CPSC 351 — Tutorial Exercise #14

Hint for the Problem in This Exercise

Consider an alphabet $\Sigma_{\text{2TM}} = \Sigma_{\text{TM}} \cup \{\#\}$. A *pair* of Turing machines M_1 and M_2 can be encoded as a string $\alpha\#\beta \in \Sigma_{\text{2TM}}^\star$ where $\alpha \in \text{TM} \subseteq \Sigma_{\text{TM}}^\star$ is the encoding for M_1 and $\beta \in \text{TM} \subseteq \Sigma_{\text{TM}}^\star$ is the encoding for M_2. Let $\text{Pair}_{\text{TM}} \subseteq \Sigma_{\text{2TM}}^\star$ be the language of encodings of pairs of Turing machines

$$M_1 = (Q_1, \Sigma, \Gamma_1, \delta_1, q_{0,1}, q_{A,1}, q_{R,1})$$

and

$$M_2 = (Q_2, \Sigma, \Gamma_2, \delta_2, q_{0,2}, q_{A,2}, q_{R,2})$$

with the same input alphabet* $\boldsymbol{\Sigma}$*. As noted in this exercise, this language is ***decidable***.

1. Let

   $$\text{Subset}_{\text{TM}} \subseteq \text{Pair}_{\text{TM}} \subseteq \Sigma_{\text{2TM}}^\star$$

   be the language including encodings of pairs of Turing machines M_1 and M_2, with the same input alphabet Σ, such that $L(M_1) \subseteq L(M_2)$. You were asked to prove that the language E_{TM} is ***undecidable***.

   Hint: Recall that it is easy to describe a Turing machine

   $$\widehat{M} = (\widehat{Q}, \Sigma, \widehat{\Gamma}, \widehat{\delta}, \widehat{q}_0, \widehat{q}_A, \widehat{q}_R)$$

   with alphabet Σ and language $\Sigma^\star$, for any alphabet Σ.