Markov Chains

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Outline

- Plan:
  - Introduce basics of Markov models
  - Define terminology for Markov chains
  - Discuss properties of Markov chains
  - Show examples of Markov chain analysis
    - On-Off traffic model
    - Markov-Modulated Poisson Process
    - Erlang B blocking formula
    - TCP congestion window evolution

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Definition: Markov Chain

- A discrete-state Markov process
- Has a set S of discrete states: $|S| > 1$
- Changes randomly between states in a sequence of discrete steps
- Continuous-time process, although the states are discrete
- Very general modeling technique used for system state, occupancy, traffic, queues, ...
- Analogy: Finite State Machine (FSM) in CS

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Some Terminology (1 of 3)

- Markov property: behaviour of a Markov process depends only on what state it is in, and not on its past history (i.e., how it got there, or when)
- A manifestation of the memoryless property, from the underlying assumption of exponential distributions

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Some Terminology (2 of 3)

- The time spent in a given state on a given visit is called the sojourn time
- Sojourn times are exponentially distributed and independent
- Each state i has a parameter q_i that characterizes its sojourn behaviour

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Some Terminology (3 of 3)

- The probability of changing from state i to state j is denoted by p_ij
- This is called the transition probability (sometimes called transition rate)
- Often expressed in matrix format
- Important parameters that characterize the system behaviour

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Desirable Properties of Markov Chains

- Irreducibility: every state is reachable from every other state (i.e., there are no useless, redundant, or dead-end states)
- Ergodicity: a Markov chain is ergodic if it is irreducible, aperiodic, and positive recurrent (i.e., can eventually return to a given state within finite time, and there are different path lengths for doing so)
- Stationarity: stable behaviour over time

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Analysis of Markov Chains

- The analysis of Markov chains focuses on steady-state behaviour of the system
- Called equilibrium, or long-run behaviour as time t approaches infinity
- Well-defined state probabilities p_i (non-negative, normalized, exclusive)
- Flow balance equations can be applied

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Examples of Markov Chains

- Traffic modeling: On-Off process
- Interrupted Poisson Process (IPP)
- Markov-Modulated Poisson Process
- Computer repair models (server farm)
- Erlang B blocking formula
- Birth-Death processes
- M/M/1 Queueing Analysis