Lecture #9: Nondeterministic Time — Speedup, Emulation, and a Nondeterministic Time Hierarchy Theorem

Exercises and Review

Additional Exercises

1. As noted in the additional exercises for Lecture #8, some authors define nondeterministic time complexity classes using ***nondeterministic nondeterministic Turing machines*** that, essentially, limit nondeterministic steps to answering Yes-No questions.

   Explain why you would have trouble proving a "Nondeterministic Linear Speedup Theorem" (and, indeed, why such a result might not even be correct) when restricted nondeterministic Turing machines are used in this way.

2. Use the Nondeterministic Time Hierarchy Theorem to prove that

$$\mathsf{NTIME}(n) \subsetneq \mathsf{NTIME}(\lceil n\sqrt{n}\,\rceil) \subsetneq \mathsf{NTIME}(n^2).$$

Questions for Review

1. State the ***Nondeterministic Linear Speedup Theorem***. How does it help to explain why nondeterministic time complexity classes are defined in the way that they are?

2. Briefly describe how a nondeterministic k-tape Turing machine can be simulated using a 3-tape nondeterministic Turing machine. How are the number of steps used by the k-tape machine, and the number of steps used by the 3-tape machine simulating it, related?

3. Briefly describe each of the following language and say what its known about its complexity (or, possibly, decidability).

   (a) L_{NTM}

   (b) $L_{\text{NTM+I}}$

   (c) $A_{\text{NTM+I}}$

   (d) $L_{\text{NTM+I+Time}}$

   (e) $A_{\text{NTM+I+Time}}$

4. State the Nondeterministic Time Hierarchy Theorem. Described what it is used for, and summary the steps you must carry out in order to use it.