Lecture #12: Rice's Theorem and the Recursion Theorem
Exercises and Review

Questions for Review

Rice's Theorem

1. Define the ***index set*** R_Γ of a set Γ of computable partial (or total) functions.
2. State ***Rice's Theorem***.
3. Describe a ***process*** that can be followed, to use Rice's Theorem to prove that the index set R_Γ of a set Γ of computable partial (and total) functions is not computable.

Recursion Theorem

4. State the ***Recursion Theorem***.
5. What other result, already described and proved in this course, is used in the proof of the Recursion Theorem? f possible, say ***briefly*** how this result is proved.
6. Describe one or two results about computability that can be proved *using* the Recursion Theorem.
7. State the ***Fixed Point Theorem***. If possible, say briefly how *it* is proved.

Rice's Theorem, Revisited

8. Describe a second proof of Rice's Theorem that uses the Recursion Theorem.

Practice Problems

These problems — which continue activities in the lecture presentation — will not be discussed during the lecture, and solutions for these problems will not generally be made available. They can be used as "practice" problems that can help you to practice skill considered in the lecture presentation for Lecture #12.

1. Use Rice's Theorem to prove that the set

$$\text{Total} = \{n \in \mathbb{N} \mid \Phi_n \text{ is a total function}\}$$

   is not computable.

2. The ***Ackerman function*** $A : \mathbb{N}^2 \to \mathbb{N}$ can be defined as follows: For all $m, n \in \mathbb{N}$,

$$A(m,n) = \begin{cases} n+1 & \text{if } m = 0, \\ A(m-1, 1) & \text{if } m > 0 \text{ and } n = 0, \\ A(m-1, A(m, n-1)) & \text{if } m > 0 \text{ and } n > 0. \end{cases}$$

   Prove that the Ackerman function is a computable total function.