CPSC 351 — Tutorial Exercise #12
Reductions and Undecidability I

This tutorial exercise will be discussed on Wednesday, October 25, Thursday, October 26 and Friday, October 27.

The questions are of the difficulty, and length, that would be appropriate for questions on a ***test*** in CPSC 351.

Problems To Be Solved

1. Let Σ be an alphabet, let $L \subseteq \Sigma^\star$, and consider the language

$$L \circ L = \{\omega_1 \cdot \omega_2 \mid \omega_1, \omega_2 \in L\} \subseteq \Sigma^\star.$$

   Prove that $L \circ L \preceq_{\mathsf{O}} L$.

2. Let Σ be an alphabet. The ***reversal***, ω^R, of a string $\omega \in \Sigma^\star$, is the string obtained by reversing the order of the symbols in the string. That is, if

$$\omega = \alpha_1 \alpha_2 \dots \alpha_{n-1} \alpha_n$$

   where $\alpha_1, \alpha_2, \dots \alpha_{n-1}, \alpha_n \in \Sigma$ (so that n is the length of ω), then

$$\omega^R = \alpha_n \alpha_{n-1} \dots \alpha_2 \alpha_1.$$

   (a) Show that the function $f : \Sigma^\star \to \Sigma^\star$ such that $f(\omega) = \omega^R$, for every string $\omega \in \Sigma^\star$, is a computable function.

   Let $L \subseteq \Sigma^\star$. Let L^R be the set of reversals of strings in L, that is,

$$L^R = \{\omega^R \mid \omega \in L\}.$$

   (b) Prove that $L^R \preceq_{\mathsf{O}} L$.

   (c) Prove that $L^R \preceq_{\mathsf{M}} L$.

   (d) Could either of the above results be used to prove that if L is ***undecidable*** then L^R is ***undecidable*** as well? If the answer if "no" then what reduction(s) could you give, to prove this, instead?