Lecture #12: Classical Reductions
Lecture Presentation

When defining Boolean formulas in ***3-conjunctive normal form***, we allowed clauses in a formula to include more than one copy of the same literal — so that, for example,

$$((\neg x_1 \vee \neg x_1 \vee x_2) \wedge (x_1 \wedge \neg x_2 \wedge x_1))$$

is a Boolean formula in 3-conjunctive normal form.

Suppose we change the definition of "3-conjunctive normal form" to make this more restrictive, by requiring that the three literals in each clause must be distinct. Then the above formula would *not* be in 3-conjunctive normal form.

Now let $\widehat{L}_{\text{3CNF}}$ be the language of instances of the "3-CNF Satisfiability" problem, and let $\widehat{L}_{\text{3CNF-SAT}}$ be the language of Yes-instances of the "3-CNF Satisfiability" problem, when this more restrictive definition of a Boolean formula in 3-conjunctive normal form is used. Then $\widehat{L}_{\text{3CNF}}$ is a proper subset of L_{3CNF}, and $\widehat{L}_{\text{3CNF-SAT}}$ is a proper subset of $L_{\text{3CNF-SAT}}$.

The goal of this exercise is to prove that $\widehat{L}_{\text{3CNF}} \in \mathcal{P}$ and $\widehat{L}_{\text{3CNF-SAT}}$ is $\mathcal{NP}$-complete.

Proving That $\widehat{L}_{\text{3CNF}} \in \mathcal{P}$:

Proving That $\widehat{L}_{\textbf{3CNF-SAT}} \in \mathcal{NP}$:

Proving That $\widehat{L}_{\text{3CNF-SAT}}$ is $\mathcal{NP}$-Hard: