Lecture #2: Modelling Computations of Functions I
Lecture Presentation

Proving that an Exponential Function is Primitive Recursive

Consider the function *exp2* : $\mathbb{N} \to \mathbb{N}$ such that, for all $x \in \mathbb{N}$,

$$\textit{exp2}(x) = 2^x.$$

A Base Case:

$$\textit{exp2}(0) =$$

Larger Values:

How is *exp2*$(t+1)$ related to *exp2*(t), for $t \in \mathbb{N}$?

How can this relationship be described, using primitive recursive functions that we already know?

Working Up To the Target Function:

Give a sequence of functions — that are either already known to be primitive recursive or can be obtained from earlier ones using *composition* or one of the forms of *primitive recursion* — ending with a function that is (equal to) the desired function, *exp2*.

Completing the Argument

The function that you described, at the end of this, probably doesn't look very much like the function, *exp2*, that you are interested in.

State a claim, about these functions, that can be used to conclude that the function *exp2* is also primitive recursive, and say how it an be proved.

Claim:

How This is Proved:

Proving That a Logarithm Function is Primitive Recursive

Consider the function *log2* : $\mathbb{N} \to \mathbb{N}$ such that, for all $x \in \mathbb{N}$,

$$\textit{log2}(x) = \begin{cases} 0 & \text{if } x = 0, \\ \lfloor \log_2 x \rfloor & \text{if } x \geq 1 \end{cases}$$

— so that, for a positive input x, *log2*(x) is the largest integer that is less than or equal to the real number $\log_2 x$.

A Base Case:

$$\textit{log2}(0) =$$

Larger Values:

How is *log2*$(t+1)$ related to *log2*(t), for $t \in \mathbb{N}$?

How can this relationship be described, using primitive recursive functions that we already know? One or more of the ***closure properties***, introduced near the end of the lecture, might be useful here.

Working Up To the Target Function:

Give a sequence of functions — that are either already known to be primitive recursive or can be obtained from earlier ones using *composition*, one of the forms of *primitive recursion*, or one of the *closure properties* for primitive recursive functions, given near the end of the lecture — ending with a function that is (equal to) the desired function, *log2*.

Completing the Argument

The function that you described, at the end of this, probably doesn't look very much like the function, *log2*, that you are interested in.

State a claim, about these functions, that can be used to conclude that the function *log2* is also primitive recursive, and say how it an be proved.

Claim:

How This is Proved:

Have You Seen Something Like This, Before?