) = O\left(B \log\left(1 + \frac{1}{\log^2 n}\right)\right)$ and $Cap(H_p) = \Omega\left(B \log\left(1 + \frac{1}{n\phi(n) \log^2 n}\right)\right)$, where $\phi(n)$ is any function with $\lim_{n\to\infty} \phi(n) = \infty$.*

Proof: We prove the upper bound first. For that we show that there exist two nodes $x, y \in V$ so that $Cap(H_p, x, y) = O\left(B \log\left(1 + \frac{1}{\log^2 n}\right)\right)$. Let e^* be the maximum length edge of MST_V. Let H'_p be a graph obtained by removing all edges with length equal or greater than $|e^*|$ from H_p. Due to the fact that the longest edge of any spanning tree of G_V has length of at least $|e^*|$, H'_p has at least 2 strongly connected components. Let $x, y \in V$ be two nodes from different components. Let (u, v) be one of the edges removed from H_p with the maximum $SINR(u, v)$. According to Corollary II.3, $d(u, v) = \Omega\left(\sqrt{\frac{\log n}{n}}\right)$. From Corollary III.1 and Lemma III.2, $SINR(u, v) = O(1/\log^2 n)$. As every path from x to y in H_p uses one of the edges which were removed we obtain: $Cap(H_p, x, y) \leq B \log(1 + SINR(u, v)) = O\left(B \log\left(1 + \frac{1}{\log^2 n}\right)\right)$.

Next, we prove the lower bound. From Corollary II.3 it follows that if H_p is strongly connected, then $p(u) = \Omega(\log n/n)$, $\forall u \in V$, and therefore the directed version of MST_V (obtained by replacing each edge with two edges in opposite directions) is a subgraph in H_p (note that all links in H_p are bidirectional as all the nodes are assigned the same power level). As a result for any pair of nodes $x, y \in V$ there is a path in H_p that uses edges with length at most $O(\sqrt{\log n/n})$. Denote such a path by $P_{x,y}$. Combining with Lemma III.3, for any edge $(u, v) \in P_{x,y}$ it holds $SINR(u, v) = \Omega\left(\frac{n/\log n}{n^2 \phi(n) \log n}\right)$, where $\phi(n)$ is any function with $\lim_{n\to\infty}\phi(n)=\infty$. As a result,

$$Cap(H_p,x,y)=\Omega\left(B\log\left(1+\frac{1}{n\phi(n)\log^2 n}\right)\right),$$

for any $x,y\in V$. ∎

Remark: Theorem III.4 analyzes the capacity of the worst link in the network. However, some nodes may have a routing with links of substantially higher capacity. It is possible to find such a routing for every source node $u\in V$, by constructing a maximum bottleneck spanning tree of H_p rooted at u. A maximum bottleneck spanning tree is a spanning tree which maximizes the minimum cost edge. In our case the cost of an edge $(u,v)\in E_p$ can be defined as $SINR(u,v)$. It is possible to compute the above tree efficiently even with an additional requirement of a hop-count [29].

IV. Heterogeneous power assignment

In this section we allow nodes to have distinct power levels. We develop a power assignment algorithm that produces p such that $c(p)=O(\log n)OPT_c$, $l(p)=\Omega(1)OPT_l$, and $Cap(H_p)=\Omega\left(B\log\left(1+\frac{1}{\sqrt{n}\log^2 n}\right)\right)$, where OPT_c and OPT_l are the minimum possible cost and maximum possible lifetime, respectively, of a power assignment that induces a strongly connected communication graph. Then we show that this algorithm can be implemented in a distributed way.

A. The power assignment algorithm

From the analysis in the previous section we can intuitively see that the closest nodes have the most impact on interference levels. The main idea of the algorithm is to create clusters of nodes with low transmission power, and consequently low interference; then connect the clusters, allowing only one node to transmit at high power level. By clustering we wish to avoid multiple nodes, within a small distance from each other, transmitting at high power levels.

The algorithm Cluster-Based-Power-Assignment (CBPA) works as follows. The algorithm has two phases: in the first phase of the algorithm (lines 4-17) the clusters are created; in the second phase (line 18-32) the nodes are assigned a transmission power. We now describe each phase in detail.

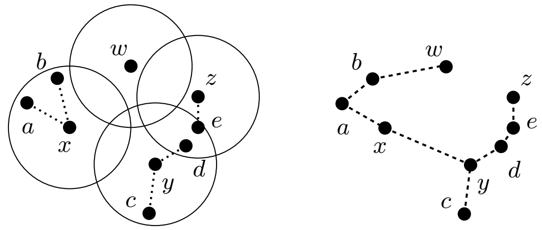
Phase I – the clusters are created iteratively. Each node should belong to some cluster and can either be a member of some cluster or its head. We use a temporary set U, which at first is initialized to be all the nodes in V. Then, while U is not empty, we pick an arbitrary node $u\in U$ (line 5). This is our new cluster head. The cluster is formed (line 6-8) from all the nodes, N, which are within a distance of $\frac{1}{\sqrt{n\sqrt{n}}}$ from u, and u itself. The nodes in M are defined as the cluster members of u (line 7). Then we check for every node $v\in M$ if it is already defined as a member of another cluster. If not, it stores u as its cluster head (line 17). Otherwise, v chooses to be the member of the cluster with the closest cluster head. Note that v cannot be a cluster head, as then u could not have been chosen from U, as the cluster head and its members are removed from U (line 8). This process continues until U becomes empty.

Cluster-Based-Power-Assignment (CBPA)

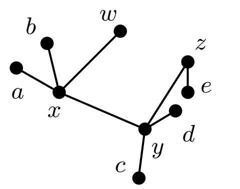
```
1  U ← V
2  foreach u ∈ V do
3      type[u] ← nil; head[u] ← nil; members[u] ← nil
4  while U ≠ ∅ do
5      take any u ∈ U
6      N ← { v ∈ V : d(u,v) ≤ 1/√(n√n) }
7      type[u] ← HEAD; head[u] ← u; members[u] ← M
8      remove u and M from U
9      foreach v ∈ M do
10         if type[v] == MEMBER then
11             if d(v, head[v]) < d(u,v) then
12                 remove v from members[u]
13             else
14                 head[v] ← u
15                 remove v from members[head[v]]
16         else
17             head[v] ← u
18 foreach u ∈ V do
19     compute MST_V = (V,E)
20     if type[u] == MEMBER then
21         p(u) ← d(u, head[u])^2
22     else
23         p(u) ← 0
24         foreach v ∈ members[u] do
25             if d(u,v)^2 > p(u) then
26                 p(u) ← d(u,v)^2
27             foreach (v,w) ∈ E do
28                 if d(u, head[w])^2 > p(u) then
29                     p(u) ← d(u, head[w])^2
30         foreach (u,x) ∈ E do
31             if d(u, head[x])^2 > p(u) then
32                 p(u) ← d(u, head[x])^2
```

Phase II – the power assignment phase is divided into cases. Each node that is a cluster member is assigned with a transmission power which is just enough to reach its cluster head (lines 20-21). Determining the power assignment for the cluster head is more complicated. A cluster head has to satisfy three requirements: (a) reach all its cluster members (lines 24-26); (b) reach all cluster heads of its neighbors in MST_V (line 30-32); (c) reach all cluster heads of the nodes which are neighbors in MST_V of its cluster members (lines 27-29). It is easy to verify that all the edges which result from these requirements are bidirectional.

An example of 4 clusters with cluster heads w, x,y, and z is given in Fig. 3. The disks outline the cluster range of $1/\sqrt{n\sqrt{n}}$, while the dotted lines identify the members of each cluster (Fig. 3(a)). Note that d and e are both within the cluster range from two cluster heads, y and z; their membership is determined by the closest cluster head. The second phase of the algorithm induces a topology based on clusters and the minimum weight spanning tree (Fig. 3(b)). The induced topology (Fig. 3(c)) has to fulfill several requirements as mentioned above: (a) cluster head connects to its members and the members connect to their cluster head (the edges $(x,a),(x,b),(y,c),(y,d),(z,e)$); (b) cluster head reaches the cluster heads of its neighbors in the minimum spanning tree (the edges $(x,w),(x,y)$); (c) cluster head reaches the cluster heads of its cluster members' neighbors in the minimum spanning tree (the edge (y,z)).



(a) Clusters with w, x,y, and z as cluster heads (b) Minimum weight spanning tree



(c) The induced topology

Fig. 3. Exposition of algorithm CBPA

B. Analysis

First we argue that H_p is strongly connected. Intuitively, we keep the connectivity of MST_V as cluster heads are able to use the edges of MST_V and cluster members do the same with the help of cluster heads. The next lemma proves that H_p is strongly connected.

Lemma IV.1. *H_p is strongly connected.*

Proof: For any pair of nodes $u, v \in V$ we show there is a path in H_p from u to v. Let $P = \langle u = z_1, z_2, \ldots, z_k = v\rangle$ be a path from u to v in MST_V. We construct a path in H_p by converting each edge $e \in P$ to a path in H_p. For each edge (z_i, z_{i+1}), $1 \le i \le k-1$ there are four cases to consider:
Case 1: z_i and z_{i+1} are both cluster heads. Therefore it follows easily that $\langle z_i, z_{i+1}\rangle \in H_p$.
Case 2: z_i is a cluster head and z_{i+1} is a cluster member. If they both belong to the same cluster, then clearly $\langle z_i, z_{i+1}\rangle \in H_p$. Otherwise, $\langle z_i, head[z_{i+1}], z_{i+1}\rangle \in H_p$, as z_i reaches the cluster head of z_{i+1}, which in turn reaches z_{i+1}.
Case 3: z_i is a cluster member and z_{i+1} is a cluster head. If they both belong to the same cluster, then clearly $\langle z_i, z_{i+1}\rangle \in H_p$. Otherwise, z_i reaches its cluster head, $head[z_i]$, which in turn reaches z_{i+1}, and therefore $\langle z_i, head[z_i], z_{i+1}\rangle \in H_p$.
Case 4: z_i and z_{i+1} are both cluster members. If they share the same cluster head, then $\langle z_i, head[z_i], z_{i+1}\rangle \in H_p$. Otherwise, z_i, reaches its cluster head, which in turn reaches the cluster head of z_{i+1}, that eventually reaches z_{i+1}. We conclude, $\langle z_i, head[z_i], head[z_{i+1}], z_{i+1}\rangle \in H_p$.

We showed how each edge in P can be converted into a path in H_p. Hence, there exists a path between any pair of nodes in H_p. ■

Following the path conversion described in Lemma IV.1 we can see that for any pair of nodes $u, v \in V$, there exists a path in H_p with two types of edges only: (a) between cluster heads; (b) between cluster members and their respective cluster heads. This gives us the next important observation.

Observation IV.2. *For any pair of nodes $u, v \in V$, there exists a path P in H_p such that for any $(x, y) \in P$ one of the following conditions holds:*

1) *Either $head[x] = y$ or $head[y] = x$.*
2) *x and y are both cluster heads.*

The next lemma analyzes the cost of the power assignment and the network lifetime.

Lemma IV.3. *Let OPT_c and OPT_l be the minimum possible cost and maximum possible lifetime, respectively, of a power assignment that induces a strongly connected communication graph. Then, $c(p) = O(\log n)OPT_c$, $l(p) = \Omega(1)OPT_l$.*

Proof: The proof is based on the fact that for every $u \in V$, $p(u) = O(\frac{\log n}{n})$. To show that we take a closer look at CBPA. The power is assigned in four different places:
Lines 21,26: from the construction of M it follows that $d(u, v) \le 1/\sqrt{n\sqrt{n}}$.
Line 29 – from Corollary II.3, $d(v, w) = O\left(\sqrt{\frac{\log n}{n}}\right)$. Note that $d(u, v), d(w, head[w]) \le 1/\sqrt{n\sqrt{n}}$. As a result,

$$d(u, head[w]) \le d(u, v) + d(v, w) + d(w, head[w]) = O\left(\sqrt{\frac{\log n}{n}}\right).$$

Line 32: following a similar reasoning,

$$d(u, head[x]) \le d(u, x) + d(x, head[x]) = O\left(\sqrt{\frac{\log n}{n}}\right).$$

Combining the above with Theorem II.4 we easily derive

$$c(p) = n \cdot O\left(\frac{\log n}{n}\right) = O(\log n) = O(\log n)OPT_c.$$

The network lifetime is given by

$$l(p) = \min_{u \in V} \frac{1}{p(u)} = \Omega\left(\frac{n}{\log n}\right).$$

From Lemma II.2 and Corollary II.3 it easily follows that $l(p) = \Omega(1)OPT_l$. ■

We now prove the main theorem of this section.

Theorem IV.4. $Cap(H_p) = \Omega\left(B \log\left(1 + \frac{1}{\sqrt{n}\log^2 n}\right)\right)$.

Proof: According to Observation IV.2, for each pair of nodes $u, v \in V$, there exists a path P in H_p with only two types of edges. We show that for each $(x, y) \in P$,

$$SINR(x, y) = \frac{p(x, y) \cdot d(x, y)^{-2}}{N_0 + I(x, y)} = \Omega\left(\frac{1}{\sqrt{n}\log^2 n}\right).$$

By definition, for each edge $(x, y) \in P$, $p(x) \ge d(x, y)^2$. As N_0 is negligible, the analysis focuses on the expression $I(x, y)$.

Recall the grid constructed in Section II, and suppose $y \in N(i, j)$. Let $R = R(i, j, 1) \cup R(i, j, 2)$. We stated in the proof of Lemma III.3 that $|R| = O(\log n)$. We distinguish between two cases:
Case 1: y is a cluster member. Then by Observation IV.2 $x =$

$head[y]$. Let M be all the cluster members in R, and let H be all the cluster heads in R. Note that $y \in R$ by definition, whereas $x \in R$ as $d(x,y) \le 1/\sqrt{n\sqrt{n}}$. Hence, the interference sensed over the link (x,y) can be expressed as:

$$I(x,y) = \sum_{z\in M\setminus\{y\}} \frac{p(z)}{d(z,y)^2} + \sum_{z\in H\setminus\{x\}} \frac{p(z)}{d(z,y)^2} + \sum_{z\in V\setminus R} \frac{p(z)}{d(z,y)^2}.$$

From Theorem II.5, the distance between any pair of nodes is at least $\Omega\left(\frac{1}{n\phi(n)}\right)$, where $\phi(n)$ is any function with $\lim_{n\to\infty}\phi(n) = \infty$. From the algorithm CBPA it follows that if z is a cluster member, then $p(z) \le \frac{1}{n\sqrt{n}}$. Therefore,

$$\sum_{z\in M\setminus\{y\}} \frac{p(z)}{d(z,y)^2} \le O\left(|M| \cdot \frac{n^2\phi^2(n)}{n\sqrt{n}}\right) = O(\sqrt{n}\phi(n)\log n).$$

The distance between y and any cluster head, which is not x, is at least $\frac{1}{2\sqrt{n\sqrt{n}}}$, as each node belongs to a cluster of the closest head (lines 10-15). In the proof of Lemma IV.3 we showed that $p(z) = O(\log n/n)$ for any $z \in V$. As a result,

$$\sum_{z\in H\setminus\{x\}} \frac{p(z)}{d(z,y)^2} \le O\left(|H| \cdot \frac{\log n \cdot n\sqrt{n}}{n}\right) = O(\sqrt{n}\log^2 n).$$

An upper bound of $\log^2 n$ for the third addend is achieved very similarly to the proof of Lemma III.3. Again we use the fact that all the nodes are assigned a transmission power which is $O(\log n/n)$. Let $R_k = R(i,j,k)$, $k \in \left\{3, ..., \sqrt{\frac{n}{16\log n}}\right\}$. Eventually,

$$\begin{aligned}\sum_{z\in V\setminus R} \frac{p(z)}{d(z,y)^2} &= \sum_{k=3}^{\sqrt{\frac{n}{16\log n}}} \sum_{z\in R_k} \frac{p(z)}{d(z,y)^2} \\ &= O\left(\sum_{k=3}^{\sqrt{\frac{n}{16\log n}}} |R_k| \cdot \frac{\log n}{n} \cdot \frac{n}{k^2\log n}\right) \\ &= O\left(\sum_{k=3}^{\sqrt{\frac{n}{16\log n}}} \frac{\log n}{k}\right) = O(\log^2 n)\end{aligned}$$

We conclude that $I(x,y) = O(\sqrt{n}\log^2 n)$.

Case 2: y is a cluster head. As in the first case, let M and H be cluster members and cluster heads in R. Then,

$$I(x,y) \le \sum_{z\in M} \frac{p(z)}{d(z,y)^2} + \sum_{z\in H} \frac{p(z)}{d(z,y)^2} + \sum_{z\in V\setminus R} \frac{p(z)}{d(z,y)^2}.$$

Similarly to the first case, we can upper bound the first addend by using Theorem II.5 and the fact that each cluster member is assigned a power of at most $\frac{1}{n\sqrt{n}}$.

$$\sum_{z\in M} \frac{p(z)}{d(z,y)^2} = O\left(|M| \cdot \frac{n^2\phi^(n)}{n\sqrt{n}}\right) = O(\sqrt{n}\phi^2(n)\log n).$$

From the construction it follows that all the cluster heads are separated by a distance of at least $1/\sqrt{n\sqrt{n}}$. Combining with the bound of $O(\log n/n)$ on the transmission power we obtain:

$$\sum_{z\in H} \frac{p(z)}{d(z,y)^2} = O\left(|H| \cdot \frac{n\sqrt{n}\log n}{n}\right) = O(\sqrt{n}\log^2 n).$$

Finally, the third addend is upper bounded exactly as in the first case. We conclude $I(x,y) = O(\sqrt{n}\log^2 n)$ in the second case as well.

Note that both cases cover the conditions of Observation IV.2, which rests our proof. ■

C. Distributed implementation

Sometimes, especially in wireless deployments, it is impossible to have a central entity that coordinates the network. In this section we present a distributed implementation of the CBPA algorithm, which only requires each node to have a unique ID and the knowledge of total number of nodes, n.

The distributed algorithm is composed of 3 phases: (a) MST_V computation; (b) clustering; (c) power assignment. Due to space constraints we briefly outline each of the phases.

Phase (a): First, each node $u \in V$ transmits its ID using power level of $p'(u) = \frac{8\log n}{\pi(n-1)}$ (recall that the total number of nodes, n, is known to all). After some timeout, u receives all the transmissions of close nodes and obtains a list of all nodes $S(u)$ which are within a distance of $2\sqrt{\frac{2\log n}{\pi(n-1)}}$ from it. Using the standard methods described in [30] it is possible to compute all distances $d(u,v)$, $v \in S(u)$ (we skip the discussion about correct reception of all these transmissions). Due to the upper bound of Theorem II.5, MST_V is a subgraph of H_p. Recall that a weight of an edge $(u,v) \in E_p$ is $d(u,v)^2$, and therefore can be computed. As all the links in H_p are bidirectional, we can now use the Distributed Algorithm for Minimum-Weight Spanning Trees [31] to compute the minimum weight spanning tree of H_p, which is exactly MST_V.

Phase (b): Before the clustering we need each node to know the IDs of other nodes in the network. For that we could use one of the data dissemination protocols, e.g. [32]. We describe the first step of the clustering procedure, which is basically a distributed implementation of the first phase of CBPA (lines 4-32). Node u with the smallest ID, becomes the first cluster head and selects the candidate member nodes $M(u) = \{v \in S(u) : d(u,v) \le 1/\sqrt{n\sqrt{n}}\}$. Then, u floods the network (using the edges of MST_V) with the information about $M(u)$. All nodes remove u and $M(u)$ from their list of nodes (line 8) and the next node with the smallest ID executes the same routine. Note that in subsequent executions, any candidate member v in $M(u)$ can either accept or decline u as a cluster head (lines 10-17). An appropriate message should always be sent to u or to current $head[v]$ to notify the change. This does not effect further execution of the algorithm.

Phase (c): The final phase is straightforward. All cluster members choose a power level required to reach their cluster heads (line 21), which can be decided based on the distances obtained in Phase (a). The cluster heads can simply be assigned a power level of $\frac{8\log n}{\pi(n-1)} + \frac{2}{n\sqrt{n}}$. Which complies with all the requirements (lines 24-32). It is easy to verify that this does not effect the asymptotic performance of the algorithm.

V. Simulation results

To the best of our knowledge we are the first to study the throughput when all the nodes transmit simultaneously. As a result, in our simulations we compare the performance of the CBPA algorithm to two naive methods: minimum weight spanning tree based power assignment (MBPA) (introduced in [14]) and homogeneous power assignment (HPA). In MBPA, each node is assigned to reach its farthest neighbor in MST_V, thus forming a strongly connected graph. We choose this power assignment due to its energy efficiency, as its cost is at most twice the cost of an optimal one [14]. In HPA, each node $u \in V$ is assigned a power of $p(u) = |e^*|^2$, where e^* is the maximum length edge in MST_V, which results in a strongly connected communication graph.

Fig. 4 shows the comparison between the power assignments. The simulations have been carried out for values of n ranging from 50 to 1050 with steps of 50. Each point in the plot is an average of 10 tries. The throughput is measured in bps, with channel bandwidth of $B = 1$. We can see that both MBPA and HPA achieve almost the same throughput, while CBPA has a significantly better one. More specifically, the ratio between CBPA and the other two methods increases with n, from ≈ 2.8 for $n = 50$, up to ≈ 12 for $n = 1050$ (the plot of the ratio is omitted due to lack of space).

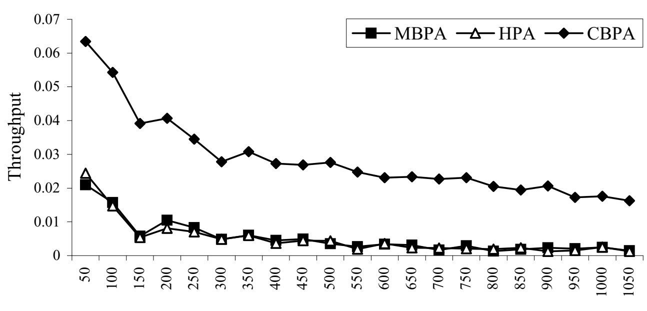


Fig. 4. Comparison between MBPA, HPA and CBPA

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