

# Topic 9: Recursion

To understand recursion, you must first understand recursion.

# Textbook

- Recommended Exercises
  - The Python Workbook, 3<sup>rd</sup> Edition: 202, 204, 205, 206 and 207; or
  - The Python Workbook, 2<sup>nd</sup> Edition: 178, 179, 180, 181 and 182
- Recommend Readings
  - The Python Workbook, 2<sup>nd</sup> or 3<sup>rd</sup> Edition: Chapter 8

# Recursion

- Definition:
  - See Recursion
  - Defining something in terms of itself
    - Generally using a smaller or simpler version
- Recursive Function
  - A function that calls itself

# A Small Example

- Compute  $n$  factorial:
  - Using a loop
    - Initialize result to 1
    - For  $i$  ranging from 1 to  $n$  (inclusive)
      - Multiply result by  $i$ , storing the result back into  $i$
  - Another solution
    - By definition,  $0!$  is 1
    - View  $n!$  as  $n * (n-1)!$

# A Small Example

# Recursion

- A well-formed recursive function normally has two cases
  - Base Case:
    - Does not make a recursive call
    - Permits function to terminate
  - Recursive Case:
    - Function calls itself
    - Generally, must be a call to a smaller or simpler version of the problem

# Useful Examples of Recursion

- Drawing fractals
- Finding a path through a maze
- Flood fill / “paint bucket” tool
- Merge sort, quick sort, binary search
- Finding the total size of all of the files in a directory and its subdirectories
- Parsing / evaluating expressions
- ...

# Greatest Common Divisor

- Finding the greatest common divisor of two positive integers,  $x$  and  $y$ :
  - If  $x$  can be evenly divided by  $y$ , then  $\gcd(x,y)$  is  $y$
  - Otherwise,  $\gcd(x,y)$  is  $\gcd(y, \text{remainder of } x/y)$

# Fibonacci Numbers

- A sequence of values:
  - 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, ...
- Defined recursively:
  - By definition:
    - $\text{fib}(0)$  is 0
    - $\text{fib}(1)$  is 1
  - Remaining values:
    - Formed by computing the sum of the previous two values in the sequence

# Fibonacci Numbers

# Advantages of Recursion

- Very well suited to some problems
  - Tree traversals
  - Flood fill
  - Fractal images
  - Quick sort / merge sort
  - ...
- Easier to implement for some problems, sometimes faster than iterative

# Advantages of Iteration

- Typically
  - Faster (but not always)
  - Requires less memory (most of the time)
- Can be more intuitive for some problems / people
- But some problems are messy to express iteratively

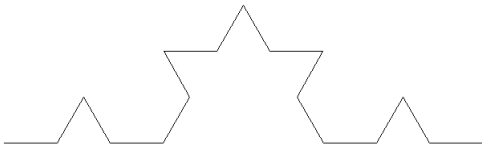
# Fractals

- Self similar images
- Often have reasonably simple recursive definitions

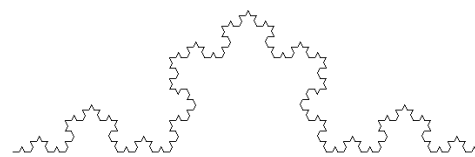
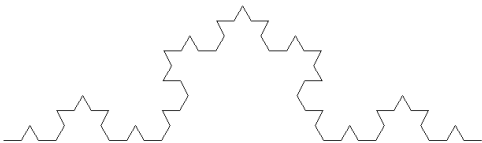
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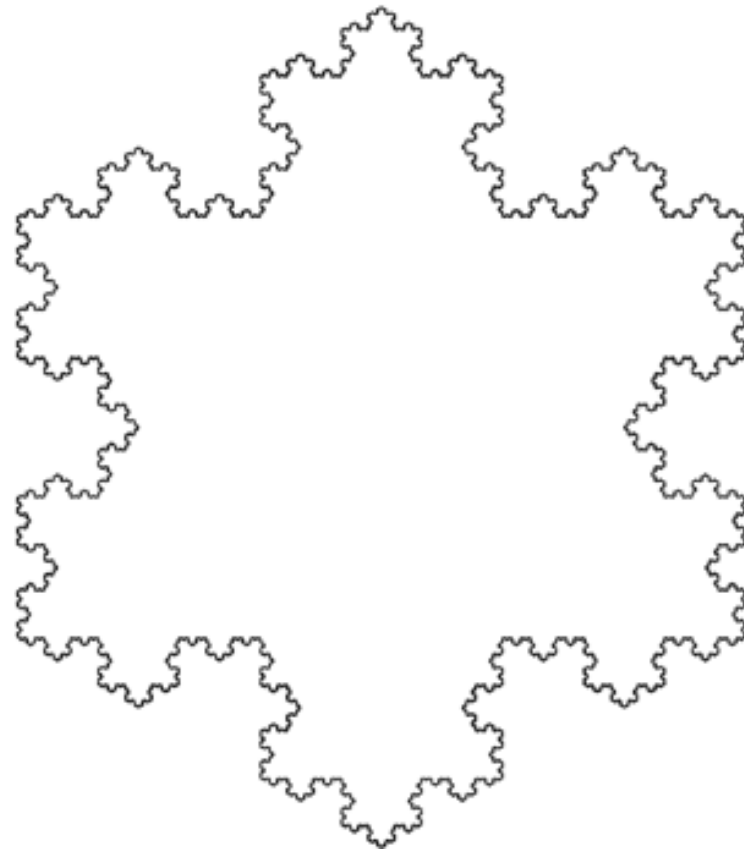
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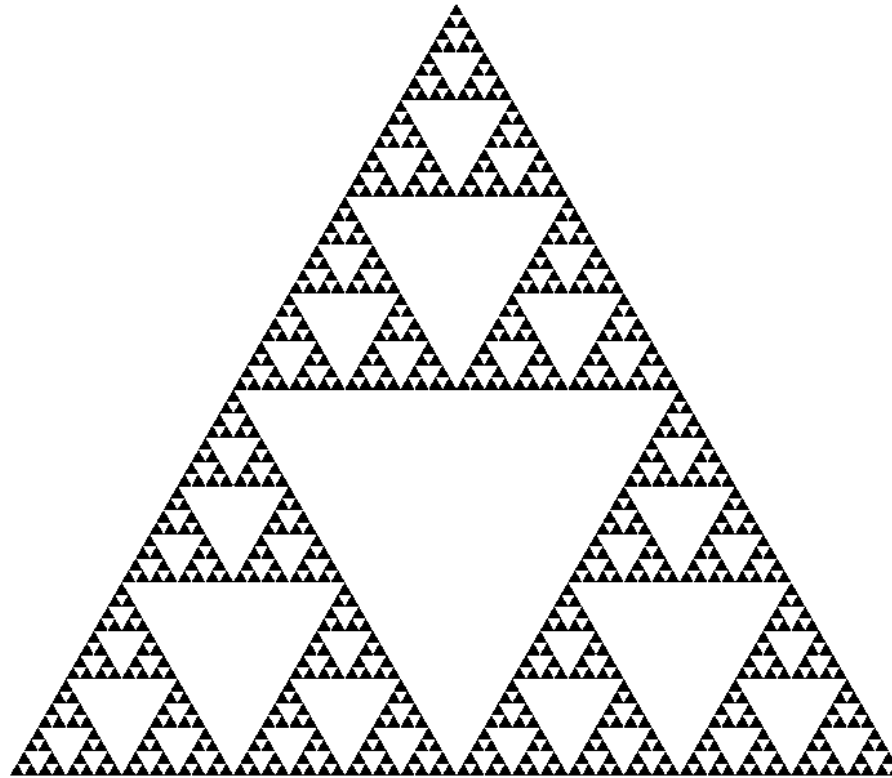
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# Koch Snowflake



# Sierpinski Triangle



Sierpinski Trianglge  
Source: <http://commons.wikimedia.org/wiki/File:Sierpinski-Trigon-7.svg>  
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# Fractal T-Square

# Fractal T-Square

# Maze Path Finding

- Consider a two-dimensional list containing 4 different values
  - Entrance for the maze
  - Exit for the maze
  - Open spaces
  - Walls
- Assume that the maze is fully enclosed

# Maze Path Finding

- Algorithm solve(map, x, y)
  - If the current square is a wall or a space we have already visited, return failure
  - If the current square is the exit point, mark it as part of the solution and return success
  - Mark the current square as part of the solution
  - If solve(map, x, y+1) is successful, return success
  - If solve(map, x, y-1) is successful, return success
  - If solve(map, x+1, y) is successful, return success
  - If solve(map, x-1, y) is successful, return success
  - Mark the current square as visited but not part of the solution
  - Return failure

# Maze Path Finding

# Recursion

- Recursion: See Recursion
  - Very useful for some problems
  - Caution:
    - Can be inefficient
    - Not a good solution for all problems – Use it when appropriate, don't abuse it