CPSC 351 — Practice Term Test #2

Name: ____

Please **DO NOT** write your ID number on this page.

Academic integrity is the foundation of the development and acquisition of knowledge and is based on values of honesty, trust, responsibility, and respect. We expect members of our community to act with integrity. Research integrity, ethics, and principles of conduct are key to academic integrity. Members of our campus community are required to abide by our institutional code of conduct and promote academic integrity in upholding the University of Calgary's reputation of excellence.

Aids Allowed: Students are asked to prepare a double-sided letter-sized page of notes in advance, and refer to this during the test. No other aids are allowed and no communication about the test, with anyone except the instructor, is allowed while the test is in progress.

Instructions:

1. Answer all questions in the space provided. Use the blank pages at the end of this test if you need more space for your answers.
2. This test is out of 30.

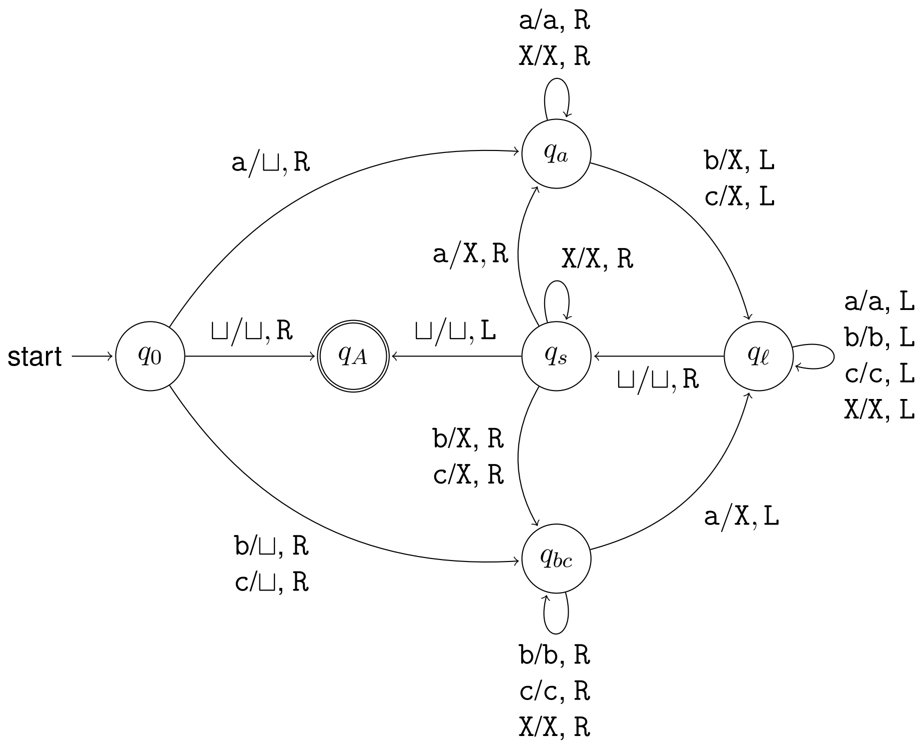
Duration: 90 minutes.

1. Consider a ***Turing machine***

$$M = (Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}})$$

whose input alphabet is $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$, whose tape alphabet is $\Gamma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}, \mathtt{X}, \sqcup\}$, and that has the following state diagram. As usual the accepting state is drawn as q_A to simplify the picture. All transitions that are not shown move to the rejecting state, leaving whatever symbol that is visible unchanged, and moving the tape head right. That is, every missing transition has the form

$$\delta(q, \sigma) = (q_{\text{reject}}, \sigma, \mathtt{R}).$$



(a) *(2 marks)* Show the sequence of configurations that M reaches when it is executed on the input ab.

Sequence of Configurations for This Execution:

(b) *(2 marks)* Show the sequence of configurations that M reaches when it is executed on the input `ca`, instead.

Sequence of Configurations for This Execution:

(c) *(2 marks)* Describe (using simple English) what happens when M is executed on the input `abc`. Does M accept this string?

What Happens when M is Executed on Input `abc`:

(d) *(2 marks)* Describe (using simple English) what happens when M is executed on the input `aacb`. Does M accept this string?

What Happens when M is Executed on Input `aacb`:

(e) *(2 marks)* State the ***language*** $L(M)$ of this Turing machine.

The Language of This Turing Machine:

2. Once again, consider ***Turing machines*** and their languages.

   (a) *(2 marks)* Say what it means for a Turing machine to ***recognize*** a language $L \subseteq \Sigma^\star$.

   What it Means for a Turing Machine to Recognize a Language:

   (b) *(2 marks)* Say what it means for a Turing machine to ***decide*** a language $L \subseteq \Sigma^\star$.

   What it Means for a Turing Machine to Decide a Language:

   (c) *(3 marks)* Say whether the following claim is ***true*** or ***false*** and sketch a ***brief*** proof of your answer.

   Claim: Let $L_1, L_2 \subseteq \Sigma^\star$ for an alphabet Σ. If L_1 and L_2 are both ***decidable*** then $L_1 \cup L_2$ is ***decidable*** as well.

   Is This Claim True?:

   Brief Proof of Your Answer:

(d) *(3 marks)* Say whether the following claim is ***true*** or ***false*** and sketch a ***brief*** proof of your answer.

> *Claim:* Let $L_1, L_2 \subseteq \Sigma^\star$ for an alphabet Σ. If L_1 and L_2 are both ***recognizable*** then $L_1 \cup L_2$ is ***recognizable*** as well.

Is This Claim True?:

Brief Proof of Your Answer:

3. *(10 marks)* Suppose $L \subseteq \Sigma^\star$ (for some alphabet Σ. Let

$$L^{(2)} = \{\mu\mu \mid \mu \in L\}.$$

For example, if $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$ and $L = \{\mathtt{a}, \mathtt{bc}\}$ then $L^{(2)} = \{\mathtt{aa}, \mathtt{bcbc}\}$.

Use a ***many-one reduction*** to prove that if L is undecidable then $L^{(2)}$ is undecidable as well.

Note: If you cannot find a many-one reduction that can be used to prove this then for part marks, you should ***say so***. You should then give the ***definition*** of a ***many-one reduction***, describe how to use these to prove that languages are undecidable, and give as much detail as you can about the properties that a many-one reduction should have in order to prove the result that is given ***in this problem*** — so that your answer does not look like something you copied from a cheat sheet.

Solution: