Optimal Layered Multicast
Mathematical Model and Empirical Studies

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Motivation

A Paradigm Shift
Why multicast?
Why network coded multicast?
Why layered coding?
Partial Layer Reception
Non-sequential Layer Reception

Our Contributions
Model for Layered Multicast

Empirical Results

Conclusion/Future Work
High-speed technologies - paradigm shift
- Increased user expectations
- Simple text pages to content rich audio/video streaming applications

Proliferation of real-time multimedia applications
- video-on-demand, distance education, live video/audio streams, video conferencing, etc ...

Most streaming applications require 1-to-many communication

Multicast!
Why multicast?

- Mechanism of choice for one-to-many data dissemination
- Advantages
  - Exploits information flow properties
  - Reduces redundancy
  - Better bandwidth utilization
  - Minimizes routing costs
Why network coded multicast?

- Traditional Multicast - Packing Steiner Trees
- Problem - Finding Steiner trees is NP-Hard
- Fundamental result of network coding:

A multicast rate of $d$ to all receivers is feasible if and only if $d$ is a feasible unicast rate to each receiver

- Corollary - Efficient multicast viewed as union of conceptual unicast flows
  - Linear program formulation
  - Polynomial-time solvable
Why layered coding?

- Internet is naturally heterogeneous
- Single rate multicast disadvantage:
  - Starve low capacity users
  - Under-utilized bandwidth for high-capacity users

- Solution? **Layered Coding**
  - Data stream encoded into layers
  - More layers - higher quality

- Non-cumulative coding
  - Independent layers
  - E.g. Multiple Description Coding

- Cumulative coding
  - Layers 1 through \( k - 1 \) needed to play \( k \)
  - E.g. MPEG-2, H.263
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Partial Layer Reception

Layer Sizes
2 2 2

Layers Received
T1: 1
T2: 1, 2, 3
Non-sequential Layer Reception

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Layer Sizes

Received Layers
T1: 1
T2: 1
T3: 1
Non-sequential Layer Reception

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Layer Sizes

Received Layers
T1: 1
T2: 1,2
T3: 1,2
Non-sequential Layer Reception

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Layer Sizes

Received Layers
T1: 1
T2: 1, 2
T3: 1, 2, 3
Our Contributions

- Mathematical model of problem
- Network flow formulation for optimal multicast throughput / routing costs
- Optimal multicast obtained via separation of
  - Network layer throughput
  - Application layer throughput
Model for Layered Multicast
Network is directed, capacitated graph, $G = (V, E)$

Distinguished nodes, $S$ - sender, $T = \{T_1 \ldots T_{|T|}\}$ - set of receivers

Data to be streamed in layers $1 \ldots k$, layer sizes $l_k$
Network layer vs Application Layer

- Conceptual separation of data received at the:
  - **Network layer** - allow routing and reception of any fraction of layer
  - **Application layer** - insist on reception of layers that are
    (i) complete, and
    (ii) sequential

- Network Layer Variables
  \[ y^i_k \] - fraction of layer \( k \) received by \( i \), \( 0 \leq y^i_k \leq 1 \)

- Application Layer Variables
  \[ x^i_k = \begin{cases} 
  1, & T_i \text{ is able to play } k \\
  0, & \text{otherwise}
  \end{cases} \]
Conceptual unicast flow in each layer, $f^i_k$

- No inter-layer coding
- Feasible multicast flow - union of unicast flows

Flow conservation

$$\sum_{v \in N(u)} [f^i_k(uv) - f^i_k(vu)] = 0 \quad \forall k, \forall i, \forall u$$

Real flow, $f_k$

$$f^i_k(uv) \leq f_k(uv) \quad \forall k, \forall i, \forall uv$$

No bandwidth contention!

Capacity constraints

$$\sum_k f_k(uv) \leq C(uv) \quad \forall uv$$
Relating flow variables to "playability"

- Received data within each layer:
  \[ f_k^i(T_iS) = l_k \cdot y_k^i \quad \forall i, \forall k \]

- "Playability" criteria 1: completion
  \[ x_k^i \leq y_k^i \quad \forall i, \forall k \]

- "Playability" criteria 2: sequential integrity
  \[ x_L^i \leq x_{L-1}^i \ldots \leq x_2^i \leq x_1^i \quad \forall i \]
Maximize \[ \sum_i \sum_k l_k x^i_k \]

Subject to:

\[
\begin{align*}
\sum_{v \in N(u)} [f^i_k(\overrightarrow{uv}) - f^i_k(\overrightarrow{vu})] &= 0 \quad \forall k, \forall i, \forall u \\
f^i_k(\overrightarrow{uv}) &\leq f_k(\overrightarrow{uv}) \quad \forall k, \forall i, \forall \overrightarrow{uv} \\
\sum_k f_k(\overrightarrow{uv}) &\leq C(\overrightarrow{uv}) \quad \forall \overrightarrow{uv} \\
x^i_{k+1} &\leq x^i_k \leq \frac{f^i_k(T_iS)}{l_k} \quad \forall k = 1..L - 1, \forall i \\
f_k(\overrightarrow{uv}), f^i_k(\overrightarrow{uv}) &\geq 0, \ x^i_k \in \{0, 1\} \quad \forall k, \forall i, \forall \overrightarrow{uv}
\end{align*}
\]
Empirical Results
Simulations

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Empirical Results

Simulations

MILP vs. LION - Throughput comparison

MILP vs. LION - Gain

MILP vs. LION - #Receivers

MILP vs. LION - #Layers

Min-cost Layered Multicast

MILP vs. LION - MinCost

Optimal Layer Size Progression

Simulated Annealing - Results

Layer Size Progression vs. Receiver Capacity

Conclusion/Future Work

Random networks - BRITE

- Random link capacities - [5,20] bps
- Uniform layer size - 6 bits
- 5 layers in total

Basis of Comparison - LION

- Layered Overlay Multicast with Network Coding (LION) framework
- Previous work by Zhao et al. (IEEE Transactions on Multimedia, 8:1021, 2006)
- Enforces routing of complete and in-sequence data only
MILP vs. LION - Throughput comparison

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MILP vs. LION - #Layers

Throughput vs. # of layers

MILP (9) vs. LION
Min-cost Layered Multicast

Minimize
\[ \sum_{uv} \sum_k f_k(\overrightarrow{uv})c(\overrightarrow{uv}) \]

Subject to:
\[
\begin{align*}
\sum_{v \in N(u)} [f_k^i(\overrightarrow{uv}) - f_k^i(\overrightarrow{vu})] &= 0 \quad \forall k, \forall i, \forall u \\
f_k^i(\overrightarrow{uv}) &\leq f_k(\overrightarrow{uv}) \quad \forall k, \forall i, \forall \overrightarrow{uv} \\
\sum_k f_k(\overrightarrow{uv}) &\leq C(\overrightarrow{uv}) \quad \forall \overrightarrow{uv} \\
x_k^i &\leq \frac{f_k^i(\overrightarrow{T_iS})}{l_k} \quad \forall k, \forall i \\
f_k(\overrightarrow{uv}), f_k^i(\overrightarrow{uv}) &\geq 0 \quad \forall k, \forall i, \forall \overrightarrow{uv}
\end{align*}
\]
MILP vs. LION - MinCost

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![Graph showing Min Cost vs. Network Size for LP (10) and LION]
In reality, layer sizes dictated by coding method used

Our goal:

- Shed insight into optimal layer size progression
- Guide design of layer codes for optimal multicast

Problem? Non-linear, non-convex optimization

Solution - Simulated Annealing
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## Layer Size Progression vs. Receiver Capacity

### Empirical Results

<table>
<thead>
<tr>
<th>Topology</th>
<th>Optimal Layer Size</th>
<th>Receiver Capacities</th>
</tr>
</thead>
<tbody>
<tr>
<td>50a</td>
<td>(5, 4, 6, 5, 10,)</td>
<td>5, 9, 10, 15, 16, 20</td>
</tr>
<tr>
<td>50b</td>
<td>(7, 5, 1, 16, 1,)</td>
<td>7, 8, 9, 12, 13, 15, 42, 48</td>
</tr>
<tr>
<td>50c</td>
<td>(13, 4, 6, 3, 4,)</td>
<td>13, 14, 17, 23, 43</td>
</tr>
<tr>
<td>50d</td>
<td>(6, 4, 9, 9, 2,)</td>
<td>6, 10, 12, 19, 32, 33, 36, 41, 46</td>
</tr>
<tr>
<td>50e</td>
<td>(11, 2, 1, 4, 12,)</td>
<td>6, 11, 13, 15, 18, 37, 49</td>
</tr>
<tr>
<td>50f</td>
<td>(11, 2, 9, 5, 3,)</td>
<td>11, 13, 14, 15, 16, 22, 27</td>
</tr>
<tr>
<td>50g</td>
<td>(10, 3, 4, 11, 2,)</td>
<td>10, 13, 14, 31, 33, 39, 41</td>
</tr>
</tbody>
</table>

**Table 1:** Optimal layer size progression vs. distinct receiver capacities
Conclusion/Future Work
Conclusions and Future Work

- Complete and accurate model for layered multicast
  - Network coding for efficient multicast
  - Conceptual separation of network vs application layer data

- Simulations
  - Proposed model shows significant performance improvement
  - Larger feasibility region than previous models

- Future
  - Integer LP formulation - intractable for very large networks
    - Approximation algorithms, heuristics
  - Layer size optimization with respect to coding protocol
Questions?