

Lecture #17: Proofs of Undecidability — Examples II

What Will Happen During the Lecture

Remember... You Had Homework!

Students were asked to work through the following set of lecture notes before this lecture.

- Lecture Notes — “Proofs of Undecidability — Examples II”.

As always, you may attend the lecture presentation if you have not worked through this material ahead of time — but it will not be repeated for you, and you might get a little bit lost, during the presentation, if you haven’t worked through this.

Problems To Be Solved

When using many-one reductions to prove that a language is undecidable, ***you do not have to use A_{TM} as the undecidable language used in your reduction.***

With that noted, let $\Sigma_{2TM} = \Sigma_{TM} \cup \{\#\}$. A pair of Turing machines M_1 and M_2 can be encoded as a string $\alpha\#\beta \in \Sigma_{2TM}^*$ where $\alpha \in TM \subseteq \Sigma_{TM}^*$ is the encoding for M_1 and $\beta \in TM \subseteq \Sigma_{TM}^*$ is the encoding for M_2 .

1. Let $\text{Pair}_{TM} \subseteq \Sigma_{2TM}^*$ be the language of encodings of pairs of Turing machines

$$M_1 = (Q_1, \Sigma, \Gamma_1, \delta_1, q_{0,1}, q_{A,1}, q_{R,1})$$

and

$$M_2 = (Q_2, \Sigma, \Gamma_2, \delta_2, q_{0,2}, q_{A,2}, q_{R,2})$$

with the same input alphabet Σ . The lecture presentation will include a sketch of a proof that the language Pair_{TM} is ***decidable***.

2. Now let

$$E_{TM} \subseteq \text{Pair}_{TM} \subseteq \Sigma_{2TM}^*$$

be the language including encodings of pairs of Turing machines M_1 and M_2 , with the same input alphabet Σ , such that $L(M_1) = L(M_2)$. During the lecture presentation, it will be shown, using a many-one reduction, that the language E_{TM} is **undecidable**.

As noted above, ***you do not have to use A_{TM} as the undecidable language used in your reduction.*** The lecture presentation will, ideally, suggest that it can be *much* easier to use a many-one reduction to prove undecidability, if the undecidable language, that you start with, is chosen with care.