Lecture #3: Modelling Computations of Functions II
Lecture Presentation

Perfect Numbers

A ***perfect number*** is an integer that is greater than or equal to 2 that is equal to the sum of its positive integer divisors — including 1 but not including itself. For example, 6 is the smallest perfect number because

$$6 = 1 + 2 + 3$$

and none of 2, 3, 4 or 5 are perfect. As of December 2018, 51 perfect numbers have been identified.

A First Problem To Be Solved

Let $p_{\text{perfect}} : \mathbb{N} \to \{0, 1\}$ such, that for $n \in \mathbb{N}$,

$$p_{\text{perfect}}(n) = \begin{cases} 1 & \text{if } n \text{ is a perfect number,} \\ 0 & \text{if } n \text{ is not a perfect number.} \end{cases}$$

Then

$$p_{\text{perfect}}(n) = \begin{cases} 1 & \text{if } n \geq 2 \text{ and } \textit{sumFactors}(n) = n, \\ 0 & \text{if } n \geq 1 \text{ or } \textit{sumFactors}(n) \neq n, \end{cases}$$

where *sumFactors* : $\mathbb{N} \to \mathbb{N}$ such that, for $n \in \mathbb{N}$,

$$\textit{sumFactors}(n) = \sum_{i=1}^{n} \textit{factorTerm}(i, n)$$

and where *factorTerm* : $\mathbb{N}^2 \to \mathbb{N}$ such that, for all $i, n \in \mathbb{N}$,

$$\textit{factorTerm}(i, n) = \begin{cases} i & \text{if } 1 \leq i \leq n - 1 \text{ and } n \bmod i = 0, \\ 0 & \text{otherwise.} \end{cases}$$

Suppose we want to show that p_{perfect} is a primitive recursive predicate.

Objective: Proving that *factorTerm* is Primitive Recursive

Objective: Proving that *sumFactors* is Primitive Recursive

Objective: Proving that p_{perfect} is Primitive Recursive

A Second Problem To Be Solved

Let $f_{\text{perfect}} : \mathbb{N} \to \mathbb{N}$ be a partial or total function such that $f_{\text{perfect}}(0) = 0$ and such that the following properties are satisfied.

- If $n \geq 1$ and there are at least n perfect numbers, then $f_{\text{perfect}}(n)$ is the n^{th} perfect number (where these are listed in increasing order) — so that, for example, $f_{\text{perfect}}(1) = 6$.
- If $n \geq 1$ and n or more perfect numbers do not exist, then $f_{\text{perfect}}(n)$ is undefined.

Suppose we wish to prove that f_{perfect} is a μ-recursive partial or total function.

Objective: Relating $f_{\text{perfect}}(n+1)$ to $f_{\text{perfect}}(n)$

Objective: Finding the Smallest Perfect Number Exceeding a Given Bound (If One Exists)

Objective: Proving that f_{perfect} is μ-Recursive