CPSC 351 — Tutorial Exercise #14
Reductions and Undecidability III

This tutorial exercise will be discussed on Wednesday, November 1, Thursday, November 2, and Friday, November 3.

The problem to be solved is of the difficulty, and length, that would be appropriate for an ***assignment*** in CPSC 351.

Problem To Be Solved

Consider an alphabet $\Sigma_{\text{2TM}} = \Sigma_{\text{TM}} \cup \{\#\}$. Recall, from the lecture presentation for Lecture #17, that a *pair* of Turing machines M_1 and M_2 can be encoded as a string $\alpha\#\beta \in \Sigma_{\text{2TM}}^\star$ where $\alpha \in \text{TM} \subseteq \Sigma_{\text{TM}}^\star$ is the encoding for M_1 and $\beta \in \text{TM} \subseteq \Sigma_{\text{TM}}^\star$ is the encoding for M_2.

As in the presentation for Lecture #17, let $\text{Pair}_{\text{TM}} \subseteq \Sigma_{\text{2TM}}^\star$ be the language of encodings of pairs of Turing machines

$$M_1 = (Q_1, \Sigma, \Gamma_1, \delta_1, q_{0,1}, q_{A,1}, q_{R,1})$$

and

$$M_2 = (Q_2, \Sigma, \Gamma_2, \delta_2, q_{0,2}, q_{A,2}, q_{R,2})$$

with the same input alphabet $\boldsymbol{\Sigma}$. It was proved, during the presentation for Lecture #17, that the language Pair_{TM} is ***decidable***.

1. Let

   $$\text{Subset}_{\text{TM}} \subseteq \text{Pair}_{\text{TM}} \subseteq \Sigma_{\text{2TM}}^\star$$

   be the language including encodings of pairs of Turing machines M_1 and M_2, with the same input alphabet Σ, such that $L(M_1) \subseteq L(M_2)$. Prove that the language $\text{Subset}_{\text{TM}}$ is ***undecidable***.

   Note: A ***hint*** for this problem is available in a separate file — but you should spend at least a little bit of time trying to solve this problem, without looking at it, before you use this hint.