Lecture #2: Modelling Computation of Functions I
Exercises and Review

Questions for Review

1. Describe — as completely and precisely as you can — how a function can be produced from a set of other functions using ***composition***. (Your answer should closely resemble the definition of "composition" in the lecture notes.)
2. Describe — as completely and precisely as you can — how a function can be produced from other functions using ***primitive recursion***. (You should describe two forms of this operations on functions and parts of your answer should closely resemble two definitions of "primitive recursion" in the lecture notes.)
3. Define each of the following ***initial functions:***
   (a) the ***successor function***
   (b) the ***nullify function***
   (c) the ***projection function*** $\boldsymbol{u_i^n}$ where $i, n \in \mathbb{N}$ and $1 \leq i \leq n$
4. Define the set of ***primitive recursive functions***.
5. Describe, in a reasonably precise way, a ***process*** that you can follow in order to show that a given function is primitive recursive.

Primitive Recursive Predicates

6. What is a ***predicate***? When do we consider the value of a predicate to be ***true***, and when do we consider the value of the predicate to be ***false***?
7. Describe a useful set of ***closure properties*** for the set of primitive recursive predicates.

Practice Problems

These problems will not be discussed during the lecture, and solutions for these problems will not generally be made available. They can be used as "practice" problems that can help you to practice skill considered in the lecture presentation for Lecture #2.

1. Prove that each of the following functions (and predicates) is primitive recursive. When doing so you may use all of the functions that were used as examples of primitive recursive functions in Lecture #2, along the the functions that are before this one on this list.

   (a) The function $f_1 : \mathbb{N}^2 \to \mathbb{N}$ such that $f_1(x, y) = x^y$ for all $x, y \in \mathbb{N}$ — where this is set to be 1 if $y = 0$, and where this is 0 if $x = 0$ and $y \geq 1$.

   (b) The predicate "$x \geq y$," that is, the function $p_1 : \mathbb{N}^2 \to \{0, 1\}$ such that, for $x, y \in \mathbb{N}$,

   $$p_1(x, y) = \begin{cases} 1 & \text{if } x \geq y \\ 0 & \text{otherwise.} \end{cases}$$

   (c) The function $f_2 : \mathbb{N}^2 \to \mathbb{N}$ such that $f_2(x, y) = \min(x, y)$ for all $x, y \in \mathbb{N}$.