Lecture #11: The Cook-Levin Theorem
Exercises and Review

Additional Exercises

Consider the languages $L_{\text{FUnsat}}, L_{\text{Falsifiable}}, L_{\text{Tautology}} \subseteq L_{\text{F}}$ that were introduced during the lecture presentation. Notes from the discussion of these languages, from the lecture presentation, certainly might be helpful as you try to complete the following exercises.

1. Prove that L_{FUnsat} is co-$\mathcal{NP}$-complete.
2. Prove that $L_{\text{Falsifiable}}$ is $\mathcal{NP}$-complete.
3. Prove that $L_{\text{Tautology}}$ is co-$\mathcal{NP}$-complete.

Questions for Review

1. What does it mean for a language $L \subseteq \Sigma^\star$ to be ***$\mathcal{NP}$-hard***?
2. What does it mean for a language $L \subseteq \Sigma^\star$ to be ***$\mathcal{NP}$-complete***?
3. What is the ***Cook-Levin Theorem***? Why is it significant?
4. How were ***Boolean formulas*** and ***truth assignments*** defined in this lecture? What does it mean for a Boolean formula to be ***satisfiable***?
5. What is a ***tableau*** for a nondeterministic one-tape Turing machine[1]

   $$M = (Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}})$$

   and an input string $\omega \in \Sigma^\star$ for M? What information, about the computation of M on input ω, does a tableau represent?

[1] satisfying additional simplifying conditions, as given in the lecture notes

6. What is an ***accepting tableau*** for a nondeterministic Turing machine $\mathcal{M}$ and an input string ω for M? Why are ***accepting tableaux*** of interest?

   In particular, how are these used to prove that the language L_{FSAT} is $\mathcal{NP}$-hard?

7. Describe the structure of the formula (given in the lecture) $\mathcal{F}_\omega$ for a given string $\omega \in \Sigma^\star$ — which is satisfiable if and only if ω is in the language of a given nondeterministic Turing machine.

8. What is a ***window*** (as used in the proof of the Cook-Levin Theorem)? Briefly describe — at a high level — how these are used to complete the proof of the Cook-Levin theorem whose beginning is given in the lecture notes.