Lecture #17: Proofs of Undecidability — Examples II
Lecture Presentation

Goal for This Lecture

Let $\Sigma_{\mathsf{2TM}} = \Sigma_{\mathsf{TM}} \cup \{\#\}$. A *pair* of Turing machines M_1 and M_2 can be encoded as a string $\alpha\#\beta \in \Sigma_{\mathsf{2TM}}^\star$ where $\alpha \in \mathsf{TM} \subseteq \Sigma_{\mathsf{TM}}^\star$ is the encoding for M_1 and $\beta \in \mathsf{TM} \subseteq \Sigma_{\mathsf{TM}}^\star$ is the encoding for M_2.

1. Let $\mathsf{Pair}_{\mathsf{TM}} \subseteq \Sigma_{\mathsf{2TM}}^\star$ be the language of encodings of pairs of Turing machines

   $$M_1 = (Q_1, \Sigma, \Gamma_1, \delta_1, q_{0,1}, q_{A,1}, q_{R,1})$$

   and

   $$M_2 = (Q_2, \Sigma, \Gamma_2, \delta_2, q_{0,2}, q_{A,2}, q_{R,2})$$

   with the same input alphabet Σ.

   Goal #1: Prove that the language $\mathsf{Pair}_{\mathsf{TM}}$ is ***decidable***.

2. Now let

   $$\mathsf{E}_{\mathsf{TM}} \subseteq \mathsf{Pair}_{\mathsf{TM}} \subseteq \Sigma_{\mathsf{2TM}}^\star$$

   be the language including encodings of pairs of Turing machines M_1 and M_2, with the same input alphabet Σ, such that $L(M_1) = L(M_2)$.

   Goal #2: Prove that the language E_{TM} is ***undecidable***.

Proving That $Pair_{TM}$ is Decidable

Useful Properties

A "High-Level" Algorithm

Implementation-Level Details

Proving That E_{TM} is Undecidable

Undecidable Languages That We Already Know About

Which Many-One Reduction Will We Try To Establish?

Useful Aspects of The Problems of Interest — and How To Use Them

Using the Decidability of a Related Language

Thinking about Turing Machines and Input Strings: What Should Our Mapping Be?

Describing This in More Detail (if Needed)

Specifying a Suitable Function f

A First Claim about This Function

A Second Claim about This Function

A Third Claim about This Function

A Useful Related Result, That You Might Establish First

The Third Result and Its Proof

Finishing Up

Something Helpful To Remember About This Problem