A Recursive Partitioning Algorithm for Space Information Flow

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Outline

1 Motivation

2 Formulation

3 New Algorithm of SIF & Simulations
   - Algorithm
   - Simulations and Discussions

4 conclusion
Motivation: Space Information Flow (SIF)

What is SIF?
Information Flow: Network Coding
Space: geometric Space, e.g. Euclidean space
SIF: network coding in space

What is new? (2011)
SIF (2011) allows introducing additional relay nodes to reduce cost

why study SIF?
Network Coding in space is strictly better than routing in space
e.g. Pentagram example
Consider Min-cost Multicast in 2-D Euclidean Space
Objective: min total length under requirement of same throughput

Terminal node

Radius = 1

Figure: 6 terminal nodes: F → {A,B,C,D,E}
Why SIF (Network Coding in Space)? Pentagram example

Using Routing in Space

Min Spanning Tree (MST)

Euclidean Steiner Min Tree (ESMT)

Cost = 5

Cost = 4.64

Figure: ESMT is optimal routing in space, using 3 Steiner nodes
Why SIF (Network Coding in Space)? Pentagram example

Using Network Coding in Space (Space Information Flow)

Figure: Pentagram: using 5 relay nodes

Cost = 9.14/2 = 4.57/bit
Why SIF? *Network Coding in space* is strictly better than *optimal routing in space*

**Single Multicast Example: Pentagram**

![Diagram of a pentagram network with nodes A, B, C, D, E, F and links between them.](image)

- **(a)** 6 terminal nodes in space
- **(b)** optimal ESMT = 4.64
- **(c)** SIF = 9.14/2 = 4.57

**Figure:** (a) 6 terminal nodes in space (b) optimal ESMT (c) SIF

Cost advantage = \( \frac{\text{Cost of routing}}{\text{cost of network coding}} = \frac{4.64}{4.57} \approx 1.015 > 1 \)
SIF vs. NIF

<table>
<thead>
<tr>
<th>Butterfly</th>
<th>NC &gt; routing</th>
<th>in Graph</th>
<th>Network Information Flow</th>
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</thead>
<tbody>
<tr>
<td>Pentagram</td>
<td>NC &gt; routing</td>
<td>in Space</td>
<td>Space Information Flow</td>
</tr>
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Figure: Space Information Flow (SIF) [1][2][3][4][5]

Butterfly network routing in Graph (NIF) since 2000
Pentagram network routing in Space (SIF) since 2011

Coding Advantage: ↑
Complexity: ↓
Cost Advantage: ↑
Complexity: ?

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Related Work

Network Coding in Space (SIF)

- Algorithm (Objective): uniform partitioning heuristic [5], but has drawback (next page ...)
- Properties of SIF: Convex Hull, Convexity, Wedge [6], ...

Routing in Space: ESMT (Euclidean Steiner Minimal Tree)

- Algorithms
  1. Exact algorithms [7]
  2. Approximation algorithms (e.g. PTAS [8], partitioning)
  3. Heuristic algorithms [9]
- Properties of ESMT [10]: FST, Wedge, Diamond, ...
- Complexity: ESMT is NP-Hard [9]
Drawback of Previous SIF algorithm that uses *uniform* partitioning

- Terminals have non-uniform density distribution, algorithm convergence slows down
- e.g. clustered terminals

**Figure:** Issue of uniform partitioning
Outline

1. Motivation

2. Formulation

3. New Algorithm of SIF & Simulations
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   - Simulations and Discussions

4. Conclusion
Formulation

Given *terminal* nodes in space + extra *relay* nodes

Single Multicast

Minimize cost = \( \sum_e w(e) f(e) \)
- \( w(e) \): Euclidean distance, i.e. \(|e|\)
- \( f(e) \): flow rate

SIF includes two aspects:
- **Topology**: connection + *flow rate* on each link \( f(uv) \)
- **Positions**: positions of relay nodes
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New Heuristic Algorithm

SIF algorithm includes two phases:

- **Phase I: Approach optimal SIF topology**
  - Non-uniform partitioning
  - Linear Programming

- **Phase II: Approach optimal SIF positions of relays**
  - Analytic geometry method
  - Equilibrium method
New Heuristic Algorithm: Phase I

Figure: Non-uniform Partitioning: Through every terminal node, a vertical line and a horizontal line are drawn to obtain a bounding box and a number of sub-rectangles; every sub-rectangle is partitioned into $q \times q$ (e.g. $q=2$) cells. The centers of the cells inside the convex hull (in red) determined by given terminal nodes are taken as the candidate relay nodes.
New Heuristic Algorithm: Phase I

**Phase I: Non-uniform Partitioning + LP**

- Non-uniform Partitioning
- sub-rectangle \( q \times q, q=2 \)
- Construct complete graph
- Apply LP

**Phase I: Procedures**

- **Minimize**
  
  \[ \text{cost}_q = \sum_{uv \in A} w(\overrightarrow{uv})f(\overrightarrow{uv}) \]

- **Subject to**:

  \[
  \begin{align*}
  \sum_{v \in V_{\uparrow}(u)} f_i(\overrightarrow{vu}) &= \sum_{v \in V_{\downarrow}(u)} f_i(\overrightarrow{uv}) \\
  f_i(\overrightarrow{T_jS}) &= r \\
  f_i(\overrightarrow{uv}) &\leq f(\overrightarrow{uv}) \\
  f(\overrightarrow{uv}) &\geq 0, f_i(\overrightarrow{uv}) \geq 0
  \end{align*}
  \]

**Figure:** Clustering network \( N=9 \)
New Heuristic Algorithm: Phase II

**Phase II: Analytic geometry**

![Diagram](image)

**Phase II: Procedures**

- Equilibrium method, if not
- Solve equations using analytic geometry, if $120^\circ$

\[
\begin{align*}
\frac{(x_1 - x)(x_2 - x) + (y_1 - y)(y_2 - y)}{\sqrt{(x_1 - x)^2 + (y_1 - y)^2} \sqrt{(x_2 - x)^2 + (y_2 - y)^2}} &= \cos 120^\circ \\
\frac{(x_1 - x)(x' - x) + (y_1 - y)(y' - y)}{\sqrt{(x_1 - x)^2 + (y_1 - y)^2} \sqrt{(x' - x)^2 + (y' - y)^2}} &= \cos 120^\circ \\
(x_1 - x)/(y_1 - y) &= (x_3 - x')/(y_3 - y') \\
(x_2 - x)/(y_2 - y) &= (x_4 - x')/(y_4 - y')
\end{align*}
\]

(1)

**Figure:** Compute balanced positions
New Heuristic Algorithm: Phase II

Figure: 1st round of Phase I: Compute optimal positions for relay using analytic geometry
New Heuristic Algorithm: Phase I (2\textsuperscript{nd} round)

- q = q + 1 = 3; Apply retention mechanism
- Construct complete graph; Apply LP

Figure: 2\textsuperscript{nd} round of Phase I
New Heuristic Algorithm: Clustering Network N=9

Figure: Uniform partitioning

Figure: Non-uniform partitioning

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New Heuristic Algorithm: Pentagram Network $N=5+1$

**Figure:** 1$^{st}$ round ($q=2$) and 2$^{nd}$ round ($q=3$)
New Heuristic Algorithm: Pentagram Network $N=5+1$

**Pentagram N=5+1**

- SIF Cost from LP (Phase I)
- SIF Cost from LP (Phase II)
- ESMT

**Figure:** Uniform partitioning

**Pentagram N=5+1**

- SIF Cost from LP (Phase I)
- SIF Cost from LP (Phase II)
- ESMT

**Figure:** Non-uniform partitioning

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New Heuristic Algorithm for Ladder Network $N=10$

Figure: Ladder network when $N=10$. (a) optimal ESMT by GeoSteiner (b) SIF result in the third round ($q=4$).
New Heuristics Algorithm for Random Network N=7

**Figure:** Random network when N=7. (a) optimal ESMT by GeoSteiner (b) SIF result in the third round (q=4).
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Contributions: Propose SIF algorithm based on non-uniform partitioning that can deal with any density distribution of terminal nodes

- Phase I: Non-uniform Partitioning + LP; Retention mechanism;
- Phase II: Analytic geometry to compute positions
Ongoing Work

- SIF can decrease complexity? (polynomial algorithm?)
- SIF algorithm in 3-D?
- SIF properties?
Welcome to our lab, HUST

Figure: http://itec.hust.edu.cn

Thank you! and Q & A
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