Lecture #13: What If...? More about Nondeterministic Computation

Exercises and Review

Additional Exercises

1. Recall that
$$\mathcal{QP} = \bigcup_{k \geq 1} \text{TIME}\left(2^{(n^k)}\right).$$
Now suppose that
$$\mathcal{NQP} = \bigcup_{k \geq 1} \text{NTIME}\left(2^{(n^k)}\right).$$
Prove that if $\mathcal{P} = \mathcal{NP}$ then $\mathcal{QP} = \mathcal{NQP}$.

2. Let $L \subseteq \Sigma^\star$ such that $L \in \mathcal{P}$. Prove that $\mathcal{P}^L = \mathcal{P}$ and $\mathcal{NP}^L = \mathcal{NP}$.

Questions for Review

1. What is an ***$\mathcal{NP}$-intermediate*** language? Under what circumstances can it be proved that $\mathcal{NP}$-intermediate languages exists?
2. Name (and describe) a language, corresponding to a decision problem, that is *believed*, by many researchers, to be $\mathcal{NP}$-intermediate.
3. Consider the complexity classed *EXPTIME* and *NEXPTIME*.
   (a) Give the definition of each of these complexity classes.
   (b) What is known about the relationship between *EXPTIME* and *NEXPTIME*? What is suspected about this, but unproved?
   (c) Suppose that $\mathcal{P} = \mathcal{NP}$. What does this imply about the relationship between *EXPTIME* and *NEXPTIME*?

4. Let $L \subseteq \Sigma^\star$ for an alphabet Σ.

   (a) Define each of the complexity classes $\mathcal{P}^L$ and $\mathcal{NP}^L$.

   (b) What is known about the relationship between $\mathcal{P}^L$ and $\mathcal{NP}^L$, for various language $L \subseteq \Sigma^\star$ (for alphabets Σ)?

   (c) What, if anything, can be concluded from this, concerning proofs that $\mathcal{P} = \mathcal{NP}$ (if this is, in fact, true)?

   (d) What, if anything, can be concluded from this, concerning proofs that $\mathcal{P} \neq \mathcal{NP}$ (if this is, in fact, true instead)?