Lecture #10: Nondeterministic Time — More about $\mathcal{NP}$, and co-$\mathcal{NP}$

Exercises and Review

Additional Exercises

1. Prove the following claim, from the lecture notes.

   Claim 3.

   (a) $\mathcal{P} \subseteq \mathcal{NP} \cap$ co-$\mathcal{NP}$.
   (b) If $\mathcal{P} = \mathcal{NP}$ *then* $\mathcal{NP} =$ co-$\mathcal{NP}$.
   (c) If either $\mathcal{NP} \subseteq$ co-$\mathcal{NP}$ *or* co-$\mathcal{NP} \subseteq \mathcal{NP}$ *then* $\mathcal{NP} =$ co-$\mathcal{NP}$.

2. Using results that are either proved in the lecture notes for this lecture, or during the lecture presentation, prove that the complexity class co-$\mathcal{NP}$ is closed under polynomial-time many-one reductions.

3. Prove that it $\mathcal{NP} \neq \mathcal{NP}$ then there exist languages $L_1, L_2 \subseteq \Sigma^\star$, for some alphabet Σ, such that $L_1 \preceq_{\mathsf{P,\,O}} L_2$ but $L_1 \not\preceq_{\mathsf{P,\,M}} L_2$.

4. Let $\Sigma = \Sigma_{\mathsf{UTM}} \cup \{\,\widehat{\#}\,\}$, $\widetilde{\Sigma} = \{\Sigma\} \cup \{\spadesuit\} = \Sigma_{\mathsf{UTM}} \cup \{\,\widehat{\#}, \spadesuit\}$, and let $L \subseteq \Sigma^\star$ be the language of the polynomial-time verification algorithm for the language $A_{\mathsf{NTM+I+Time}}$, described in the lecture notes — so that $L \in \mathcal{P}$.

   As a writing exercise (and practice for assignments and tests), use the information about this, from the lecture presentation, to prove that the associated language, $\widetilde{L} \subseteq \widetilde{\Sigma}^\star$, is $\mathcal{NP}$-complete.

Questions for Review

1. Name (and describe) a language that is known to be $\mathcal{NP}$-complete.
2. Describe a process that can (in principle) be used to prove that a language $L \subseteq \Sigma^\star$ is $\mathcal{NP}$-complete, if you already know of at least one other $\mathcal{NP}$-complete language $\widehat{L} \subseteq \widehat{\Sigma}^\star$.
3. What is the ***complement*** of a language $L \subseteq \Sigma^\star$?
4. What is the complexity class co-$\mathcal{NP}$?
5. What does it mean for a language to be ***co-$\mathcal{NP}$-hard***? What does it mean for a language to be ***co-$\mathcal{NP}$-complete***?
6. Explain why the set of co-$\mathcal{NP}$-complete languages you know about also grows, as more $\mathcal{NP}$-complete languages are identified.